Application of Mechanism Design to Pose Synchronization

FL08 -26-1

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Outline

- Preliminary, Introduction, Objective
- Introduction to the results of Literature[11]
- Pose Synchronization
  - Some Results
  - Passivity-like Property
- Application of Mechanism Design
  - Derivation of Objective Functions
  - Mechanism Design Problem
    - Centralized
    - Decentralized
- Conclusion and Future Work

Preliminary

- Graph : A set of connections (Edges) of between Objects (Vertice)

  Vertex (node) : Agent Edge : Information Flow

  -Directed Graph (Fig. 1) : the information flows from agent j to i

  -Undirected Graph (Fig. 2) : the information flows to both directions

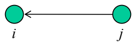

Fig. 1 Directed Graph

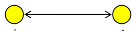

Fig. 2 Undirected Graph

- Directed Graph

  -strongly connected (Fig. 3) :

  there is a directly path connecting any two distinct nodes

  -balanced (Fig. 3) :

  the total number of edges entering a node and leaving the same node are equal for all nodes.

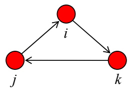

Fig. 3 Strongly Connected Graph

- Undirected Graph

  -connected :

  there is a path between any two distinct nodes

Preliminary

$G=(V,E)$: directed (or undirected) graph

$V=\{1,\ldots,n\}$: set of nodes (agents)

$E\subseteq V\times V$: set of edges (an edge of G : $e_{ij}=(i,j)$)

$N_i=\{j\in V\,|\,(j,i)\in E\}$: set of neighbors of node i

(the set of nodes whose information node i can get)

Graph Laplacian : $L=[l_{ij}]=\begin{cases}|N_i| & i=j\\ -1 & j\in N_i\\ 0 & j\notin N_i\end{cases}$

$|N_i|$: the number of neighbors of agent i

Introduction

Mechanism Design

The study of designing rules of a game or system to achieve a specific outcome, even though each agent may be self-interested.

- Mechanism design is done by setting up a structure in which agents have an incentive to behave according to the rules.
- The resulting mechanism is then said to implement the desired outcome.

Application

- Creation of markets, auctions and so on
- Design of matching algorithms
- Provision of public goods
- Optimal design of taxation schemes by governments

etc…

Fig. 4 Auction(※)

※http://graphics8.nytimes.com/images/2006/11/08/arts/08auction_CA0.600.jpg

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Introduction

Cooperative Control

A distributed control strategy that achieves specified tasks in multi-agent system

- Analysis of emergent and self-organized swarming behaviors in biological groups with distributed agent-to-agent interaction.
- Interest in a group behavior of animals, formulation control of multi-vehicle systems and so on

Application

- Mobile sensor networks
- Robot networks
- Formation control
- many other Multi-agent systems…

Fig. 5 Formation(※)

※ http://www.grahamowen.com/images8/B-Jet-Formation.jpg

Objective of My Study

Mechanism Design $\xrightarrow{\text{apply}}$ Pose Synchronization

Mechanism Design for Pose Synchronization

To design and impose individual objectives and control rules on each agent of a group in order to achieve pose synchronization

Rules
Self-interested Utilities
Group Objective

Group objective : to converge all agents' poses to a desired value
Rule : to have each objective function(nonnegative)
Utility : to minimize each objective function

What function does each agent minimize?
Analyze the function by using Passivity-like Property

⇨ Apply the result of literature[11]

How to apply Mechanism Design?

To derive the function which the control input for pose synchronization minimizes

⇨ Use the result of literature [11]

In literature [11],

There is a theorem which shows the relation between output feedback control law and its minimizing nonnegative scalar function of a passive system.

Method:
Consider to apply kinematic model of each agent and its passivity-like property to this theorem.

