Visual-feedback based Attitude Synchronization with Visibility Maintenance and Experimental Validation

Takahide Goto

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Cooperative Control

- Objective
  - Achieving specified tasks in multi-agent systems
- Motivation
  - Interests in group behavior of animals
  - Engineering applications (Sensor network etc.)
- Major problems
  - Consensus problem
  - Flocking problem
  - Synchronization problem
    - Attitude Synchronization
    - Pose Synchronization
  - Coverage problem

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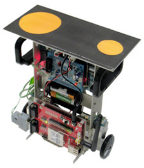

Two-wheel Robot

Earlier Studies

- In the interest of the easiness of implementation and the simulation of animals, the control laws **using the relative information of neighbors** are studied intensively
- But, early in the study, the method of information acquisition is not studied sufficiently

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Earlier Studies

- Recently, **vision-based** information acquisition have been studied intensively
- Problems of the earlier studies
  - They rely on only the image-recognition techniques, and they use only **information obtained directly from camera image**
  - They have not considered dynamical characteristics of the camera and the target sufficiently

In this study, We design the **visual observer** to estimate the position and the attitude of the target object

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- Introduction
- Setup
  - Kinematics of Robot
  - Information Graph Structure
  - Attitude Synchronization
- Visual Observer
- Visibility Maintenance
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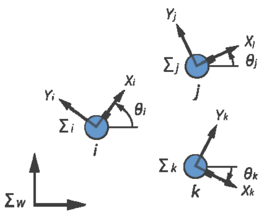
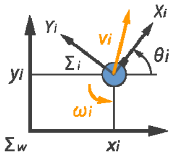
Kinematics of Omni-directional Robot

$$\begin{bmatrix}\dot{x}_i\\ \dot{y}_i\\ \dot{\theta}_i\end{bmatrix} = \begin{bmatrix}\cos\theta_i & -\sin\theta_i & 0\\ \sin\theta_i & \cos\theta_i & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}v_i\\ \omega_i\end{bmatrix}$$

$$i = 1, 2, \ldots, n$$



(x_i, y_i) : Position
θ_i : Rotation Angle
v_i, ω_i : Body Velocity
N_i : Set of Neighbors

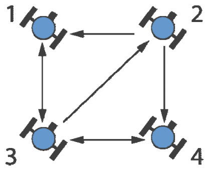


Information Graph Structure

- Information graph structure is expressed by the digraph, and there are some classes of them:
  - **Fixed**

    Topology of the graph never changes
  - **Strongly connected**

    The graph contains a directed path from *i* to *j* and a directed path from *j* to *i* for every pair of agents *i, j*



Attitude Synchronization

- Goal

  To achieve attitude synchronization of the system with *n* agents:

$$v_i = v_j, \quad \lim_{t\to\infty}(\theta_i - \theta_j) = 0 \quad \forall i, j$$

- Proposition

  Under the follower assumptions, this control inputs achieve the attitude coordination:

Control Input:

Relative Attitude of neighbor

$$\begin{cases} v_i = v_c \\ \omega_i = k_i \sum_{j\in N_i} \sin\boxed{(\theta_j - \theta_i)} \end{cases} \quad \forall i$$

Assumptions:

- $|\theta_i(0)| \le \frac{\pi}{2} \quad \forall i$
- Information graph is fixed and strongly connected

To achieve attitude synchronization, **the neighbors' relative attitude is required**

Outline

- Introduction
- Setup
- Visual Observer
  - Formulation
  - Components
  - Estimation Error System
- Visibility Maintenance
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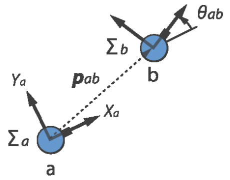
Formulation

- Relative Position and Attitude of Σ_b from Σ_a ($\boldsymbol{p}_{ab}$, R_{ab}):

$$p_{ab} := \begin{bmatrix}x_{ab}\\ y_{ab}\end{bmatrix}$$

$$R_{ab} := \begin{bmatrix}\cos\theta_{ab} & -\sin\theta_{ab}\\ \sin\theta_{ab} & \cos\theta_{ab}\end{bmatrix}$$



- Using ($\boldsymbol{p}_{ab}$, R_{ab}), body velocity of Σ_b from static frame Σ_a is expressed by the following equation:

$$V_{ab} := \begin{bmatrix}v_{ab}\\ \omega_{ab}\end{bmatrix} = \begin{bmatrix}R_{ab}^T\dot{p}_{ab}\\ (R_{ab}^T\dot{R}_{ab})^\vee\end{bmatrix}$$

- Wedge $\wedge$:

$$\hat{\omega} := \begin{bmatrix}0 & -\omega\\ \omega & 0\end{bmatrix} = W\omega$$

