

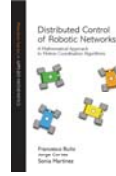
Introduction to Distributed Control of Robotic Networks



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outline

• Robotic network models and complexity notions

- Physical components
- Robotic network
- Example
- Control and communication laws
- Evolution of a robotic network
- The agree pursue control and communication law
- Coordination tasks
- complexity
- Connectivity maintenance and rendezvous
- Deployment



Physical components

continuous-time continuous-space dynamical system

$\text{tuple}(X, U, X_0, f)$

• X : **state space**
 $X \in \mathfrak{R}^d, X \in \mathcal{S}^d, X \in \mathfrak{R}^d \times \mathcal{S}^d$

• U : **input space**
• X_0 : **allowable initial states**
 $X_0 \in X$

• f is a continuously differentiable control vector field on X
 f determines the **robot motion**

via the control system
 $\dot{x}(t) = f(x(t), u(t))$

$f : X \times U \rightarrow \mathfrak{R}^d$

Example

Omni direction

$$R = \{\mathfrak{R}^2, [-u_{\max}, u_{\max}]^2, \mathfrak{R}^2, (0_2, e_1, e_2)\}$$

Planner Vehicle

$$\dot{x} = v \cos \theta$$

$$\dot{y} = v \sin \theta$$

$$\dot{\theta} = \omega$$

$$R = \{\mathfrak{R}^2 \times \mathcal{S}^1, \begin{bmatrix} -v_{\max} & v_{\max} \\ -\omega_{\max} & \omega_{\max} \end{bmatrix}, \mathfrak{R}^2 \times \mathcal{S}^1, \begin{pmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{pmatrix}\}$$



Robotic network

Definition Robotic network S consist of a $\text{tuple}(I, R, E_{\text{cmm}})$

$$I = \{1, \dots, n\}$$

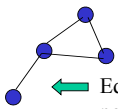
→ Set of unique identifiers

$$R = \{R^{[i]}\}_{i \in I} = \{(X^{[i]}, U^{[i]}, X_0^{[i]}, f^{[i]})\}_{i \in I}$$

→ Set of mobile robots

$$E_{\text{cmm}} : \prod_{i \in I} X^{[i]} \rightarrow I \times I$$

→ Communication edge map



→ Edge depend on agents position

pair (i, j) is an edge in $E_{\text{cmm}}(x)$



i, j can communicate



$(I, E_{\text{cmm}}(x))$

→ Communication graph at x

If $R^{[i]} = (X, U, X_0, f)$
for all i (identical)

→ Robotic network is uniform



Example

1. First-order robots with range - limited communication

Robot locations
 $p = \{p^{[1]}, p^{[2]}, \dots, p^{[n]}\}$
 $(p^{[i]} \in R^d)$

$$\dot{p}^{[i]}(t) = u^{[i]}(t)$$

$$(u^{[i]} \in [-u_{\max}, u_{\max}]^d)$$

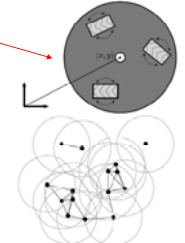
Identical robots

$$R = \{\mathfrak{R}^d, [-u_{\max}, u_{\max}]^d, \mathfrak{R}^d, (0_d, e_1, \dots, e_d)\}$$

input space initial states Omni direction

Sensible own position
Communicate with robots within distance r

These data → Robotic network S_{disk}



$$\rightarrow g_{\text{disk}}(r)$$



Example

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2. Planner vehicle robots with Delaunay communication

Robot physical state

$$\{(p^{[1]}, \theta^{[1]}), (p^{[2]}, \theta^{[2]}), \dots, (p^{[n]}, \theta^{[n]})\}$$

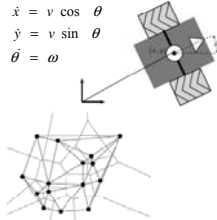
$$p^{[i]} = (x^{[i]}, y^{[i]}), \theta^{[i]} \in S^1$$

Planner vehicle model

$$\dot{x} = v \cos \theta$$

$$\dot{y} = v \sin \theta$$

$$\dot{\theta} = \omega$$



Communication graph $\rightarrow g_D$

Robots move allowable environment Q

These data \rightarrow Robotic network $S_{vehicles}$

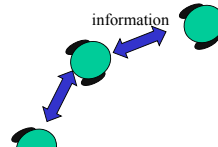
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Control and Communication laws

