Visual-feedback based Attitude Synchronization with Visibility Maintenance and Experimental Validation

Takahide Goto

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Outline

- Introduction
  - Cooperative Control
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Cooperative Control

- Objective
  - Achieving specified tasks in multi-agent systems
- Motivation
  - Interests in group behavior of animals
  - Engineering applications (Sensor network etc.)
- Major problems
  - Consensus problem
  - Flocking problem
  - Synchronization problem
    - Attitude Synchronization
    - Pose Synchronization
    - Coverage problem

Earlier Studies

- In the interest of the easiness of implementation and the simulation of animals, the control laws using the relative information of neighbors are studied intensively.
- But, early in the study, the method of information acquisition is not studied sufficiently.

Earlier Studies

- Recently, vision-based information acquisition have been studied intensively.
- Problems of the earlier studies
  - They rely on only the image-recognition techniques, and they use only information obtained directly from camera image
  - They have not considered dynamical characteristics of the camera and the target sufficiently

In this study, We design the visual observer to estimate the position and the attitude of the target object.
Outline

- Introduction
- Setup
  - Kinematics of Robot
  - Information Graph Structure
  - Attitude Synchronization
- Visual Observer
- Visibility Maintenance
- Simulation
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Kinematics of Omni-directional Robot

\[
\begin{bmatrix}
\dot{x}_i \\
\dot{y}_i \\
\dot{\theta}_i
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_i - \sin \theta_i & 0 & \psi_i \\
\sin \theta_i & \cos \theta_i & 0 \\
0 & 0 & 1
\end{bmatrix} \\
\begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix}
\]

\(i = 1, 2, \ldots, n\)

\(\{x_i, y_i\} \) Position
\(\theta_i \) Rotation Angle
\(\nu_i, \omega_i \) Body Velocity
\(N_i \) Set of Neighbors

Information Graph Structure

- Information graph structure is expressed by the digraph, and there are some classes of them:
  - Fixed
    - Topology of the graph never changes
  - Strongly connected
    - The graph contains a directed path from \(i\) to \(j\) and a directed path from \(j\) to \(i\) for every pair of agents \(i, j\)

Attitude Synchronization

- Goal
  To achieve attitude synchronization of the system with \(n\) agents:
  \(\psi_i = \psi_j, \lim_{t \to \infty} (\theta_i - \theta_j) = 0 \quad \forall i, j\)

- Proposition
  Under the following assumptions, this control input achieves the attitude coordination:

\[
\begin{aligned}
\nu_i &= \nu_j \\
\omega_i &= \sum_{j \in N_i} \sin(\theta_i - \theta_j) \quad \forall i
\end{aligned}
\]

Control Input:

- \(\theta_i(0) \leq \frac{\pi}{2} \quad \forall i\)
- Information graph is fixed and strongly connected

To achieve attitude synchronization, the neighbors' relative attitude is required

Formulation

- Relative Position and Attitude of \(\Sigma_b\) from \(\Sigma_a\) \((\psi_{ab}, \theta_{ab})\):

\[
\begin{bmatrix}
\psi_{ab} \\
\theta_{ab}
\end{bmatrix} := \\
\begin{bmatrix}
\cos \theta_{ab} - \sin \theta_{ab} \\
\sin \theta_{ab} \\
\theta_{ab}
\end{bmatrix}
\]

- Using \((\psi_{ab}, \theta_{ab})\), body velocity of \(\Sigma_b\) from static frame \(\Sigma_a\) is expressed by the following equation:

\[
V_{ab} := \\
\begin{bmatrix}
\nu_{ab} \\
\omega_{ab}
\end{bmatrix} = \\
\begin{bmatrix}
R_{ab}^T \psi_{ab} \\
(R_{ab}^T R_{ab}) \gamma
\end{bmatrix}
\]

- Wedge: \(\Lambda := \begin{bmatrix} 0 & -\omega_z \\ \omega_z & 0 \end{bmatrix} = W \omega\)

- Vee: Inverse Operator to \(\Lambda\)
**Formulation**

In this section, assume that there are an agent mounted with camera (Camera), and an agent with feature points (Object) on the field.

- Relative position and attitude of the object from camera frame:
  \[ p_{CO} = R_{EO}^{T}(p_{WO} - p_{WE}) \]
  \[ R_{CO} = R_{EO}^{T}R_{WO} \]

**Perspective Projection**

- Perspective projection is defined by the following equation:
  \[ f = \frac{\lambda x}{y} \]

- The reason why the image plane is 1D:
  In the inverted pendulum or the vehicle, the vertical coordinate is affected by the uncontrolled motions.