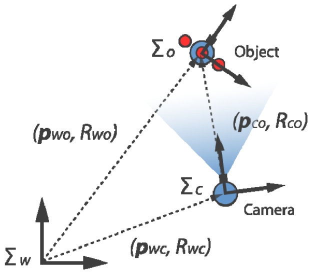
$$W := \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} (= R_{\frac{\pi}{2}})$$

- Vee $\vee$: Inverse Operator to $\wedge$

Formulation

- In this section, assume that there are an agent mounted with camera (Camera), and an agent with feature points (Object) on the field
- Relative position and attitude of the object from camera frame:

$$\boldsymbol{p}_{co} = R_{wc}^T(\boldsymbol{p}_{wo} - \boldsymbol{p}_{wc})$$
$$R_{co} = R_{wc}^T R_{wo}$$



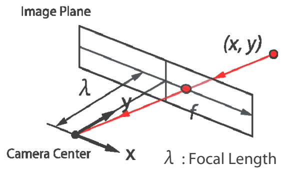
Perspective Projection

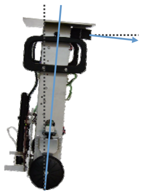
- Perspective projection is defined by the following equation:

$$f = \frac{\lambda}{y}x$$

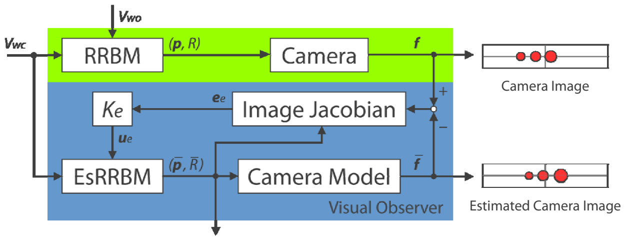
- The reason why the image plane is 1D:

  In the inverted pendulum or the vehicle, the vertical coordinate is affected by the uncontrolled motions





Visual Observer



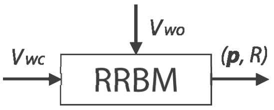
Block Diagram of Visual Observer

RRBM: Relative Rigid Body Motion
EsRRBM: Estimated Relative Rigid Body Motion
Image Jacobian: Matrix describing the relationship between the image information error and the estimation error

RRBM

- RRBM represents the relation between the body velocities of three coordinate frames
- It is calculated from temporal differentiation of (***p***, *R*) :

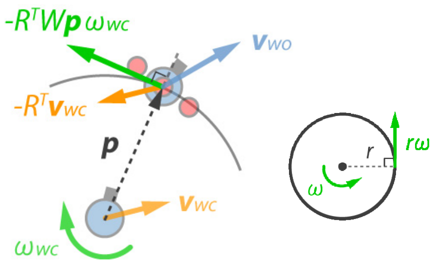
$$\boxed{V = \begin{bmatrix} R^T\dot{\boldsymbol{p}} \\ (R^T\dot{R})^\vee \end{bmatrix} = -\begin{bmatrix} R^T & R^T W\boldsymbol{p} \\ 0 & 1 \end{bmatrix} V_{wc} + V_{wo}}$$



Physical Meaning of RRBM

- RRBM calculates the relative velocity of object from camera frame:

$$\boldsymbol{v} = \boldsymbol{v}_{wo} \boxed{- R^T\boldsymbol{v}_{wc}} \boxed{- R^T W\boldsymbol{p}\omega_{wc}}$$
$$\omega = \omega_{wo} \boxed{- \omega_{wc}}$$



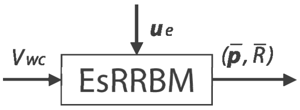
Estimated RRBM

- RRBM in the observer:

$$\boxed{\bar{V} = \begin{bmatrix} \bar{R}^T\dot{\bar{\boldsymbol{p}}} \\ (\bar{R}^T\dot{\bar{R}})^\vee \end{bmatrix} = -\begin{bmatrix} \bar{R}^T & \bar{R}^T W\bar{\boldsymbol{p}} \\ 0 & 1 \end{bmatrix} V_{wc} + \boldsymbol{u}_e}$$

$(\bar{\boldsymbol{p}}, \bar{R})$: Estimated position and attitude of object from camera frame
$\bar{V}$: Estimated body velocity of object from camera frame

The new input $\boldsymbol{u}_e$ is to be determined in order to drive the estimated values to their actual values



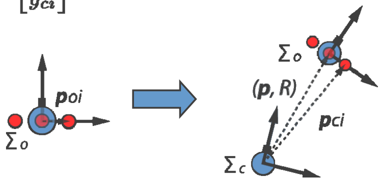
Camera Model

- Assume that the target has *m* feature points
  - The position of feature point *i* in the object frame:

$$\boldsymbol{p}_{oi} = \begin{bmatrix} x_{oi} \\ y_{oi} \end{bmatrix} \qquad i = 1, 2, \ldots, m$$