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Continuous time

sense own position ,evolves

discrete time

execute state machine ,exchange information

Definition Control and Communication laws (CC)

sets

1. A : communication alphabet elements of A : messages
2. $W^i, i \in I$: prosser state sets
3. $W_0^i \subset W^i, i \in I$: allowable initial values

maps

1. $msg^{[i]}: X^{[i]} \times X^{[i]} \times I \rightarrow A$ message generation functions
2. $stf^{[i]}: X^{[i]} \times W^{[i]} \times A^n \rightarrow W^{[i]}$ processor state transition functions
3. $ctl^{[i]}: X^{[i]} \times X^{[i]} \times W^{[i]} \times A^n \rightarrow U^{[i]}$ motion control functions

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Evolution of a robotic network

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Definition Evolution of a robotic network

Evolution of (S, CC)

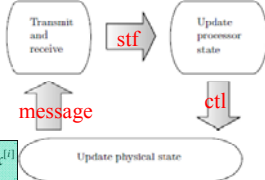
$$\dot{x}^{[i]}(t) = f(x^{[i]}(t), ctl^{[i]}(x^{[i]}(t), x^{[i]}(\lfloor t \rfloor), w^{[i]}(\lfloor t \rfloor), y^{[i]}(\lfloor t \rfloor)))$$

$$w^{[i]}(l) = stf^{[i]}(x^{[i]}(l), w^{[i]}(l-1), y^{[i]}(l))$$

$$(x^{[i]}(0) = x_0^{[i]}, w^{[i]}(-1) = w_0^{[i]})$$

$$y_j^{[i]}(l) = msg^{[j]}(x^{[j]}(l), w^{[j]}(l-1), i)$$

if $(j, i) \in E_{comm}(x^{[1]}(l), \dots, x^{[n]}(l))$



* data-sampled

$ctl^{[i]}(x^{[i]}, x_{sample}^{[i]}, w^{[i]}, y^{[i]})$ is independent of $x^{[i]}$

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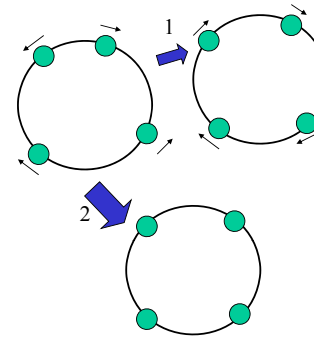
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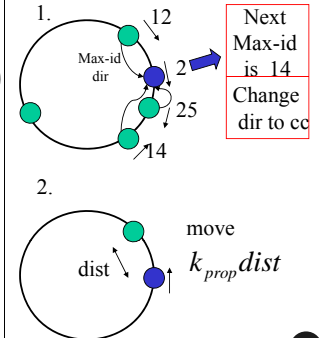
the agree and pursue control and communication law

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object 1. agree direction 2. equally angularly spaced



idea 1. Leader election algorithms 2. Cyclic pursuit problem



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algorithm

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Robotic Network: S_{robots} , first-order agents in S^2 with absolute sensing of own position, and with communication range r

Distributed Algorithm: AGREE & PURSUE

Alphabet: $A = S^1 \times \{c, cc\} \times J \cup \{\text{null}\}$

Processor State: $w = (\text{dir}, \text{max-id})$, where

$\text{dir} \in \{c, cc\}$, initially: $\text{dir}^{[i]}$ unspecified

$\text{max-id} \in I$, initially: $\text{max-id}^{[i]} = i$ for all i

Standard message-generation function

function $msg(\theta, w, i)$

1. return (θ, w)

function $ctl(\theta_{mpdt}, w, y)$

1. for each non-null message $(\theta_{recvd}, \text{dir}_{recvd}, \text{max-id}_{recvd})$ in y do

2. if $(\text{max-id}_{recvd} > \text{max-id})$ AND $(\text{dist}_{cc}(\theta, \theta_{recvd}) \leq r)$ AND $\text{dir}_{recvd} = c$ OR $(\text{dist}_c(\theta, \theta_{recvd}) \leq r)$ AND $\text{dir}_{recvd} = cc$ then