**Visual Observer**

- RRBM: Relative Rigid Body Motion
- EsRRBM: Estimated Relative Rigid Body Motion
- Image Jacobian: Matrix describing the relationship between the image information error and the estimation error

**Physical Meaning of RRBM**

- RRBM calculates the relative velocity of object from camera frame:
  \[ \dot{v} = v_{wo} - R_{EO}^{T}W_{eo}p_{we} \]
  \[ \dot{\omega} = \omega_{we} + \omega_{we}' \]

**RRBM**

- RRBM represents the relation between the body velocities of three coordinate frames.
- It is calculated from temporal differentiation of \((p, R)\):
  \[ V = \begin{bmatrix} R_{EO}^{T}\dot{p} \\ (R_{EO}^{T}R)\dot{R} \end{bmatrix} = \begin{bmatrix} R_{EO}^{T} & R_{EO}^{T}W_{eo}p \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{wc} \\ v_{eo} \end{bmatrix} \]

**Estimated RRBM**

- RRBM in the observer:
  \[ \dot{\hat{V}} = \begin{bmatrix} R_{EO}^{T}\hat{\dot{p}} \\ (R_{EO}^{T}R)\hat{\dot{R}} \end{bmatrix} = \begin{bmatrix} R_{EO}^{T} & R_{EO}^{T}W_{eo}p \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{v}_{wc} \\ \hat{v}_{eo} \end{bmatrix} \]

The new input \(u_{e}\) is to be determined in order to drive the estimated values to their actual values.
Camera Model

- Assume that the target has \( m \) feature points
- The position of feature point \( i \) in the object frame:
  \[
  \mathbf{p}_{oi} = \begin{bmatrix} x_{oi} \\ y_{oi} \end{bmatrix} \quad i = 1, 2, \ldots, m
  \]
- The relative position of feature point \( i \) in the camera frame:
  \[
  \mathbf{p}_{ci} = \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} = \mathbf{R} \mathbf{p}_{oi} + \mathbf{p} \quad i = 1, 2, \ldots, m
  \]

Image Jacobian

- The relation between all the feature points on the image and the estimation error is expressed as
  \[
  \mathbf{f} - \mathbf{f} = J \mathbf{e}_e
  \]
  \[
  J := \begin{bmatrix} \frac{x_{ci}}{y_{ci}} - \frac{x_{pe}}{y_{pe}} \\ \frac{y_{ci}}{y_{ci}} - \frac{y_{pe}}{y_{pe}} \end{bmatrix} \mathbf{R} \begin{bmatrix} 1 & W \mathbf{p}_{oi} \end{bmatrix}
  \]
  \( J \): Image Jacobian
- If Image Jacobian \( J \) is full column rank, the estimation error can be uniquely defined by the image feature vector:
  \[
  \mathbf{e}_e = J^T (\mathbf{f} - \mathbf{f}) = (J^T J)^{-1} J^T (\mathbf{f} - \mathbf{f})
  \]

Estimation Error System and Feedback

- The estimation error system is calculated from temporal differentiation of \((\mathbf{p}_{pe}, \mathbf{R}_{pe})\):
  \[
  \mathbf{V}_{ee} = \begin{bmatrix} \mathbf{R}_{pe}^T \mathbf{p}_{pe} \\ \mathbf{R}_{pe}^T \mathbf{R}_{pe} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{pe}^T \mathbf{R}_{pe} W \mathbf{p}_{pe} \\ 0 \end{bmatrix} \mathbf{u}_e + \mathbf{V}_{eso}
  \]
- Based on passivity of the visual observer, we consider the following control law
  \[
  \mathbf{u}_e = \mathbf{K}_v \mathbf{e}_e
  \]
  \( \mathbf{K}_v := \text{diag} \{ k_{v1}, k_{v2}, k_{v3} \} > 0 \) (Positive Definite)
- If \( V_e = 0 \), the equilibrium point \( \mathbf{e}_e = 0 \) for the visual observer is asymptotic stable

Visual-Feedback Cooperative Control System

- Introduction
- Setup
- Visual Observer

Visibility Maintenance
- Observable Range of Camera
- Artificial Potential
- New Control Law
- Simulation
- Experiment
- Summary
Actually, the range where a camera can be observed is limited.

**Angle of View**
- The angular extent of a given scene that is imaged by a camera.
- It depends on the focal length and the size of imaging area.

**Maximum Distance**
- The distance extent where feature points can be detected.
- It depends on the optical performance of camera and image acquisition algorithm.

If the observed object departs from the observable range of camera, no longer its position and attitude cannot be observed.

It is required to control the agent mounted with camera to maintain visibility of observed object.

With the artificial potential, I propose a new control law.