  - The relative position of feature point *i* in the camera frame:

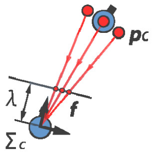
$$\boldsymbol{p}_{ci} = \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} = R\boldsymbol{p}_{oi} + \boldsymbol{p} \qquad i = 1, 2, \ldots, m$$



Camera Model

- Perspective projection of $\boldsymbol{p}_{ci}$:

$$f_i = \frac{\lambda}{y_{ci}} x_{ci} \qquad \boldsymbol{p}_c := \begin{bmatrix} \boldsymbol{p}_{c1} \\ \vdots \\ \boldsymbol{p}_{cm} \end{bmatrix} \qquad \boldsymbol{f} := \begin{bmatrix} f_1 \\ \vdots \\ f_m \end{bmatrix}$$



- Estimated camera model is also almost the same as camera model

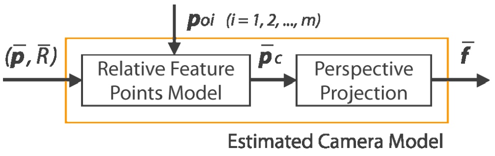


Image Jacobian

- The relation between all the feature points on the image and the estimation error is expressed as

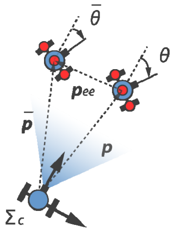
$$\boldsymbol{f} - \bar{\boldsymbol{f}} = J\boldsymbol{e}_e$$

$$J_i := \begin{bmatrix} \frac{\lambda}{\bar{y}_{ci}} & -\frac{\lambda \bar{x}_{ci}}{\bar{y}_{ci}^2} \end{bmatrix} \bar{R} \begin{bmatrix} I & W\boldsymbol{p}_{oi} \end{bmatrix}$$

J : Image Jacobian

- If Image Jacobian *J* is full column rank, the estimation error can be uniquely defined by the image feature vector :

$$\boxed{\begin{aligned} \boldsymbol{e}_e &= J^{+}(\boldsymbol{f} - \bar{\boldsymbol{f}}) \\ &= (J^T J)^{-1} J^T (\boldsymbol{f} - \bar{\boldsymbol{f}}) \end{aligned}}$$



$$\boldsymbol{e}_e := \begin{bmatrix} \boldsymbol{p}_{ee} \\ e_R(R_{ee}) \end{bmatrix}$$

$$\boldsymbol{p}_{ee} := \bar{R}^T(\boldsymbol{p} - \bar{\boldsymbol{p}})$$

$$R_{ee} := \bar{R}^T R$$

Estimation Error System and Feedback

- The estimation error system is calculated from temporal differentiation of ($\boldsymbol{p}_{ee}$, R_{ee}) :

$$\boxed{V_{ee} = \begin{bmatrix} R_{ee}^T \dot{\boldsymbol{p}}_{ee} \\ (R_{ee}^T \dot{R}_{ee})^{\vee} \end{bmatrix} = -\begin{bmatrix} R_{ee}^T & R_{ee}^T W \boldsymbol{p}_{ee} \\ 0 & 1 \end{bmatrix} \boldsymbol{u}_e + V_{wo}}$$

- Based on passivity of the visual observer, we consider the following control law

$$\boldsymbol{u}_e = K_v \boldsymbol{e}_e$$

$$K_v := \mathrm{diag}\{k_{v1}, k_{v2}, k_{v3}\} > 0 \quad \text{(Positive Definite)}$$

- If V_o=0, the equilibrium point $\boldsymbol{e}_e$=0 for the visual observer is asymptotic stable

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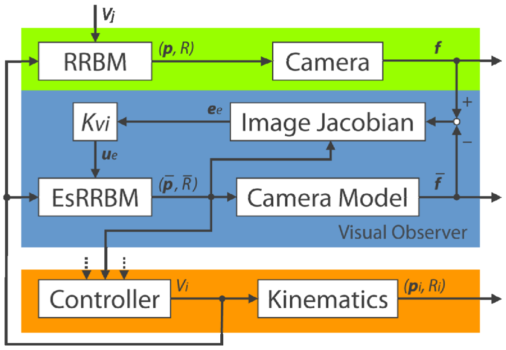
Visual-Feedback Cooperative Control System