3. $\text{new-dir} := \text{dir}_{recvd}$

4. $\text{new-id} := \text{max-id}_{recvd}$

5. return $(\text{new-dir}, \text{new-id})$

function $ctl(\theta_{mpdt}, w, y)$

1. $d_{\text{emp}} := r$

2. for each non-null message $(\theta_{recvd}, \text{dir}_{recvd}, \text{max-id}_{recvd})$ in y do

3. if $(\text{dir} = cc)$ AND $(\text{dist}_{cc}(\theta_{mpdt}, \theta_{recvd}) < d_{\text{emp}})$ then

4. $d_{\text{emp}} := \text{dist}_{cc}(\theta_{mpdt}, \theta_{recvd})$

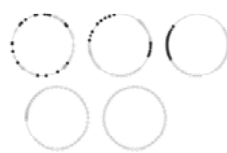
5. if $(\text{dir} = c)$ AND $(\text{dist}_c(\theta_{mpdt}, \theta_{recvd}) < d_{\text{emp}})$ then

6. $d_{\text{emp}} := \text{dist}_c(\theta_{mpdt}, \theta_{recvd})$

7. $d_{\text{emp}} := -k_{\text{prop}} d_{\text{emp}}$

8. return d_{emp}

Simulation result



Leader election

Neighbor?

Moving toward?

Cyclic pursuit

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Coordination tasks

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Definition

Coordination task

$$\text{map } T: \prod_{i \in I} X^{[i]} \times W^n \rightarrow \{true, false\}$$

(if W is singleton $T: \prod_{i \in I} X^{[i]} \rightarrow \{true, false\}$)

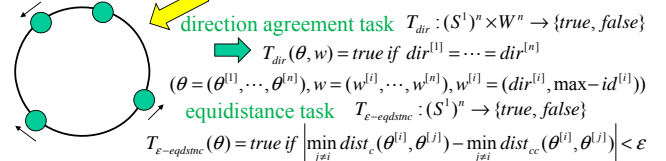
if $W^{[i]} = W$, the law CC is compatible with the task

law CC achieves the task $T \rightarrow T(x(t), w(t)) = true$ for all $t \geq T$

Ex) dir c or cc Processor state take Values in W

Ex) Direction agreement and equidistance task

processor state taking value in $W = \{cc, c\} \times I$



direction agreement task $T_{dir}: (S^1)^n \times W^n \rightarrow \{true, false\}$

$T_{dir}(\theta, w) = true$ if $\text{dir}^{[1]} = \dots = \text{dir}^{[n]}$

$(\theta = (\theta^{[1]}, \dots, \theta^{[n]}), w = (w^{[1]}, \dots, w^{[n]}), w^{[i]} = (\text{dir}^{[i]}, \text{max-id}^{[i]}))$

equidistance task $T_{e-eqdistmc}: (S^1)^n \rightarrow \{true, false\}$

$T_{e-eqdistmc}(\theta) = true$ if $|\min_{j \neq i} \text{dist}_c(\theta^{[i]}, \theta^{[j]}) - \min_{j \neq i} \text{dist}_{cc}(\theta^{[i]}, \theta^{[j]})| < \epsilon$

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Complexity

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Definition Time complexity

Time complexity to achieve T with CC from initial condition

$$TC(T, CC, x_0, w_0) = \inf_{\{T\}} \{T(x(k), w(k)) = true, for all k \geq l\}$$

Time complexity to achieve T with CC

$$TC(T, CC) = \sup_{\{T\}} TC(T, CC, x_0, w_0) | (x_0, w_0) \in \prod_{i \in I} X_0^{[i]} \times \prod_{i \in I} W_0^{[i]}$$

Time complexity to achieve T worst case

$$TC(T) = \inf_{\{T\}} \{TC(T, CC) | CC compatible with T\}$$

Set of non-null message

Definition Space complexity

SC(T, CC)

Maximum number of basic memory units

Definition Mean and Total Communication complexity

$$MCC(T, CC, x_0, w_0) = \frac{|A|_{basic}}{\tau} \sum_{l=0}^{\tau-1} |M(x(l), w(l))|$$

$$TCC(T, CC, x_0, w_0) = |A|_{basic} \sum_{l=0}^{\tau-1} |M(x(l), w(l))|$$

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outline

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- Robotic network models and complexity notions
- Connectivity maintenance and rendezvous
 - Averaging control and communication law
 - Circumcenter control and communication laws
 - Correctness and complexity of circumcenter laws
 - The circumcenter law in nonconvex environments
- Deployment