⇨ First, introduce the results of literature [11]

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Literature[11]

Optimal Stabilizing Control[11]

Nonlinear System

$$\begin{cases} \dot{\xi} = f(\xi) + g(\xi)u(\xi) \\ \zeta = k(\xi) \end{cases} \quad \cdots(7)$$

$\xi \in \Re^n$: states $\quad f(.) : \Re^n \to \Re^n$
$u \in \Re^m$: inputs $\quad g(.) : \Re^n \to \Re^{n\times m}$
$\zeta \in \Re^m$: outputs $\quad k(.) : \Re^n \to \Re^m$

Cost Function

$$J = \int_0^\infty \left(l(\xi) + u^T(\xi)u(\xi) \right) dt \quad \cdots(8)$$

$l(\xi) \geq 0 \in \Re$

Definition

We say that the control law u optimally stabilize for the cost function (8) if the following properties are satisfied:
(i) $u(\xi)$ achieves asymptotic stability of the equilibrium $\xi = \mathbf{0}_n$
(ii) $u(\xi)$ minimizes the cost functional J

$\mathbf{0}_i = [0 \ \cdots \ 0]^T \in \Re^i$

Literature[11]

Optimal Stabilizing Control[11]

Theorem 3 (Theorem 3.23[11])

The control law $u = -k(\xi)$ is optimal stabilizing for the cost functional (8) if and only if the system (7) is Zero State Detectable and Output Feedback Passive$\left(-\frac{1}{2}\right)$ with a C^1 storage function $S(\xi) \geq 0 \in \Re$.

Zero State Detectability(ZSD)

Let there be a system with zero input and $Z \subset \Re^n$ be its largest positively invariant set contained in $\{\xi \in \Re^n \mid \zeta = k(\xi) = \mathbf{0}_m\}$. Then we say that the system is zero state detectable if $\xi = \mathbf{0}_n$ is asymptotically stable conditionally to Z.

Output Feedback Passivity(OFP)

We say that the system is output feedback passive if the following inequality is satisfied for some $\rho \in \Re$:

$$\dot{S}(\xi) \leq u^T\zeta - \rho\zeta^T\zeta$$

Literature[11]

In the proof of theorem 3.23[11], the following property is used:

The system is OFP$\left(-\frac{1}{2}\right)$ $\Leftrightarrow$

There exists a C^1, positive semidefinite, function $V(\xi)$ such that $\begin{cases} k(\xi) = \frac{1}{2}(L_gV)^T \\ l(\xi) = \frac{1}{4}L_gV(L_gV)^T - L_fV \end{cases}$.

$\left(L_fV(\xi) = \frac{\partial V}{\partial \xi} f(\xi) : \text{Lie derivative} \right)$

HJB equation

With $S(\xi) = \frac{1}{2}V(\xi)$,

$$\begin{cases} L_gS(\xi) = k^T(\xi) \\ L_fS(\xi) = -\frac{1}{2}l(\xi) + \frac{1}{2}k^T(\xi)k(\xi) \end{cases} \quad \cdots(10)$$

⇨ If we can solve the equation (10), then we will find $S(\xi)$ and $l(\xi)$ from $k(\xi)$ (inputs).

⇨ apply this theorem to the system and control input of pose synchronization

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Review(Pose Synchronization)

Pose Synchronization [5,14]

Kinematic Model

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 \\ \sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}, i = \{1,\dots,n\} \quad \cdots(1)$$

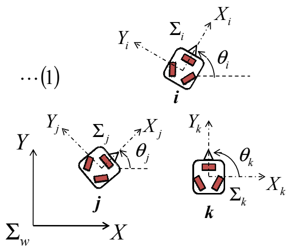

Fig. 6 Rigid Body Motion

Control Input : v_i, ω_i

$x_i, y_i \in \Re$: position

$\theta_i \in \Re$: rotation angle

$v_i \in \Re^2$: body velocity

$\omega_i \in \Re$: body angular velocity

Σ_w : inertial coordinate frame

Σ_i : body - fixed coordinate frame

Review(Pose Synchronization)

Pose Synchronization

A group of agents is said to pose synchronize when all agents converge to the same position and orientation between the agents.

