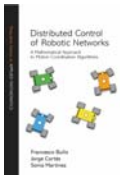
Introduction to Distributed Control of Robotic Networks

FL09-08-1

6,July,2009

Goto Tatsuhiko

author



Francesco Bullo
Professor
Department of Mechanical Engineering
University of California at Santa Barbara

outline

- Robotic network models and complexity notions
  - Physical components
  - Robotic network
  - Example
  - Control and communication laws
  - Evolution of a robotic network
  - The agree pursue control and communication law
  - Coordination tasks
  - complexity
- Connectivity maintenance and rendezvous
- Deployment

Physical components

continuous-time continuous-space dynamical system

$tuple(X, U, X_0, f)$

- X : state space

$X \in \Re^d, X \in S^d, X \in \Re^d \times S^d$

- U : input space
- X0 : allowable initial states

$X_0 \in X$

- f is a continuously differentiable control vector field on X

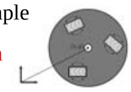
f determines the robot motion

via the control system $\dot{x}(t) = f(x(t), u(t))$

$f : X \times U \to \Re^d$

Example

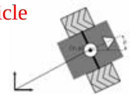
Omni direction



$R = \{\Re^2, [-u_{max}, u_{max}]^2, \Re^2, (0_2, e_1, e_2)\}$

Planner Vehicle

$\dot{x} = v \cos \theta$
$\dot{y} = v \sin \theta$
$\dot{\theta} = \omega$



$R = \{\Re^2 \times S^1, \begin{bmatrix} -v_{max} & v_{max} \\ -\omega_{max} & \omega_{max} \end{bmatrix}, \Re^2 \times S^1, \left(\begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right)\}$

Robotic network

Definition　Robotic network S consist of a $tuple(I, R, E_{cmm})$

$I = \{1, \ldots, n\}$

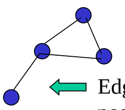
→ Set of unique identifiers

$R = \{R^{[i]}\}_{i \in I} = \{(X^{[i]}, U^{[i]}, X_0^{[i]}, f^{[i]})\}_{i \in I}$

→ Set of mobile robots

$E_{cmm} : \prod_{i \in I} X^{[i]} \to I \times I$

→ Communication edge map



← Edge depend on agents position

pair (i,j) is an edge in $E_{cmm}(x)$

↓

i,j can communicate

↓

$(I, E_{cmm}(x))$ →

Communication graph at x

If $R^{[i]} = (X, U, X_0, f)$ for all i (identical)

→ Robotic network is uniform

Example

1． First-order robots with range - limited communication

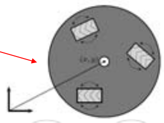
Robot locations

$p = \{p^{[1]}, p^{[2]}, \ldots, p^{[n]}\}$

$(p^{[i]} \in R^d)$

$\dot{p}^{[i]}(t) = u^{[i]}(t)$

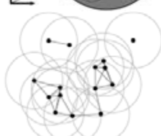
$(u^{[i]} \in [-u_{max}, u_{max}]^d)$



Identical robots

$R = \{\Re^d, [-u_{max}, u_{max}]^d, \Re^d, (0_2, e_1, \ldots, e_d)\}$

input space　initial states　Omni direction



Sensible own position
Communicate with robots within distance r

→ $g_{disk}(r)$

These data → Robotic network S_{disk}

Example

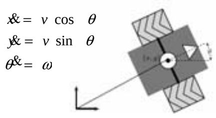
2 . Planner vehicle robots with Delaunay communication