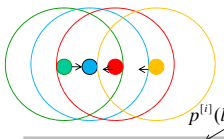
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Averaging control and communication law

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1. Compute the average point
2. Move toward average point

$$p^{[i]}(l+1) = p^{[i]}(l) + u^{[i]}(l) \quad \text{Simulation result}$$

Robotic Network: S_{disk} with motion model (4.1.1), with absolute sensing of own position, and with communication range r in \mathbb{R}^2

Distributed Algorithm: AVERAGING

Alphabet: $A = \mathbb{R}^2 \cup \{\text{null}\}$

function msg(p, i)

return p

function ctrl(p, i)

return avg([p] \cup {p_{jcd} | p_{jcd} is non-null message to i}) - p

$$T_{rendvs}(x^{[1]}, \dots, x^{[n]}) = true \text{ if } x^{[i]} = x^{[j]} \text{ for all } (i, j) \in E_{conn}$$

Theorem Correctness and time complexity of averaging law

the law $CC_{averaging}$ achieves the task T_{rendvs} with time complexity

$$TC(T_{rendvs}, CC_{averaging}) \in O(n^5) \quad TC(T_{rendvs}, CC_{averaging}) \in \Omega(n)$$

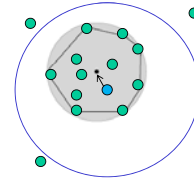
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Circumcenter control and communication laws

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1. Compute the circumcenter point
2. Move toward circumcenter point

Maintaining connectivity
use constraint set
constraint set

$$X_{disk}(p^{[i]}, q^{[j]}) = \overline{B}\left(\frac{p^{[j]} + q^{[j]}}{2}, \frac{r}{2}\right)$$

$$X_{disk}(p^{[i]}, P) = \{x \in X_{disk}(p^{[i]}, q) | q \in P \setminus \{p^{[i]}\}, s.t. \|q - p^{[i]}\|_2 \leq r\}$$

Theorem Maintaining network connectivity

$$u^{[i]}(l) \in X_{disk}(p^{[i]}(l), P(l)) - p^{[i]}(l)$$

Maintaining edge

$$p^{[i]}(l+1) \in X_{disk}(p^{[i]}(l), P(l))$$

$$\|p^{[i]}(l) - p^{[j]}(l)\|_2 \leq r \rightarrow \|p^{[i]}(l+1) - p^{[j]}(l+1)\|_2 \leq r$$

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Circumcenter control and communication laws

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algorithm

Robotic Network: S_{disk} with discrete-time motion model (4.1.1), with absolute sensing of own position, and with communication range r in \mathbb{R}^2

Distributed Algorithm: CIRCUMCTR

Alphabet: $A = \mathbb{R}^2 \cup \{\text{null}\}$

function msg(p, i)

return p

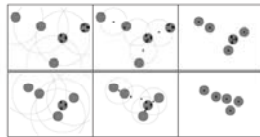
function ctrl(p, i)

1: $p_{jcd} := CC([p] \cup \{p_{jcd} | \text{for all non-null } p_{jcd} \in y\})$

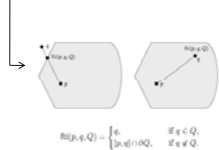
2: $X := \bigcap_{j \in E_{conn}(p, P_{jcd})} X_{disk}(p, p_{jcd})$ for all non-null $p_{jcd} \in y$

3: return $U([p], p_{jcd}, X) - p$

Simulation result



constraint set



Circumcenter law with control bounds and relaxed connectivity constraints

function ctrl(p, i)

% Includes control bound and relaxed G-connectivity constraint

1: $p_{jcd} := CC([p] \cup \{p_{jcd} | \text{for all non-null } p_{jcd} \in y\})$

2: $X := \bigcap_{j \in E_{conn}(p, P_{jcd})} X_{disk}(p, p_{jcd})$ for all non-null $p_{jcd} \in y$

3: return $U([p], p_{jcd}, X) - p$

relax constrain sets Agents have compact input space

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Correctness and complexity of circumcenter laws

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Theorem correctness of the circumcenter laws

1. on the network S_{disk} , the law $CC_{circumctr}$ achieves the exact rendezvous task T_{rendvs}