**Angle of View**
- Bearing $\beta$ is the relative angle between $p$ and optical axis of camera (y-axis of camera frame):
  \[ \beta = \tan^{-1}(y, x) - \frac{\pi}{2} \]
- In order to keep the observed object in the angle of view, following condition must be satisfied:
  \[ |\beta| < R \]

**Maximum Distance**
- $|p|$ is distance between the observed object and the origin of camera frame.
- In order to keep the distance less than maximum distance, following condition must be satisfied:
  \[ |p| < D \]
Control Law for Visibility Maintenance

- New control law for attitude synchronization and visibility maintenance:

\[ V_i = k_{e_i} \sum_{j \in N_i} \sin(\theta_j - \theta_i) \]

- Attitude synchronization input

\[ - \sum_{j \in N_i} K_{\beta j} \frac{\partial U_{\beta j}}{\partial \beta} \frac{\partial \beta_j}{\partial t} \]

- Visibility maintenance input (bearing) based on gradient of \( U_{\beta} \)

\[ - \sum_{j \in N_i} K_{pd j} \frac{\partial U_{pd j}}{\partial d} \frac{\partial d_j}{\partial t} \]

- Visibility maintenance input (distance) based on gradient of \( U_d \)

\( \beta > 0 \)

\( \beta < 0 \)

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Setup

- There are 3 agents on the field (Agent 0: Leader / Agent 1, 2: Followers)

(a)

(b)

- Observable Range of Camera is as follows:
  - Angle of View:
    \( R = \pi / 4 \) [rad]
    \( r = \pi / 8 \) [rad]
  - Maximum Distance:
    \( D = 0.75 \) [m]
    \( d = 0.60 \) [m]

Control Input

- Agent 0:

\[ V_0 = \begin{bmatrix} v_e \\ \dot{A} \cos \omega t \end{bmatrix} \]

- Agent 1, 2:

\[ V_i = k_{e_i} \sum_{j \in N_i} \sin(\theta_j - \theta_i) = \sum_{j \in N_i} K_{\beta j} \frac{\partial U_{\beta j}}{\partial \beta} \frac{\partial \beta_j}{\partial t} - \sum_{j \in N_i} K_{pd j} \frac{\partial U_{pd j}}{\partial d} \frac{\partial d_j}{\partial t} \]

\( \theta_{ab} \) : Estimated relative attitude of agent a from agent b

To test the effectiveness of visibility maintenance, compare the results of control laws with and without visibility maintenance.
Attitude synchronization has been achieved with a slight delay, but bearings of 0th agent from 1st and 2nd agent sometimes deviated the angle of view.

Attitude synchronization has been achieved with a slight delay virtually, and bearings of 0th agent from 1st and 2nd agent were kept in the angle of view.
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Attitude synchronization has been achieved with a slight delay virtually, and bearings of 0th agent from 1st and 2nd agent were kept in the angle of view.
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- Introduction
- Setup
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- Visibility Maintenance
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- Experiment
  - Experiment System
  - Kinematics of Two-wheel Robot
- Results
- Summary

Experiment System

- Onboard Camera Feature Points

Kinematics of Two-wheel Robot

\[
\begin{bmatrix}
\dot{x}_i \\
\dot{y}_i \\
\dot{\theta}_i \\
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_i & 0 & v_i \\
\sin \theta_i & 0 & \omega_i \\
0 & 1 & 0 \\
\end{bmatrix}
\]

Constraint: \( \dot{x} \sin \theta - \dot{y} \cos \theta = 0 \)

Modification of Control Law

- Because of constraint, the control law must be modified
- Remove the drift velocity term from the control input

\[
V_i = k_{ei} \sum_{j \in N_i} \frac{\theta_i - \theta_j}{\sin(\theta_j - \theta_i)} - \sum_{j \in N_i} k_{ji} \frac{\theta_{ij} \dot{\theta}_j}{\theta_{ij}} - \sum_{j \in N_i} k_{pi} \frac{\theta_{ij} \dot{\theta}_j}{\theta_{ij}}
\]

Setup

- There are 3 agents on the field (Agent 0: Leader / Agent 1, 2: Followers)
- Observable Range of Camera is as follows:
  - Angle of View:
    \[ R = \frac{\pi}{6} \text{ (rad)} \]
    \[ \alpha = 2 \pi / 15 \text{ (rad)} \]
  - Maximum Distance:
    \[ D = 0.75 \text{ [m]} \]
    \[ d = 0.60 \text{ [m]} \]
Experiment Result

**Attitude synchronization problem**
- Achievement of attitude synchronization requires neighbors’ information
- Visual observer
  - By the use of visual observer, the position and the attitude of the observed agent can be estimated
  - The estimation error feedback system has asymptotic stability
- Visibility Maintenance
  - The observable range of camera is determined by the optical factors and the image acquisition algorithm
  - The new control law based on artificial potential keeps the observed object in the observable range

**Visual observer**
- By the use of visual observer, the position and the attitude of the observed agent can be estimated
- The estimation error feedback system has asymptotic stability

**Visibility Maintenance**
- The observable range of camera is determined by the optical factors and the image acquisition algorithm
- The new control law based on artificial potential keeps the observed object in the observable range

**Simulation**
- We confirmed the effectiveness of visual observer
- Attitude synchronization is achieved by use of the estimates of visual observer
- With the new control law, attitude synchronization and visibility maintenance are both achieved

**Experiment**
- Because two-wheel robot has a constraint of velocity, the control law is modified
- The experiments show that the effectiveness of visual observer and visibility maintenance
Future Works

- Proof of convergence
- Simulation of many agents
- Experiments under many circumstances