Block Diagram of Agent *i*

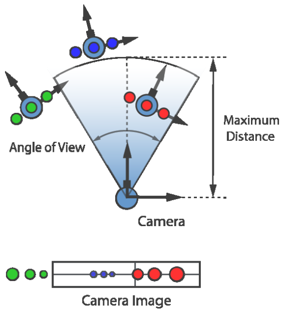
Outline

- Introduction
- Setup
- Visual Observer
- Visibility Maintenance
  - Observable Range of Camera
  - Artificial Potential
  - New Control Law
- Simulation
- Experiment
- Summary

Observable Range of Camera

Actually, the range where a camera can be observed is limited

- Angle of View
  - The angular extent of a given scene that is imaged by a camera
  - It depends on the focal length and the size of imaging area
- Maximum Distance
  - The distance extent where feature points can be detected
  - It depends on the optical performance of camera and image acquisition algorithm

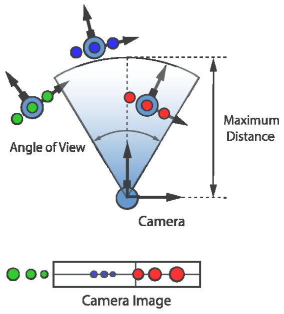


Observable Range of Camera

- If the observed object departs from the observable range of camera, no longer its position and attitude cannot be observed

⬇

- It is required to control the agent mounted with camera to **maintenance visibility** of observed object
- With the artificial potential, I propose a new control law



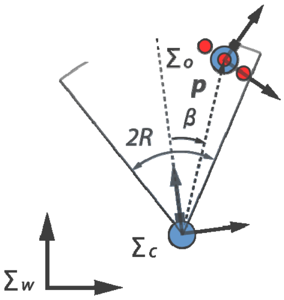
Angle of View

- Bearing β is the relative angle between $\boldsymbol{p}$ and optical axis of camera (y-axis of camera frame):

$$\beta = \mathrm{atan2}(y, x) - \frac{\pi}{2}$$

- In order to keep the observed object in the angle of view, following condition must be satisfied:

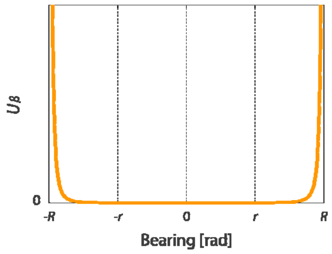
$$|\beta| < R$$

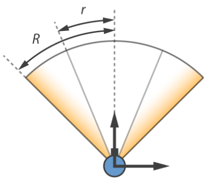


Angle of View

- Artificial potential function of bearing U_β:

$$U_\beta = \left(\min \left(0, \frac{\beta^2 - r^2}{\beta^2 - R^2} \right) \right)^2$$

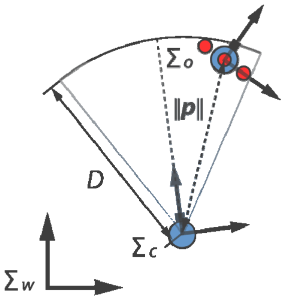




Maximum Distance

- $\|\boldsymbol{p}\|$ is distance between the observed object and the origin of camera frame
- In order to keep the distance less than maximum distance, following condition must be satisfied:

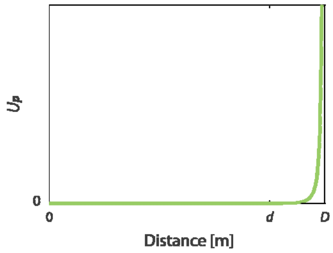
$$\|\boldsymbol{p}\| < D$$

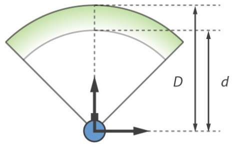


Maximum Distance

- Artificial potential function of distance U_p:

$$U_{\boldsymbol{p}} = \left(\min \left(0, \frac{\|\boldsymbol{p}\|^2 - d^2}{\|\boldsymbol{p}\|^2 - D^2} \right) \right)^2$$





Control Law for Visibility Maintenance

- New control law for attitude synchronization and visibility maintenance:

$$V_i = \begin{bmatrix} v_c \\ k_{ei} \sum_{j \in \mathcal{N}_i} \sin(\theta_j - \theta_i) \end{bmatrix}$$

Attitude synchronization input

$$- \sum_{j \in \mathcal{N}_i} K_{\beta i} \begin{bmatrix} \frac{\partial U_{\beta ij}}{\partial p_{wi}} \\ \frac{\partial U_{\beta ij}}{\partial \theta_{wi}} \end{bmatrix}$$