2. on the network S_{LD} , the law $CC_{circumctr}$ achieves the ϵ -rendezvous task $T_{\epsilon-rendvs}$

$$T_{\epsilon-rendvs}(P) = true \iff \|p^{[i]} - \text{avg}(\{p^{[j]} | (i, j) \in E_{conn}(P)\})\|_2 < \epsilon, \quad i \in \{1, \dots, n\}$$

3. if any two agents belong to the same connected component at l

continue to belong to the same connected component for all subsequent time

4. $P^* = (p_1^*, \dots, p_n^*)$
(a) evolution asymptotically approaches

Theorem time complexity (d=1)

$$S_{disk} \Rightarrow TC(T_{rendvs}, CC_{circumctr}) \in \Theta(n)$$

$$S_{LD} \Rightarrow TC(T_{\epsilon-rendvs}, CC_{circumctr}) \in \Theta(n^2 \log(n\epsilon^{-1}))$$

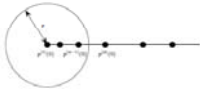
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Proof:Theorem1

Dimension 1 $g_{disk}(r)$ is connected



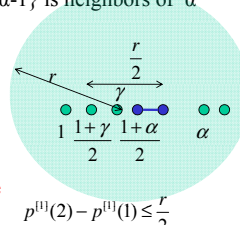
$\{2, \dots, \alpha-1\}$ is neighbors of 1
Worst case
 $\{2, \dots, \alpha-1\}$ is neighbors of α

$$p^{[1]}(1) = \frac{p^{[1]}(0) + p^{[\alpha-1]}(0)}{2}$$

$$p^{[r]}(1) \in \left[\frac{p^{[1]}(0) + p^{[\alpha]}(0)}{2}, \frac{p^{[1]}(0) + p^{[r]}(0) + r}{2} \right]$$

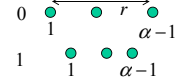
$$p^{[1]}(1) - p^{[1]}(0) = \frac{p^{[\alpha-1]}(0) - p^{[1]}(0)}{2} \leq \frac{r}{2} \quad \text{deduce} \quad p^{[1]}(2) - p^{[1]}(1) \leq \frac{r}{2}$$

$$\Rightarrow p^{[1]}(2) - p^{[1]}(0) \leq r$$



Proof:Theorem1

$$p^{[1]}(2) \in \left[\frac{p^{[1]}(1) + p^{[\alpha-1]}(1)}{2}, * \right]$$



$$\begin{aligned} p^{[1]}(2) - p^{[1]}(0) &\geq \frac{p^{[1]}(1) + p^{[\alpha-1]}(1)}{2} - p^{[1]}(0) \\ &\geq \frac{p^{[\alpha-1]}(1) - p^{[1]}(0)}{2} \geq \frac{1}{2} \left(\frac{p^{[1]}(0) + p^{[\alpha]}(0)}{2} - p^{[1]}(0) \right) \\ &\geq \frac{1}{4} (p^{[\alpha]}(0) - p^{[1]}(0)) \geq \frac{r}{4} \end{aligned}$$

Move greater than $r/4$ in two time steps

$$\frac{1}{r} \text{diam}(co(P_0)) \leq TC(T_{mdvcs}, CC_{CRCMCNTR}, P_0) \leq \frac{4}{r} \text{diam}(co(P_0))$$



Proof:Theorem1

$g_{disk}(r)$ is not connected

Connected component don't change



$$TC(T_{mdvcs}, CC_{CRCMCNTR}, P_0) \leq \frac{4}{r} \text{diam}(co(C))$$

$$\text{diam}(co(C)) \leq (n-1)r \quad \leftarrow co(C) \text{ is connected}$$

$$\Rightarrow TC(T_{mdvcs}, CC_{CRCMCNTR}) \in O(n)$$

Lower bound

$$\text{diam}(co(p_0)) = (n-1)r \quad \Rightarrow TC(T_{mdvcs}, CC_{CRCMCNTR}, P_0) \geq n-1$$

$$TC(T_{mdvcs}, CC_{CRCMCNTR}) \in \Theta(n)$$



The circumcenter law in nonconvex environments

1. adapt connectivity constraints
2. Restrict robot motion
3. Move towards the circumcenter of the constraint set

algorithm

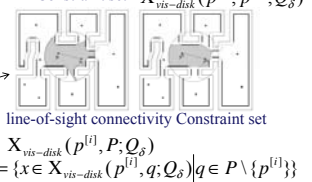
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Robotic Network:  $S_{vis-disk}$  with discrete-time motion model (4.1.1),
accurate sensing of own position and of  $q_j$ , and
communication range  $r$  within line-of-sight ( $S_{vis-disk}$ ).