Robot physical state — Planner vehicle model

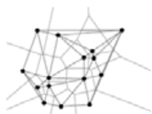
$$\{(p^{[1]},\theta^{[1]}),(p^{[2]},\theta^{[2]}),\Lambda,(p^{[n]},\theta^{[n]})\}$$

$$p^{[i]}=(x^{[i]},y^{[i]}),\theta^{[i]}\in S^1$$

$$\dot{x}=v\cos\theta,\quad \dot{y}=v\sin\theta,\quad \dot{\theta}=\omega$$



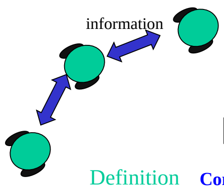
Communication graph → g_D



Robots move allowable environment Q

These data → Robotic network $S_{vehicles}$

Control and Communication laws



Continuous time

sense own position ,evolves

discrete time

execute state machine ,exchange information

Definition **Control and Communication laws (CC)**

sets
1. A : communication alphabet elements of A : messages
2. $W^{[i]}, i\in I$: prosser state sets
3. $W_0^{[i]}\subseteq W^{[i]}, i\in I$: allowable initial values

maps
1. $msg^{[i]}: X^{[i]}\times X^{[i]}\times I\to A$ message generation functions
2. $stf^{[i]}: X^{[i]}\times W^{[i]}\times A^n\to W^{[i]}$ processor state transition functions
3. $ctl^{[i]}: X^{[i]}\times X^{[i]}\times W^{[i]}\times A^n\to U^{[i]}$ motion control functions

Evolution of a robotic network

Definition Evolution of a robotic network

Evolution of (S,CC)

$$\dot{x}^{[i]}(t)=f(x^{[i]}(t),ctl^{[i]}(x^{[i]}(t),x^{[i]}(\lfloor t\rfloor),w^{[i]}(\lfloor t\rfloor),y^{[i]}(\lfloor t\rfloor)))$$

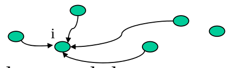
$u^{[i]}(t)$; A

$$w^{[i]}(l)=stf^{[i]}(x^{[i]}(l),w^{[i]}(l-1),y^{[i]}(l))$$

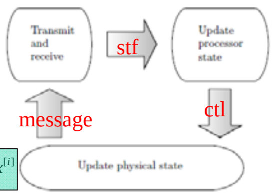
$$(x^{[i]}(0)=x_0^{[i]},w^{[i]}(-1)=w_0^{[i]})$$

$$y_j^{[i]}(l)=msg^{[j]}(x^{[j]}(l),w^{[j]}(l-1),i)$$

$$if\quad (j,i)\in E_{cmm}(x^{[1]}(l),\Lambda,x^{[n]}(l))$$



* data-sampled

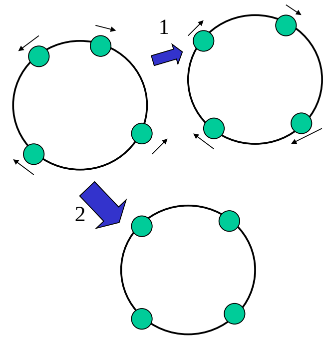


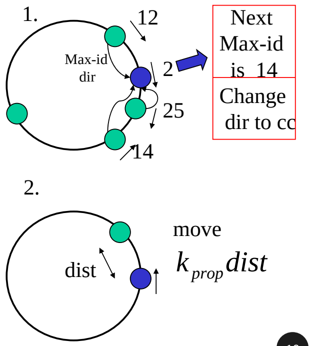
$ctl^{[i]}(x^{[i]},x^{[i]}_{smpld},w^{[i]},y^{[i]})$ is independent of $x^{[i]}$