Assumptions

A1 : $|\theta_i(t)| < \frac{\pi}{2} \quad \forall i, t$

A2 : Information graph is fixed, directed and strongly connected

A3 : Information graph is fixed, undirected and connected

Theorem 1

Consider the system with n rigid bodies represented by (1). Under the assumptions A1 and A2, the following control input achieves pose synchronization.

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \sum_{j \in N_i} \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_j - x_i \\ y_j - y_i \\ \sin(\theta_j - \theta_i) \end{bmatrix} \quad , \quad i \in \{1,\cdots,n\} \quad \cdots(5)$$

Review(Pose Synchronization)

Convergent Value

convergent values : $\alpha_x, \alpha_y, \alpha_\theta$

Information graph is undirected and connected

$$\begin{cases} (\alpha_x \quad \alpha_y) = \left(\frac{1}{n}\sum_i x_i(0) \quad \frac{1}{n}\sum_i y_i(0) \right) \\ \alpha_\theta = \frac{1}{n}\sum_i \theta_i(0) \end{cases} \quad \forall i$$

Convergent values are the averages of all agents' initial states.

Information graph is directed and strongly connected

$$(\alpha_x \quad \alpha_y) = \left(\frac{\sum_i \gamma_i x_i(0)}{\sum_i \gamma_i} \quad \frac{\sum_i \gamma_i y_i(0)}{\sum_i \gamma_i} \right) \quad \forall i$$

Convergent value is the weighted average of all agents' initial positions.

$\gamma := [\gamma_1 \quad \cdots \quad \gamma_n]^T, \forall \gamma_i > 0$: a left eigenvectors of L corresponding to eigenvalue 0 whose all elements are positive [4]

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Review(Passivity-like Property)

Energy Function

$$\begin{cases} \psi_i(p_i) := \frac{1}{2}\|p_i\|_2^2 (\geq 0) \\ \phi_i(\theta_i) := 1 - \cos\theta_i (\geq 0) \end{cases} \quad i \in \{1,\cdots,n\} \quad \cdots(3)$$

$$\xrightarrow{\text{differentiate}} \begin{cases} \dot{\psi}_i = v_i^T R_i^{-1} p_i \\ \dot{\phi}_i = \omega_i \sin\theta_i \end{cases} \quad \cdots(4)$$

$p_i = [x_i \quad y_i]^T$

$$R_i := \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix}$$

If we virtually define v_i, ω_i as inputs and $R_i^{-1} p_i, \sin\theta_i$ as outputs respectively, then the virtually-defined system (1) is passivity[11].

We call this property passivity-like property.

Control input (5) is developed by using this property(use output synchronization[10]).

$$\psi := \sum_{i=1}^{n} \psi_i \geq 0 \quad \xrightarrow{\text{Condition of graph}} \quad \dot{\psi} \leq 0$$

$$\begin{aligned} \frac{d\psi}{dt} &= \sum_{i=1}^{n} p_i^T \dot{p}_i \\ &= \sum_{i=1}^{n} \sum_{j \in N_i} p_i^T (p_j - p_i) \\ &= \sum_{i=1}^{n} \sum_{j \in N_i} \left(\underline{-\frac{1}{2}\|p_i\|^2 + \frac{1}{2}\|p_j\|^2} - \frac{1}{2}\|p_i - p_j\|^2 \right) \\ &= -\sum_{i=1}^{n} \sum_{j \in N_i} \frac{1}{2}\|p_i - p_j\|^2 \leq 0 \end{aligned}$$

Does the control input (5) minimize $\dot{\psi}$?

Analysis with theorem 3.

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Derivation of Objective Functions

First, consider the inertial model.