Distributed Algorithm: NONCONVEX CIRCUMCTR
Alphabet:  $A = \mathbb{R}^2 \setminus \{null\}$ 
function  $msg(p, i)$ 
: return  $p$ 
function  $ctrl(p, i)$ 
:  $X_i := \text{circum}(P_i)$  for all non-null  $P_{i-1} \in \{p\} \cup Q_i$ 
:  $X_i := \text{null}$  for all non-null  $P_{i-1} \in \{p\} \cup Q_i$ 
:  $p_{i+1} := \text{Circ}(X_i, r, Q_i)$ 
: return  $(p_{i+1}, \text{Vis}(P_{i-1}, p_{i+1})) = p$ 

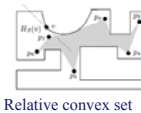
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pair wise line-of-sight connectivity Constraint set $X_{vis-disk}(p^{[i]}, p^{[j]}, Q_\delta)$



line-of-sight connectivity Constraint set

$$X_{vis-disk}(p^{[i]}, P; Q_\delta) = \{x \in X_{vis-disk}(p^{[i]}, q; Q_\delta) \mid q \in P \setminus \{p^{[i]}\}\}$$



On the network $S_{vis-disk}$ the law $CC_{nonconvex-circumctr}$ achieve the task $T_{\epsilon-mdvcs}$



outline

- Robotic network models and complexity notions
- Connectivity maintenance and rendezvous
- **Deployment**
 - Voronoi-centroid control and communication law
 - Voronoi-centroid law on planar vehicles
 - Voronoi-circumcenter control
 - Voronoi-incenter control
 - Limited-Voronoi-normal control
 - Limited-Voronoi-centroid control



Deployment

Robotic network : $S_D, S_{LD}, S_{vehicles}$

Assume : no two agents are initially at the same point

Deployment algorithms

1. Transmit own position ,receive neighbors position
2. Compute a notion of the geometric center of its own cell
determined according to some notion of partition
3. Move toward this center

Difference each algorithm \Rightarrow Geometric center Partition of the environment

Voronoi-centroid control and communication law

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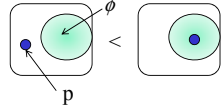
CC_{VRN-CNTRD} algorithm

Robotic Network: S_C with discrete-time motion model (4.1.1) in Q , with absolute sensing of own position
 Distributed Algorithm: VRN-CNTRD
 Alphabet: $A = \mathbb{R}^2 \cup \{\text{null}\}$
 function msg(p, q)
 1: return p
 function ctrl(p, q)
 1: $V := Q \cap (\cap_{i=1}^n H_{p_i, r_i})$ (for all non-null $p_{i+1} \in \mathcal{V}$)
 2: return $CM_\phi(V) - p$

expected-value multicenter function

$$H_{dist}(p_1, \dots, p_n) = -\sum_{i=1}^n \int_{V_i(p)} \|q - p_i\|_2^2 \phi(q) dq$$

Make voronoi cell



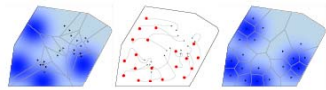
$$CM_\phi(s) = \frac{1}{A_\phi(s)} \int_s q \phi(q) dq, \quad A_\phi(s) = \int_s \phi(q) dq$$

Direction of motion

Gradient of the distortion multicenter function H_{dist}

Optimize H_{dist} (maximize)

Simulation result



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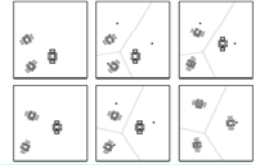
Voronoi-centroid law on planar vehicles

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Robotic network $S_{vehicles}$

$$\begin{aligned} \hat{p}^{(i)}(t) &= v^{(i)}(t) (\cos(\theta^{(i)}(t)), \sin(\theta^{(i)}(t))) \\ \dot{\theta}^{(i)}(t) &= \omega^{(i)}(t) \end{aligned}$$