Visibility maintenance input (bearing)
Based on gradient of *U β*

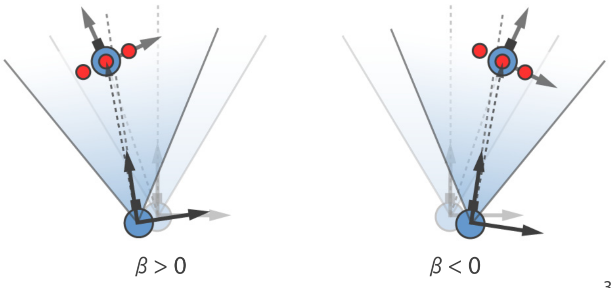
$$- \sum_{j \in \mathcal{N}_i} K_{pi} \begin{bmatrix} \frac{\partial U_{pij}}{\partial p_{wi}} \\ \frac{\partial U_{pij}}{\partial \theta_{wi}} \end{bmatrix}$$

Visibility maintenance input (distance)
Based on gradient of *U**p***

Control Law for Visibility Maintenance

$$- \sum_{j \in \mathcal{N}_i} K_{\beta i} \begin{bmatrix} \frac{\partial U_{\beta ij}}{\partial p_{wi}} \\ \frac{\partial U_{\beta ij}}{\partial \theta_{wi}} \end{bmatrix}$$

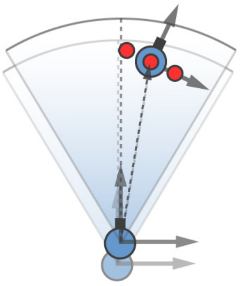
Visibility maintenance input (bearing)
Based on gradient of *U β*



Control Law for Visibility Maintenance

$$- \sum_{j \in \mathcal{N}_i} K_{pi} \begin{bmatrix} \frac{\partial U_{pij}}{\partial p_{wi}} \\ \frac{\partial U_{pij}}{\partial \theta_{wi}} \end{bmatrix}$$

Visibility maintenance input (distance)
Based on gradient of *U**p***



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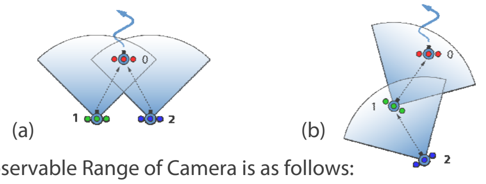
Setup

- There are 3 agents on the field (Agent 0: Leader / Agent 1, 2: Followers)

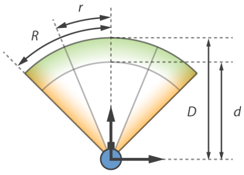


- Observable Range of Camera is as follows:
  - Angle of View:

R = π / 4 [rad]
r = π / 8 [rad]

  - Maximum Distance:

D= 0.75 [m]
d = 0.60 [m]



Setup

- Control Input
  - Agent 0:

$$V_0 = \begin{bmatrix} v_c \\ A \cos \omega t \end{bmatrix}$$

  - Agent 1, 2:

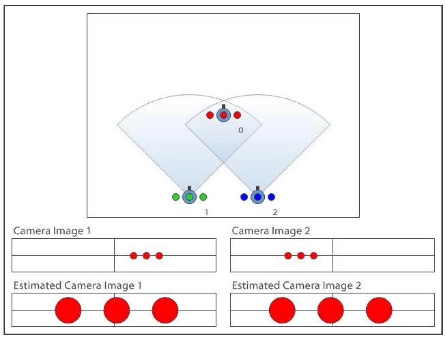
$$V_i = \begin{bmatrix} v_c \\ k_{ei} \sum_{j \in \mathcal{N}_i} \sin \bar{\theta}_{ij} \end{bmatrix} \boxed{- \sum_{j \in \mathcal{N}_i} K_{\beta} \begin{bmatrix} \frac{\partial U_{\bar{\beta} ij}}{\partial p_{wi}} \\ \frac{\partial U_{\bar{\beta} ij}}{\partial \theta_{wi}} \end{bmatrix} - \sum_{j \in \mathcal{N}_i} K_p \begin{bmatrix} \frac{\partial U_{\bar{p} ij}}{\partial p_{wi}} \\ \frac{\partial U_{\bar{p} ij}}{\partial \theta_{wi}} \end{bmatrix}}$$

$\bar{\theta}_{ab}$: Estimated relative attitude
of agent *a* from agent *b*

To test the effectiveness of visibility maintenance, compare the results of control laws with and without visibility maintenance

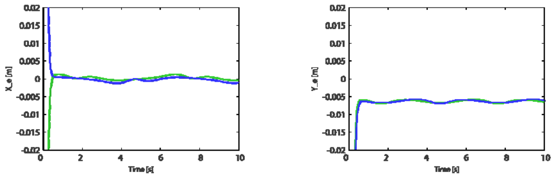
Simulation Result (a)