$\tilde{v}_i \in \Re^2$: velocity on inertial coordinate frame

Kinematic Model

$$\begin{bmatrix} \dot{p}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \tilde{v}_i \\ \omega_i \end{bmatrix}, i = \{1, \dots, n\} \quad \cdots (11)$$

Control Input

$$\begin{bmatrix} \tilde{v}_i \\ \omega_i \end{bmatrix} = \sum_{j \in N_i} \begin{bmatrix} p_j - p_i \\ \sin(\theta_j - \theta_i) \end{bmatrix}, i = \{1, \dots, n\} \quad \cdots (12)$$

Position System

$$\begin{cases} \dot{p} = \tilde{v}_i \\ \zeta_p = k_p(p) \end{cases} \quad \cdots (13)$$

$p := \begin{bmatrix} p_1^T & \cdots & p_n^T \end{bmatrix}^T$

$\tilde{v} := \begin{bmatrix} v_1^T & \cdots & v_n^T \end{bmatrix}^T$

$\zeta_p \in \Re^{2n}$: output of (13)

$\otimes$: kronecker product

$I_i \in \Re^{i \times i}$: unit matrix

$$\underline{k_p(p) = \begin{bmatrix} -\sum_{j \in N_1} (p_j - p_1) \\ \vdots \\ -\sum_{j \in N_n} (p_j - p_n) \end{bmatrix}} \Rightarrow \boxed{k_p(p) = L \otimes I_2 \cdot p}$$

for the inputs

Apply this system to theorem 3 neglecting ZSD for simplicity(consider afterwards).

Derivation of Objective Functions

Objective Function of Position System

$$J_p := \int_0^\infty \left(l_p(p) + \|\tilde{v}\|_2^2 \right) dt$$

$l_p(.) \geq 0 \in \Re$

Compare system (7) with (13), then $f(.) = \mathbf{0}_n, \quad g(.) = I_n$.

Thus, when let $S_p(p) \in \Re$ be nonnegative storage function for system (13),

$$L_g(S_p) = \frac{\partial S_p}{\partial p}, \quad L_f(S_p) = 0.$$

Substitute these equations to equations (10), then

$$\begin{cases} \dfrac{\partial S_p}{\partial p} = (L \otimes I_2 p)^T \\ l_p = (L \otimes I_2 p)^T L \otimes I_2 p \end{cases} \quad \cdots (16)$$

If the information graph is undirected and connected, then

$$\begin{cases} S_p(p) = \dfrac{1}{4} \displaystyle\sum_{i=1}^{n} \sum_{j \in N_i} \|p_j - p_i\|_2^2 \\ l_p(p) = \displaystyle\sum_{i=1}^{n} \left\| \sum_{j \in N_i} (p_j - p_i) \right\|_2^2 = \|L \otimes I_2 p\|_2^2 \end{cases} \Rightarrow \boxed{J_p = \int_0^\infty \left(\|L \otimes I_2 p\|_2^2 + \|\tilde{v}\|_2^2 \right) dt}$$

Derivation of Objective Functions

Attitude System

$$\begin{cases} \dot{\theta} = \omega \\ \zeta_\theta = k_\theta(\theta) \end{cases}$$

$\theta := \begin{bmatrix} \theta_1 & \cdots & \theta_n \end{bmatrix}^T$

$\omega := \begin{bmatrix} \omega_1 & \cdots & \omega_n \end{bmatrix}^T$

$\zeta_\theta \in \Re^n$: output of the left system

$$\underline{k_\theta(\theta) = \begin{bmatrix} -\sum_{j \in N_1} \sin(\theta_j - \theta_1) \\ \vdots \\ -\sum_{j \in N_n} \sin(\theta_j - \theta_n) \end{bmatrix}}$$

for the inputs

Objective Function of Attitude System

$$J_\theta := \int_0^\infty \left(l_\theta(\theta) + \|\tilde{v}\|_2^2 \right) dt$$

$l_\theta(.) \geq 0 \in \Re$

$$\Rightarrow \begin{cases} \dfrac{\partial S_\theta}{\partial \theta} = k_\theta^T \\ l_\theta = k_\theta^T k_\theta \end{cases} \quad \cdots (17)$$