the agree and pursue control and communication law

object
1. agree direction
2. equally angularly spaced

idea
1. Leader election algorithms
2. Cyclicpursuit problem

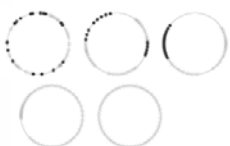




algorithm


```
Robotic Network: S_circle, first-order agents in S^1
                 with absolute sensing of own position, and
                 with communication range r

Distributed Algorithm: AGREE & PURSUE
Alphabet: A = S^1 × {c, cc} × I ∪ {null}
Processor State: w = (dir, max-id), where
  dir     ∈ {c, cc},  initially: dir^[i] unspecified
  max-id  ∈ I,        initially: max-id^[i] = i for all i

% Standard message-generation function
function msg(θ, w, i)
   return (θ, w)

function stf(θ, w, y)
   for each non-null message (θ_rcvd, (dir_rcvd, max-id_rcvd)) in y do
     if (max-id_rcvd > max-id) AND (dist_cc(θ, θ_rcvd) ≤ r AND dir_rcvd =
     c) OR (dist_c(θ, θ_rcvd) ≤ r AND dir_rcvd = cc) then
       new-dir := dir_rcvd
       new-id := max-id_rcvd
   return (new-dir, new-id)

function ctl(θ_smpld, w, y)
   d_tmp := r
   for each non-null message (θ_rcvd, (dir_rcvd, max-id_rcvd)) in y do
     if (dir = cc) AND (dist_cc(θ_smpld, θ_rcvd) < d_tmp) then
       d_tmp := dist_cc(θ_smpld, θ_rcvd)
       u_tmp := k_prop d_tmp
     if (dir = c) AND (dist_c(θ_smpld, θ_rcvd) < d_tmp) then
       d_tmp := dist_c(θ_smpld, θ_rcvd)
       u_tmp := −k_prop d_tmp
   return u_tmp
```

Simulation result



Leader election

Neighbor?

Moving toward?

Cyclic pursuit

Coordination tasks

Definition Coordination task

map $T:\prod_{i\in I}X^{[i]}\times W^n\to\{true, false\}$

$(if$ W is singleton $T:\prod_{i\in I}X^{[i]}\to\{true, false\})$

Ex) dir c or cc

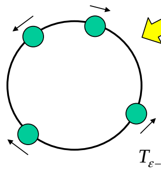
Processor state take Values in W

$if\ W^{[i]}=W$, the law CC is compatible with the task

law CC achieves the task T → T(x(t), w(t)) = true for all t ≥ T

Ex)Direction agreement and equidistance task

processor state taking value in $W=\{cc,c\}\times I$



direction agreement task $T_{dir}:(S^1)^n\times W^n\to\{true, false\}$

$$T_{dir}(\theta,w)=true\ if\ dir^{[1]}=\Lambda=dir^{[n]}$$

$$(\theta=(\theta^{[1]},\Lambda,\theta^{[n]}),w=(w^{[1]},\Lambda,w^{[n]}),w^{[i]}=(dir^{[i]},max-id^{[i]}))$$

equidistance task $T_{\varepsilon-eqdstnc}:(S^1)^n\to\{true, false\}$

$$T_{\varepsilon-eqdstnc}(\theta)=true\ if\ \left|\min_{j\neq i}dist_c(\theta^{[i]},\theta^{[j]})-\min_{j\neq i}dist_{cc}(\theta^{[i]},\theta^{[j]})\right|<\varepsilon$$

Complexity

Definition Time complexity

Time complexity to achieve T with CC from initial condition

$TC(T,CC,x_0,w_0)=\inf\{l \mid T(x(k),w(k))=true,\ for\ all\ k\geq l\}$

Time complexity to achieve T with CC

$TC(T,CC)=\sup\{TC(T,CC,x_0,w_0) \mid (x_0,w_0)\in \prod_{i\in I} X_0^{[i]}\times\prod_{i\in I} W_0^{[i]}\}$

Time complexity to achieve T worst case

$TC(T)=\inf\{TC(T,CC) \mid CC \text{ compatible with T}\}$

Set of non-null message

Definition Space complexity

$SC(T,CC)$

Maximum number of basic memory units

Definition Mean and Total Communication complexity

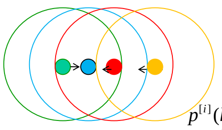
$MCC(T,CC,x_0,w_0)=\frac{|A|_{basic}}{\tau}\sum_{l=0}^{\tau-1}|M(x(l),w(l))|$