Simulation result



CC_{VRN-CNTRD-DYNMCS} algorithm

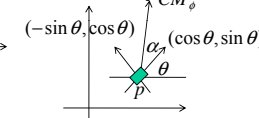
Robotic Network: $S_{vehicles}$ with motion model (5.2.1) in Q , with absolute sensing of own position
 Distributed Algorithm: VRN-CNTRD-DYNMCS
 Alphabet: $A = \mathbb{R}^2 \cup \{\text{null}\}$
 function msg(p, q)
 1: return p
 function ctrl(p, q)
 1: $V := Q \cap (\cap_{i=1}^n H_{p_i, r_i})$ (for all non-null $p_{i+1} \in \mathcal{V}$)
 2: $v := v_{opt}(\cos \theta, \sin \theta)$ ($v = CM_\phi(V)$)
 3: $\omega := 2\pi \text{random}(\text{uniform}(0, 2\pi))$ ($\theta = CM_\phi(V)$)
 4: return (v, ω)

$T_{\varepsilon\text{-distor-dply}}(P) = \text{true}$

if $\|p^{(i)} - CM_\phi(V^{(i)}(P))\|_2 \leq \varepsilon$

Theorem correctness

On the network S_D , the law $CC_{VRN-CNTRD}$ achieves the task $T_{\varepsilon\text{-distor-dply}}$ and optimizes H_{dist}



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Voronoi-circumcenter control

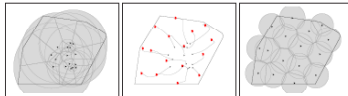
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Robotic network S_D

Simulation result

CC_{VRN-CRCMCTR} algorithm

Robotic Network: S_D with discrete-time motion model (4.1.1) in Q , with absolute sensing of own position
 Distributed Algorithm: VRN-CRCMCTR
 Alphabet: $A = \mathbb{R}^2 \cup \{\text{null}\}$
 function msg(p, q)
 1: return p
 function ctrl(p, q)
 1: $V := Q \cap (\cap_{i=1}^n H_{p_i, r_i})$ (for all non-null $p_{i+1} \in \mathcal{V}$)
 2: return $CC(V) - p$



How to cover a region with disks of minimum radius?

$$H_{dc}(p_1, \dots, p_n) = \max_{q \in S} \min_{i \in \{1, \dots, n\}} \|q - p_i\|_2$$

Maximum over the network of each robot's individual cost

not think about ϕ

$$H_{dc}(p_1, \dots, p_n) = \max_{i \in \{1, \dots, n\}} \max_{q \in \partial V_i(p_i)} \|q - p_i\|_2$$

Move toward furthest vertex minimize

Theorem correctness

On the network S_D , the law $CC_{VRN-CRCMCTR}$ optimizes H_{dc}

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Voronoi-incenter control

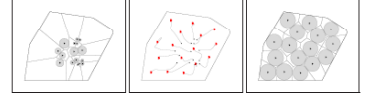
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Robotic network S_D

Simulation result

CC_{VRN-NCNTR} algorithm

Robotic Network: S_D with discrete-time motion model (4.1.1) in Q , with absolute sensing of own position
 Distributed Algorithm: VRN-NCNTR
 Alphabet: $A = \mathbb{R}^2 \cup \{\text{null}\}$
 function msg(p, q)
 1: return p
 function ctrl(p, q)
 1: $V := Q \cap (\cap_{i=1}^n H_{p_i, r_i})$ (for all non-null $p_{i+1} \in \mathcal{V}$)
 2: return $c \in \partial V$



Maximize smallest radius

$$H_{sp}(p_1, \dots, p_n) = \min_{i \in \{1, \dots, n\}} \left\{ \frac{1}{2} \|q - p_i\|_2, \text{dist}(p_i, \partial S) \right\}$$

Maximize the cost given by the minimum distance to the boundary of V

not think about ϕ

$$H_{sp}(p_1, \dots, p_n) = \min_{i \in \{1, \dots, n\}} \min_{q \in \partial V_i(p_i)} \|q - p_i\|_2$$

away from closet neighbor maximize

Theorem correctness

On the network S_D , the law $CC_{VRN-NCNTR}$ optimizes H_{sp}

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Limited-Voronoi-normal control