If the information graph is undirected and connected, then

$$\begin{cases} S_\theta(\theta) = \dfrac{1}{2} \displaystyle\sum_{i=1}^{n} \sum_{j \in N_i} \left(1 - \cos(\theta_j - \theta_i) \right) \\ l_\theta(\theta) = \displaystyle\sum_{i=1}^{n} \left(\sum_{j \in N_i} \sin(\theta_j - \theta_i) \right)^2 \end{cases} \Rightarrow \boxed{J_\theta = \int_0^\infty \left(\sum_{i=1}^{n} \left(\sum_{j \in N_i} \sin(\theta_j - \theta_i) \right)^2 + \|\omega\|_2^2 \right) dt}$$

Derivation of Objective Functions

Discussion

$$S_p(p) = \frac{1}{4} \sum_{i=1}^{n} \sum_{j \in N_i} \|p_j - p_i\|_2^2$$

$$(3):\ \psi_i(p_i) = \frac{1}{2} \|p_i\|_2^2$$

$$\Rightarrow \tilde{\psi}_i(p_i) = \frac{1}{2} \sum_{j \in N_i} \|p_j - p_i\|_2^2$$

$$\Rightarrow \underline{S_p(p) = \frac{1}{2} \sum_{i=1}^{n} \tilde{\psi}_i}$$

$$S_\theta(\theta) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j \in N_i} \left(1 - \cos(\theta_j - \theta_i) \right)$$

$$(3):\ \phi_i(\theta_i) = 1 - \cos\theta_i$$

$$\Rightarrow \tilde{\phi}_i(\theta_i) = \sum_{j \in N_i} \left(1 - \cos(\theta_j - \theta_i) \right)$$

$$\Rightarrow \underline{S_\theta(\theta) = \frac{1}{2} \sum_{i=1}^{n} \tilde{\phi}_i}$$

We have neglected ZSD until now.
Next, modify the system (11) for satisfying ZSD.

Derivation of Objective Functions

Equilibrium Points : $\alpha_x,\ \alpha_y,\ \alpha_\theta (\neq 0)$

⇒ Control input (12) doesn’t satisfy ZSD.

⇒ Need to modify the system (11)

Approach

Define the following new states and change the system (11) by using them.

$$\begin{cases} z_x = x - \alpha_x \mathbf{1} \\ z_y = y - \alpha_y \mathbf{1} \\ z_\theta = \theta - \alpha_\theta \mathbf{1} \end{cases} \quad \cdots (18)$$

$\mathbf{1} = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}^T \in \Re^n$

Even though we use these virtual states, control inputs and objective functions don’t change. Thus this alternation changes only equilibrium points. So, if we use this alternation, we can apply theorem 3 to pose synchronization.

Review(Passivity-like Property)

Passivity Property

$x_i \in \Re$: state
$v_{xi} \in \Re$: input
$\zeta_{xi} \in \Re$: output
$x = [x_1 \ \cdots \ x_n]^T$
$v_x = [v_{x1} \ \cdots \ v_{xn}]^T$
$\zeta_x = [\zeta_{x1} \ \cdots \ \zeta_{xn}]^T$
$k = [k_{x1} \ \cdots \ k_{xn}]^T$

Consider the simplest system(consider only x_i).

$$\begin{cases} \dot{x}_i = v_{xi} \\ \zeta_{xi} = k_{xi}(x) \end{cases}, \quad i \in \{1, \cdots, n\} \quad \cdots (*)$$

- Let $S(x) \geq 0 \in \Re$ be the storage function of the system (∗).
- Let $v_{xi} = -\zeta_{xi}$ (Output Feedback).