$TCC(T,CC,x_0,w_0)=|A|_{basic}\sum_{l=0}^{\tau-1}|M(x(l),w(l))|$

outline

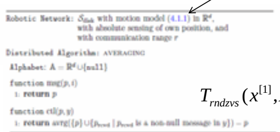
- Robotic network models and complexity notions
- Connectivity maintenance and rendezvous
  - Averaging control and communicaton law
  - Circumcenter control and communicaton laws
  - Correctness and complexity of circumcenter laws
  - The circumcenter law in nonconvex environments
- Deployment

Averaging control and communication law

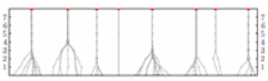


1.Compute the average point
2.Move toward average point

$p^{[i]}(l+1)=p^{[i]}(l)+u^{[i]}(l)$



Simulation result



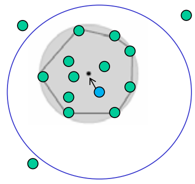
$T_{rndzvs}(x^{[1]},\Lambda,x^{[n]})=true$ if $x^{[i]}=x^{[j]}$ for all $(i,j)\in E_{cmm}$

Theorem
Correctness and time complexity of averaging law

the law $CC_{averaging}$ achives the task T_{rndzvs} with time comlexity

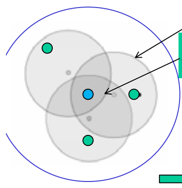
$TC(T_{rndzvs},CC_{averaging})\in O(n^5)$ $TC(T_{rndzvs},CC_{averaging})\in \Omega(n)$

Circumcenter control and communication laws



1.Compute the circumcenter point
2.Move toward circumcenter point
Maintaining connectivity
use constraint set

constraint set



$X_{disk}(p^{[i]},q^{[j]})=\bar{B}(\frac{p^{[j]}+q^{[i]}}{2},\frac{r}{2})$

$X_{disk}(p^{[i]},P)=\{x\in X_{disk}(p^{[i]},q) \mid q\in P\setminus\{p^{[i]}\}s.t.\|q-p^{[i]}\|_2\leq r\}$

Theorem Maintaining network connectivity

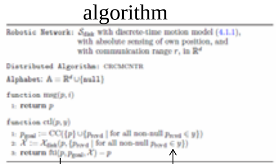
$u^{[i]}(l)\in X_{disk}(p^{[i]}(l),P(l))-p^{[i]}(l)$

$p^{[i]}(l+1)\in X_{disk}(p^{[i]}(l),P(l))$ Maintaining edge

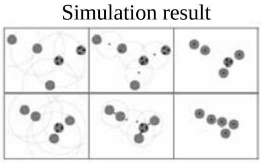
$\|p^{[i]}(l)-p^{[j]}(l)\|_2\leq r\rightarrow\|p^{[i]}(l+1)-p^{[j]}(l+1)\|_2\leq r$

Circumcenter control and communication laws

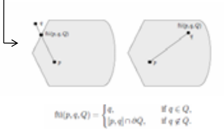
algorithm



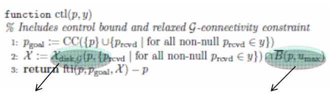
Simulation result



constrain set



Circumcenter law with control bounds and relaxed connectivity constraints



relax constrain sets

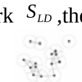
Agents have compact input space

Correctness and complexity of circumcenter laws

Theorem correctness of the circumcenteer laws

1.on the network S_{disk} ,the law $CC_{crcmcntr}$ achieves the exact rendezvous task T_{rndzvs}

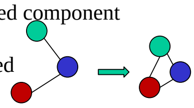
2.on the network S_{LD} ,the law $CC_{crcmcntr}$ achieves the $\varepsilon-$ rendezvous task $T_{\varepsilon-rndzvs}$



$T_{\varepsilon\text{-rndzvs}}(P)=\text{true}$
$\iff \|p^{[i]}-\text{avrg}(\{p^{[j]} \mid (i,j)\in E_{cmm}(P)\})\|_2<\varepsilon,\quad i\in\{1,\ldots,n\}.$