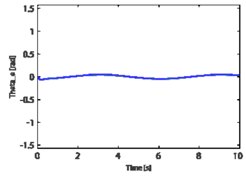
- Without visibility maintenance



Simulation Result (a)

- Without visibility maintenance





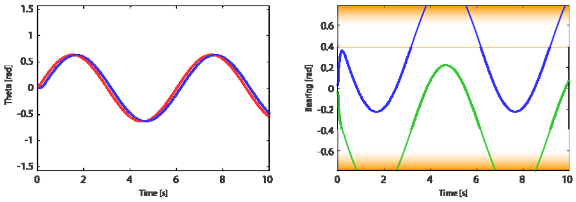
$$\boldsymbol{p}_e := \begin{bmatrix} x_e \\ y_e \end{bmatrix} = \bar{\boldsymbol{p}} - \boldsymbol{p}$$

$$\theta_e := \bar{\theta} - \theta$$

🟩 Agent 1
🟦 Agent 2

Simulation Result (a)

- Without visibility maintenance



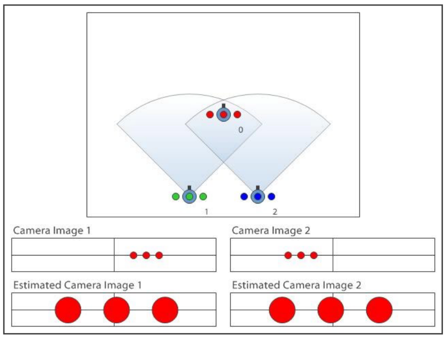
Attitude of Agents　　Bearing of Agents

Attitude synchronization has been achieved with a slight delay, but bearings of 0th agent from 1st and 2nd agent sometimes deviated the angle of view

🟥 Agent 0
🟩 Agent 1
🟦 Agent 2

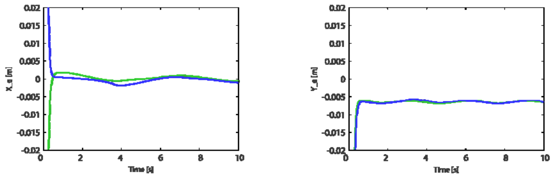
Simulation Result (a)

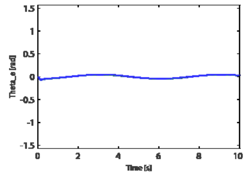
- With visibility maintenance



Simulation Result (a)

- With visibility maintenance





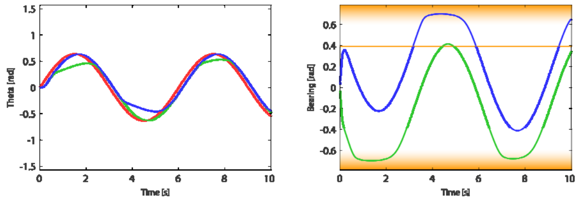
$$\boldsymbol{p}_e := \begin{bmatrix} x_e \\ y_e \end{bmatrix} = \bar{\boldsymbol{p}} - \boldsymbol{p}$$

$$\theta_e := \bar{\theta} - \theta$$

🟩 Agent 1
🟦 Agent 2

Simulation Result (a)

- With visibility maintenance



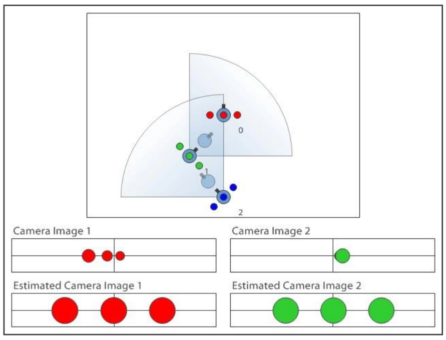
Attitude of Agents　　Bearing of Agents

Attitude synchronization has been achieved with a slight delay virtually, and bearings of 0th agent from 1st and 2nd agent were kept in the angle of view

🟥 Agent 0
🟩 Agent 1
🟦 Agent 2

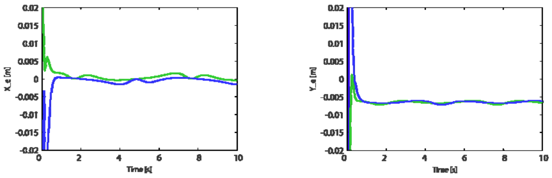
Simulation Result (b)

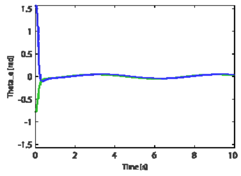
- Without visibility maintenance



Simulation Result (b)