- $S(x) = \frac{1}{2}\|x\|_2^2 \left(\dot{S}(x) \leq v_x \zeta_x \right)$

$$\begin{aligned} \dot{S}(x) &= \dot{x}^T x \\ &= v_x^T x \\ &= -\zeta_x x \\ &\leq -\zeta_x^T \zeta_x \end{aligned}$$

⇨ $\zeta_x = ax, \ (0 \leq a \leq 1)$

$$v_{xi} = -ax_i, \ (0 \leq a \leq 1)$$

- $k_{xi}(x) = \sum_{j \in N_i} (x_i - x_j)$

$$\begin{aligned} \dot{S}(x) &\leq v_x^T \zeta_x \\ &= \dot{x}^T k_x(x) \\ &= \sum_{i=1}^{n} \sum_{j \in N_i} \dot{x}_i (x_i - x_j) \end{aligned}$$

If graph is undirected and connected, then

$$= \frac{1}{2} \sum_{i=1}^{n} \sum_{j \in N_i} (\dot{x}_i - \dot{x}_j)(x_i - x_j)$$

$$S(x) = \frac{a}{4} \sum_{i=1}^{n} \sum_{j \in N_i} (x_i - x_j)^2, \ (0 \leq a \leq 1)$$

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Mechanism Design Problem(Centralized)

Application to Pose Synchronization

System Objective : Achievement of pose synchronization at α_x, α_y, α_θ

System Rule : Having all agents' objective functions J_p, J_θ

Utility : Decision of optimal control inputs (v, ω) for J_p, J_θ and choice of each agent's input (v_i, ω_i)

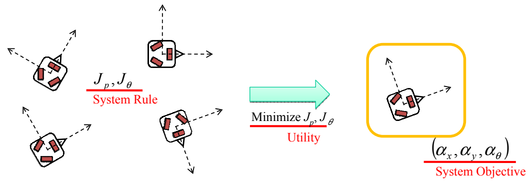

Fig. 7 Pose Synchronization

Mechanism Design Problem(Centralized)

Mechanism Design Problem 1

Consider the system with n rigid bodies represented by (11). Under the assumptions A1, A3 and that each agent chooses its input in accordance with its utility(optimality). And Suppose that each agent knows all agents' states in choosing its control input. Then, design a system rule(J_p, J_θ) such that the designer achieve the system objective(pose synchronization).

Note:
- This problem is not decentralized!! All agents have the same objective functions J_p, J_θ and must know all agents' states.
- All agents must know states of inertial coordinate frame.

⇨ Next, modify the problem

Mechanism Design Problem(Decentralized)

Kinematic Model

$$\begin{bmatrix} \dot{p}_i \\ \theta_i \end{bmatrix} = \begin{bmatrix} R_i & \mathbf{0}_2 \\ \mathbf{0}_2^T & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} \cdots (1)$$

Control Input

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \sum_{j \in N_i} \begin{bmatrix} R_i^{-1}(p_j - p_i) \\ \sin(\theta_j - \theta_i) \end{bmatrix} \cdots (5)$$

Output

$$\begin{cases} \zeta_{pi} = k_{pi}(p) \\ \zeta_{\theta i} = k_{\theta i}(\theta) \end{cases}, i \in \{1, \cdots, n\}$$

$$\begin{cases} k_{pi}(p) = -\sum_{j \in N_i} R_i^{-1}(p_j - p_i) \\ k_{\theta i}(\theta) = -\sum_{j \in N_i} \sin(\theta_j - \theta_i) \end{cases}, i \in \{1, \cdots, n\}$$

for the inputs

⇨ Theorem 3(for each agent)

$$\begin{cases} S_{pi}(p) = \frac{1}{2} \sum_{j \in N_i} \|p_j - p_i\|_2^2 = \tilde{\psi}_i(p) \\ J_{pi} = \int_0^\infty \left(\left\| \sum_{j \in N_i} R_i^{-1}(p_j - p_i) \right\|_2^2 + \|v_i\|_2^2 \right) dt \end{cases}, \begin{cases} S_{\theta i}(\theta) = \sum_{j \in N_i} (1 - \cos(\theta_j - \theta_i)) = \tilde{\phi}_i(\theta) \\ J_{\theta i} = \int_0^\infty \left(\left(\sum_{j \in N_i} \sin(\theta_j - \theta_i) \right)^2 + \|\omega_i\|_2^2 \right) dt \end{cases}, i \in \{1, \cdots, n\}$$