3.if any two agents belong to the same connected component at l

continue to belong to the same connected component for all subsequent time



4. $P^*=(p_1^*,\Lambda,p_n^*)$
(a)evolution asymptotically approaches
(b) $p_i^*=p_j^*$ or $\|p_i^*-p_j^*\|_2>r$

Theorem time complexity (d=1)

S_{disk} ⇨ $TC(T_{rndzvs},CC_{crcmcntr})\in\Theta(n)$

S_{LD} ⇨ $TC(T_{r\varepsilon-rndzvs},CC_{crcmcntr})\in\Theta(n^2\log(n\varepsilon^{-1}))$

Proof:Theorem1

Dimension 1 $g_{disk}(r)$ is connected

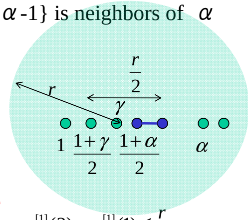
$\{2,\ldots,\alpha\text{-}1\}$ is neighbors of 1

Worst case

$\{2,\ldots,\alpha\text{-}1\}$ is neighbors of α

$$p^{[1]}(1)=\frac{p^{[1]}(0)+p^{[\alpha-1]}(0)}{2}$$

$$p^{[\gamma]}(1)\in\left[\frac{p^{[1]}(0)+p^{[\alpha]}(0)}{2},\frac{p^{[1]}(0)+p^{[\gamma]}(0)+r}{2}\right]$$



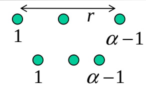
$$p^{[1]}(1)-p^{[1]}(0)=\frac{p^{[\alpha-1]}(0)-p^{[1]}(0)}{2}\le\frac{r}{2}$$

deduce ⇒ $p^{[1]}(2)-p^{[1]}(1)\le\frac{r}{2}$

⇒ $p^{[1]}(2)-p^{[1]}(0)\le r$

Proof:Theorem1

$$p^{[1]}(2)\in\left[\frac{p^{[1]}(1)+p^{[\alpha-1]}(1)}{2},*\right]$$



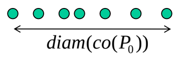
$$p^{[1]}(2)-p^{[1]}(0)\ge\frac{p^{[1]}(1)+p^{[\alpha-1]}(1)}{2}-p^{[1]}(0)$$

$$\ge\frac{p^{[\alpha-1]}(1)-p^{[1]}(0)}{2}\ge\frac{1}{2}\left(\frac{p^{[1]}(0)+p^{[\alpha]}(0)}{2}-p^{[1]}(0)\right)$$

$$\ge\frac{1}{4}(p^{[\alpha]}(0)-p^{[1]}(0))\ge\frac{r}{4}$$

⇒ Move greater than r/4 in two time steps

$$\frac{1}{r}diam(co(P_0))\le TC(T_{rndzvs},CC_{CRCMCNTR},P_0)\le\frac{4}{r}diam(co(P_0))$$



Proof:Theorem1

$g_{disk}(r)$ is not connected

C_1 C_2

⇒ Connected component don't change

⇒ $TC(T_{rndzvs},CC_{CRCMCNTR},P_0)\le\frac{4}{r}\underset{C\in C(P_0)}{diam}(co(C))$

$diam(co(C))\le(n-1)r$ ⇐ $co(C)$ is connected

⇒ $TC(T_{rndzvs},CC_{CRCMCNTR})\in O(n)$

Lower bound

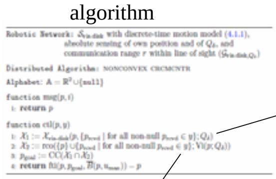
$diam(co(p_0))=(n-1)r$ ⇒ $TC(T_{rndzvs},CC_{CRCMCNTR},P_0)\ge n-1$

$TC(T_{rndzvs},CC_{crcmcntr})\in\Theta(n)$

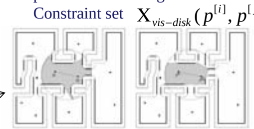
The circumcenter law in nonconvex environments