- Without visibility maintenance





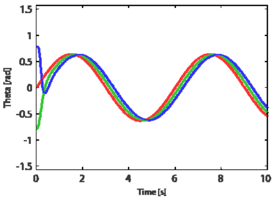
$$p_e := \begin{bmatrix} x_e \\ y_e \end{bmatrix} = \bar{p} - p$$

$$\theta_e := \bar{\theta} - \theta$$

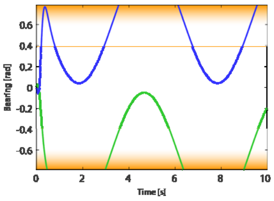
🟩 Agent 1
🟦 Agent 2

Simulation Result (b)

- Without visibility maintenance



Attitude of Agents



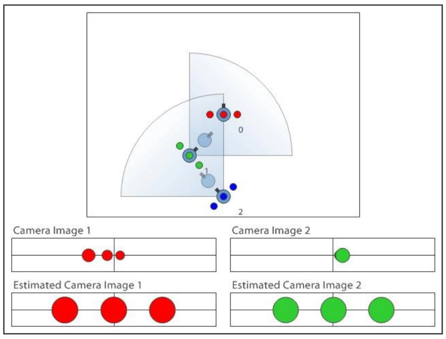
Bearing of Agents

Attitude synchronization has been achieved with a slight delay, but bearings of 0th agent from 1st and 2nd agent sometimes deviated the angle of view

🟥 Agent 0
🟩 Agent 1
🟦 Agent 2

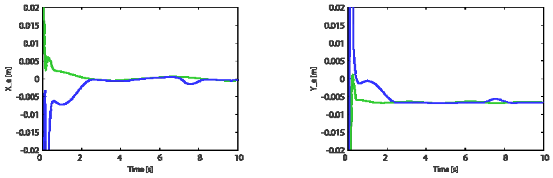
Simulation Result (b)

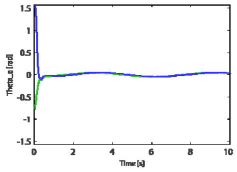
- With visibility maintenance



Simulation Result (b)

- With visibility maintenance





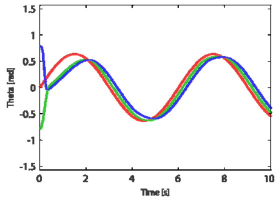
$$p_e := \begin{bmatrix} x_e \\ y_e \end{bmatrix} = \bar{p} - p$$

$$\theta_e := \bar{\theta} - \theta$$

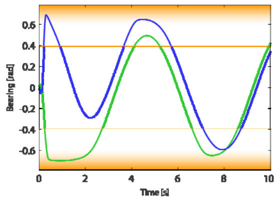
🟩 Agent 1
🟦 Agent 2

Simulation Result (b)

- With visibility maintenance



Attitude of Agents



Bearing of Agents

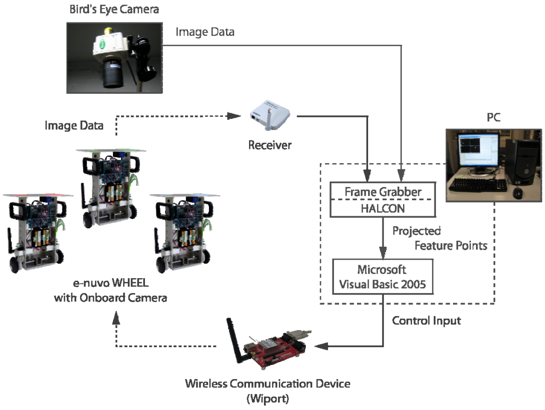
Attitude synchronization has been achieved with a slight delay virtually, and bearings of 0th agent from 1st and 2nd agent were kept in the angle of view

🟥 Agent 0
🟩 Agent 1
🟦 Agent 2

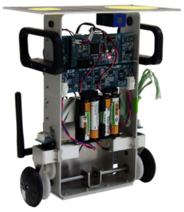
Outline

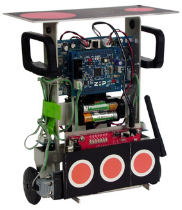
- Introduction
- Setup
- Visual Observer
- Visibility Maintenance
- Simulation
- Experiment
  - Experiment System
  - Kinematics of Two-wheel Robot
  - Results
- Summary