$S_{pi}(.), S_{\theta i}(.) \geq 0 \in \Re$: storage function for agent i

$J_{pi}(.), J_{\theta i}(.) \geq 0 \in \Re$: objective function

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Mechanism Design Problem(Decentralized)

Calculation of The Optimal Control

$$\begin{cases} J_{pi} = \int_0^\infty \left(\left\| \sum_{j \in N_i} R_i^{-1}(p_j - p_i) \right\|_2^2 + \|v_i\|_2^2 \right) dt \\ J_{\theta i} = \int_0^\infty \left(\left(\sum_{j \in N_i} \sin(\theta_j - \theta_i) \right)^2 + \|\omega_i\|_2^2 \right) dt \end{cases}, i \in \{1, \cdots, n\}$$

Difficulty: Each agent can't know about the evolution of the states of its neighbors

⇨ Naïve Assumption(the same necessity as [6])

Assumptions

A4 : When each agent calculates the optimal control input, the states of its neighbors are constant over the planning.

⇨ Use Receding Horizon Control

Mechanism Design Problem(Decentralized)

Mechanism Design Problem 2

Consider the system with n rigid bodies represented by (1). Under the assumptions A1, A3, A4 and that each agent chooses its input in accordance with its utility(optimality). Then, design a system rule(J_{pi}, $J_{\theta i}$) such that the designer achieve the system objective(pose synchronization).

Note: •The amount of necessary information for this problem is the same as one for the pose synchronization.(only relative states of neighbors and oneself)
•Assumption A4 and results of this problem(J_{pi}, $J_{\theta i}$) are the same as the problem in [6].

Application : rendezevous(for information exchanging, battery charge)

Other Consensus Protocol

Consensus Problem : D. Bauso et al. [6] (Last Seminar)

The same results !!

Condition
- Information graph is undirected and connected
- u_i minimizes J_i

In [6], use the Pontryagin minimum principle

design ➡ $l_x(x)$	calculate ➡ $v_{xi}(x)$	α_x	Convergent value
$\left(\sum_{j\in N_i}(x_j-x_i)\right)^2$	$\sum_{j\in N_i}(x_j-x_i)$	$\frac{1}{n}\sum_i x_i(0)$	Arithmetic Mean
$\left(x_i\sum_{j\in N_i}(x_j-x_i)\right)^2$	$x_i\sum_{j\in N_i}(x_j-x_i)$	$\sqrt[n]{\prod_i x_i(0)}$	Geometric Mean
$\left(x_i^2\sum_{j\in N_i}(x_j-x_i)\right)^2$	$-x_i^2\sum_{j\in N_i}(x_j-x_i)$	$\frac{n}{\sum_i \frac{1}{x_i(0)}}$	Harmonic Mean
$\left(\frac{1}{2x_i}\sum_{j\in N_i}(x_j-x_i)\right)^2$	$\frac{1}{2x_i}\sum_{j\in N_i}(x_j-x_i)$	$\sqrt{\frac{1}{n}\sum_i x_i^2(0)}$	Mean of Order 2

Outline

- Preliminary, Introduction, Objective
- Introduction to the results of Literature[11]
- Pose Synchronization
  - Some Results
  - Passivity-like Property
- Application of Mechanism Design
  - Derivation of Objective Functions
  - Mechanism Design Problem
    - Centralized
    - Decentralized
- Conclusion and Future Work

Conclusion and Future Work

Conclusion

- Introduced results of the literature [11]
- Applied mechanism design to pose synchronization
- Get the same results about position as literature [6] from other approach

Future Work

- Weakening assumptions(graph structure, naïve assumption)
- Application of mechanism design to various consensus problem
- Research of studies of other area
- Experiment of robust control with a wheeled inverted pendulum

✓Verification experiment of Azwirman's study of coverage problems

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