1.adapt connectivity constraints
2.Restrict robot motion
3.Move towards the circumcenter of the constraint set

algorithm



pair wise line-of-sight connectivity Constraint set $X_{vis-disk}(p^{[i]},p^{[j]};Q_\delta)$



line-of-sight connectivity Constraint set

$$X_{vis-disk}(p^{[i]},P;Q_\delta)=\{x\in X_{vis-disk}(p^{[i]},q;Q_\delta)\mid q\in P\setminus\{p^{[i]}\}\}$$

Relative convex set

On the network $S_{vis-disk}$ the law $CC_{nonconvex-crcmcntr}$

⇒ achieve the task $T_{\varepsilon-rndzvs}$

outline

- Robotic network models and complexity notions
- Connectivity maintenance and rendezvous
- Deployment
- Voronoi-centroid control and communication law
- Voronoi-centroid law on planar vehicles
- Voronoi-circumcenter control
- Voronoi-incenter control
- Limited-Voronoi-normal control
- Limited-Voronoi-normal control

Deployment

Robotic network : S_D S_{LD} $S_{vehicles}$

Assume : no two agents are initially at the same point

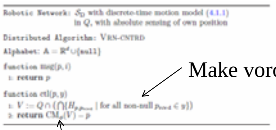
Deployment algorithms

1.Transmit own position ,receive neighbors position
2.Compute a notion of the geometric center of its own cell
determined according to some notion of partition
3.Move toward this center

Difference each algorithm ⇒ Geometric center, Partition of the environment

Voronoi-centroid control and communication law

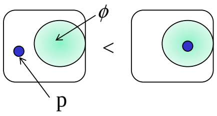
$CC_{VRN-CNTRD}$ algorithm



Make voronoi cell

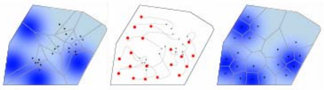
expected-value multicenter function

$$H_{dist}(p_1,\Lambda,p_n) = -\sum_{i=1}^{n}\int_{V_i(P)}\|q-p_i\|_2^2\,\phi(q)dq$$



$$CM_\phi(s) = \frac{1}{A_\phi(s)}\int_s q\phi(q)dq \quad , \quad A_\phi(q) = \int_s \phi(q)dq$$

Simulation result



Direction of motion

||

Gradient of the distortion multicenter function H_{dist}

Optimize H_{dist} (maximize)

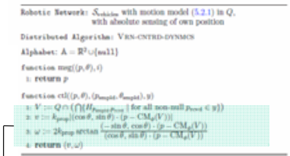
Voronoi-centroid law on planar vehicles

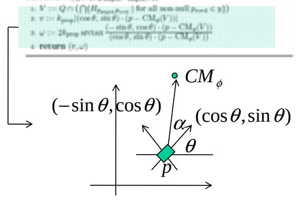
Robotic network $S_{vehicles}$

$\dot{p}^{[i]}(t) = v^{[i]}(t)(\cos(\theta^{[i]}(t)), \sin(\theta^{[i]}(t)))$

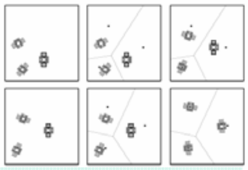
$\dot{\theta}^{[i]}(t) = \omega^{[i]}(t)$

$CC_{VRN-CNTRD-DYNMCS}$ algorithm





Simulation result



$T_{\varepsilon-distor-dply}(P) = true$

$if \left\| p^{[i]} - CM_\phi(V^{[i]}(P)) \right\|_2 \le \varepsilon$

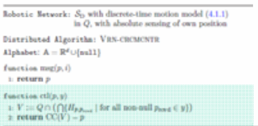
Theorem correctness

On the network S_D, the law $CC_{VRN-CNTRD}$ achieves the task $T_{\varepsilon-distor-dply}$ and optimizes H_{dist}

Voronoi-circumcenter control

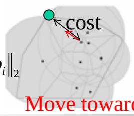
Robotic network S_D

$CC_{VRN-CRCMCENTR}$ algorithm



not think about ϕ

$$H_{dc}(p_1,\Lambda,p_n) = \max_{i\in\{1,\Lambda,n\}}\max_{q\in\partial V_i(P)}\|q-p_i\|_2$$