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Robotic network S_{LD}

Simulation result

CC_{LMTD-VRN-NRML} algorithm

Robotic Network: S_{LD} with discrete-time motion model (4.1.1) with absolute sensing of own position, and with communication range r , in Q
 Distributed Algorithm: LMTD-VRN-NRML
 Alphabet: $A = \mathbb{R}^2 \cup \{\text{null}\}$
 function msg(p, q)
 1: return p
 function ctrl(p, q)
 1: $V := Q \cap (\cap_{i=1}^n H_{p_i, r_i})$ (for all non-null $p_{i+1} \in \mathcal{V}$)
 2: $r := \int_{\partial V} \phi(q) dq$
 3: $\lambda := \max \{ \lambda \mid \int_{\partial V \cap B(p, \lambda)} \phi(q) dq \text{ is strictly increasing on } [0, \lambda] \}$
 4: return $\lambda \hat{e}$

$$H_{area, r/2}(p_1, \dots, p_n) = \sum_{i=1}^n \int_{V_i(p) \cap \bar{B}(p_i, r/2)} \phi(q) dq$$

$T_{\varepsilon\text{-r-area-dply}}(P) = \text{true}$

$$\text{if } \left\| \int_{V_i(p) \cap \bar{B}(p_i, r/2)} n_{out}(q) \phi(q) dq \right\|_2 \leq \varepsilon$$

Line search

Theorem correctness

On the network S_{LD} , the law $CC_{LMTD-VRN-NRML}$ optimizes $H_{area, r/2}$ And achieves task $T_{\varepsilon\text{-r-area-dply}}$

maximize

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Limited-Voronoi-centroid control

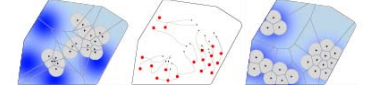
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Robotic network S_{LD}

Simulation result

CC_{LMTD-VRN-CNTRD} algorithm

Robotic Network: S_{LD} with discrete-time motion model (4.1.1) with absolute sensing of own position, and with communication range r , in Q
 Distributed Algorithm: LMTD-VRN-CNTRD
 Alphabet: $A = \mathbb{R}^2 \cup \{\text{null}\}$
 function msg(p, q)
 1: return p
 function ctrl(p, q)
 1: $V := Q \cap (\cap_{i=1}^n H_{p_i, r_i}) \cap (\cap_{j=1}^n H_{p_j, r_j})$ (for all non-null $p_{i+1} \in \mathcal{V}$)
 2: return $CM_\phi(V) - p$



$T_{\varepsilon\text{-r-distor-area-dply}}(P) = \text{true}$

if $\|p^{(i)} - CM_\phi(V_{r/2}^{(i)}(P))\|_2 \leq \varepsilon$

Theorem correctness

On the network S_{LD} , the law $CC_{LMTD-VRN-CNTRD}$ optimizes $H_{dist-area, r/2}$ And achieves task $T_{\varepsilon\text{-r-distor-area-dply}}$

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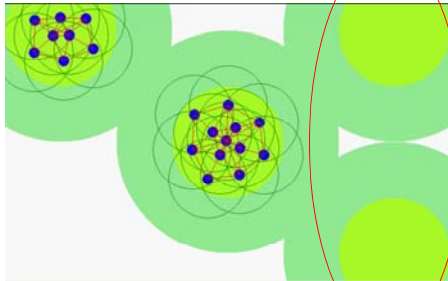
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problem1

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deployment



Don't reach and search this area

Move density function

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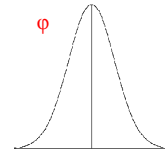
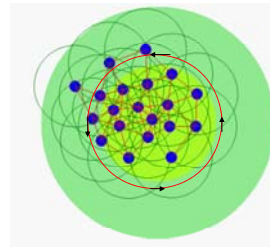
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problem1

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example



Move density function on circle

Achieve the search and deployment

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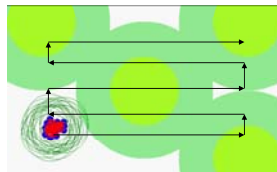
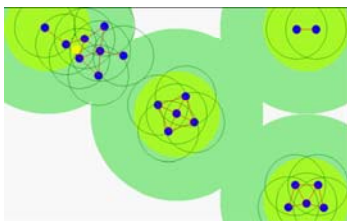
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problem1

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Applies this method to before environment



Search trajectory

Achieve search of all impotent place and deployment
But don't flock all robots

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problem2

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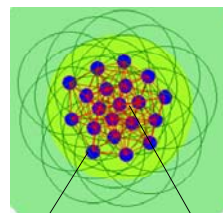
Each robot don't get the Same reward

Inner > outer

Outer side robots say lie

In this position you can get many reward!

Change inner robots density function



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