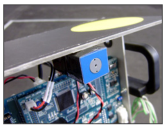
Experiment System



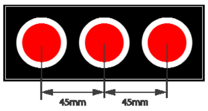
Experiment System







Onboard Camera



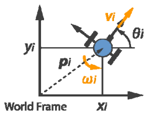
Feature Points

Kinematics of Two-wheel Robot

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}$$

$$i = 1, 2, \ldots, n$$

Constraint: $\dot{x}\sin\theta - \dot{y}\cos\theta = 0$

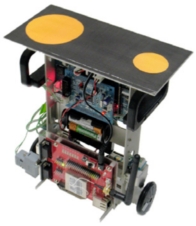


(x_i, y_i) : Position

θ_i : Rotation Angle

v_i, ω_i : Body Velocity

N_i : Set of Neighbors

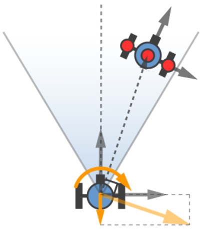


Wheeled Robot
(Mobile Inverted Pendulum)

Modification of Control Law

- Because of constraint, the control law must be modified
- Remove the drift velocity term from the control input

$$V_i = \begin{bmatrix} \overset{v_c}{k_{ei} \sum_{j \in N_i} \sin(\theta_j - \theta_i)} \end{bmatrix} - \sum_{j \in N_i} k_{\beta i} \begin{bmatrix} \frac{\partial U_{\beta ij}}{\partial y_{wi}} \\ \frac{\partial U_{\beta ij}}{\partial \theta_{wi}} \end{bmatrix} - \sum_{j \in N_i} k_{pi} \begin{bmatrix} \frac{\partial U_{pij}}{\partial y_{wi}} \\ \frac{\partial U_{pij}}{\partial \theta_{wi}} \end{bmatrix}$$



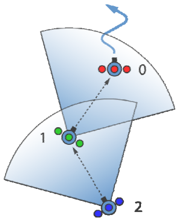
Setup

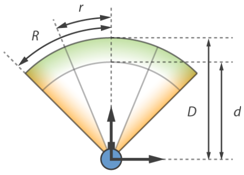
- There are 3 agents on the field
  (Agent 0: Leader / Agent 1, 2: Followers)
- Observable Range of Camera is as follows:
  - Angle of View:

    R = π / 6 [rad]
    r = 2 π / 15 [rad]

  - Maximum Distance:

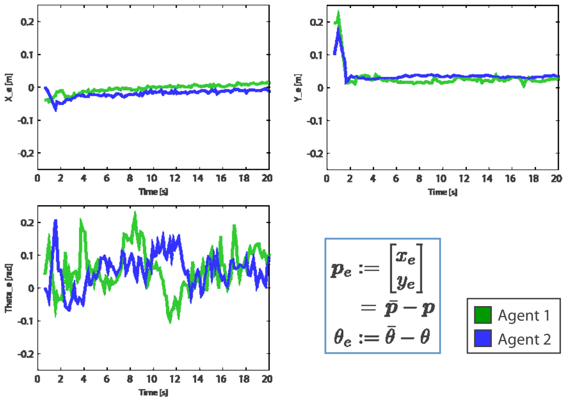
    D= 0.75 [m]
    d = 0.60 [m]





Experiment Result

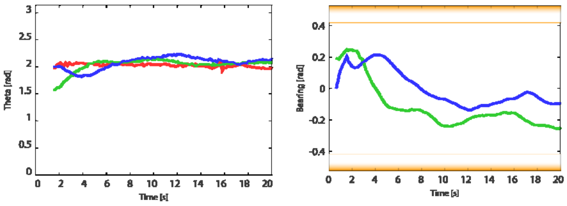
Experiment Result



$$\boldsymbol{p}_e := \begin{bmatrix} x_e \\ y_e \end{bmatrix} = \bar{\boldsymbol{p}} - \boldsymbol{p}$$

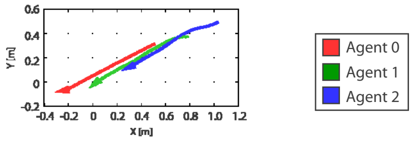
$$\theta_e := \bar{\theta} - \theta$$

Experiment Result



Attitude of Agents

Bearing of Agents



Outline

- Introduction
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- Visual Observer
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Summary

- Attitude synchronization problem
  - Achievement of attitude synchronization requires neighbors' information
- Visual observer
  - By the use of visual observer, the position and the attitude of the observed agent can be estimated
  - The estimation error feedback system has asymptotic stability
- Visibility Maintenance
  - The observable range of camera is determined by the optical factors and the image acquisition algorithm
  - The new control law based on artificial potential keeps the observed object in the observable range

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Summary

- Simulation
  - We confirmed the effectiveness of visual observer
  - Attitude synchronization is achieved by use of the estimates of visual observer
  - With the new control law, attitude synchronization and visibility maintenance are both achieved
- Experiment
  - Because two-wheel robot has a constraint of velocity, the control law is modified
  - The experiments show that the effectiveness of visual observer and visibility maintenance

Future Works

- Proof of convergence
- Simulation of many agents
- Experiments under many circumstances