Simulation result

How to cover a region with disks of minimum radius ?

$$H_{dc}(p_1,\Lambda,p_n) = \max_{q\in S}\min_{i\in\{1,\Lambda,n\}}\|q-p_i\|_2$$

Maximum over the network of each robot's individual cost

Theorem correctness

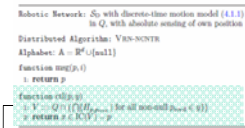
On the network S_D, the law $CC_{VRN-CRCMCNTR}$ optimizes H_{dc}

Move toward furthest vertex minimize

Voronoi-incenter control

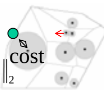
Robotic network S_D

$CC_{VRN-NCNTR}$ algorithm



not think about ϕ

$$H_{sp}(p_1,\Lambda,p_n) = \min_{i\in\{1,\Lambda,n\}}\min_{q\in\partial V_i(P)}\|q-p_i\|_2$$



Simulation result

Maximize smallest radius

$$H_{sp}(p_1,\Lambda,p_n) = \min_{i\neq j\in\{1,\Lambda,n\}}\{\frac{1}{2}\|q-p_i\|_2, dist(p_i,\partial S)\}$$

Maximize the cost given by the minimum distance to the boundary of V

Theorem correctness

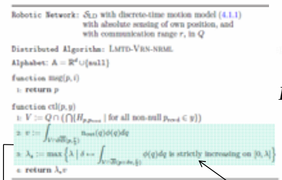
On the network S_D, the law $CC_{VRN-NCNTR}$ optimizes H_{sp}

away from closet neighbor maximize

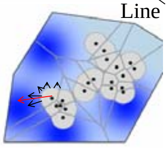
Limited-Voronoi-normal control

Robotic network S_{LD}

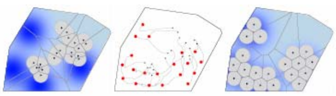
$CC_{LMTD-VRN-NRML}$ algorithm



Line search



Simulation result



$$H_{area,r/2}(p_1,\Lambda,p_n) = \sum_{i=1}^{n}\int_{V_i(P)\cap\bar{B}(p_i,r/2)}\phi(q)dq$$

$T_{\varepsilon-r-area-dply}(P) = true$

$$if \left\| \int_{V^{[i]}(P)\cap\partial\bar{B}(p^{[i]},r/2)} n_{out}(q)\phi(q)dq \right\|_2 \le \varepsilon$$

Theorem correctness

On the network S_{LD}, the law $CC_{LMTD-VRN-NRML}$ optimizes $H_{area,r/2}$

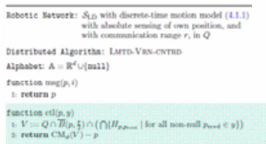
And achieves task $T_{\varepsilon-r-area-dply}$

maximize

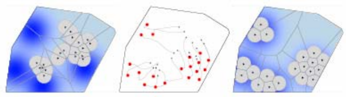
Limited-Voronoi-normal control

Robotic network S_{LD}

$CC_{LMTD-VRN-CNTRD}$ algorithm



Simulation result



$T_{\varepsilon-r-distor-area-dply}(P) = true$

$if \left\| p^{[i]} - CM_\phi(V_{r/2}^{[i]}(P)) \right\|_2 \le \varepsilon$

$$H_{dist-area,r/2}(p_1,\Lambda,p_n) = -\sum_{i=1}^{n}\int\|q-p_i\|_2^2\,\phi(q)dq + \int_{V_i(P)\cap\bar{B}(p_i,r/2)}\phi(q)dq$$

Theorem correctness

On the network S_{LD}, the law $CC_{LMTD-VRN-CNTRD}$ optimizes $H_{dist-area,r/2}$

And achieves task $T_{\varepsilon-r-distor-area-dply}$