Distributed Predictive Control of Linear Stochastic System with Information Structures : An Extension to Distributed Kalman Filtering

FL08-22-2
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Outline

Background

- **Background**
  - **In the 1970s**
    - Mounting expectation and demand for control methodology for **Large Scale Systems**
  - **In recent years**
    - **Energy・Environmental Problems** and **Security**
    - **New Applications**

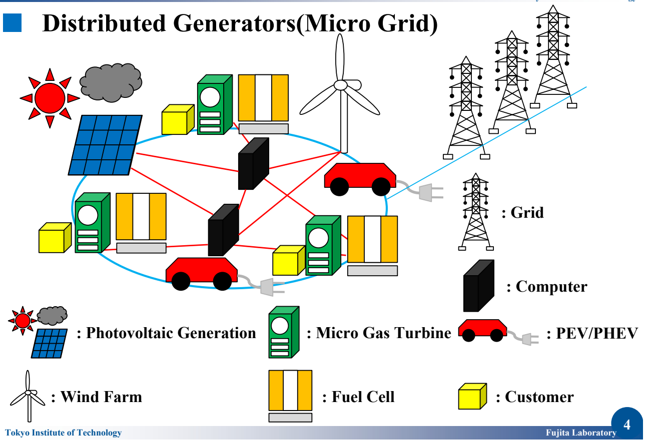

Power Plant
http://gazone.morrie.biz/


Transportation System
http://marukosugi.com/

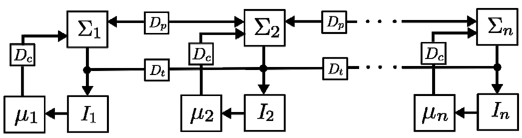

Clean Energy Network
http://www.net.ed.jp/...

Distributed Generators(Micro Grid)

- **Distributed Generators(Micro Grid)**



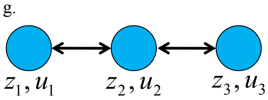
Information Structure with Delays



Σ_i : i th subsystem
D_p : propagation delay
D_t : transmission delay
D_c : computational delay

μ_i : controller of i th subsystem
z_i : state of i th subsystem
u_i : input of i th subsystem
I_i : information that i th subsystem can obtain

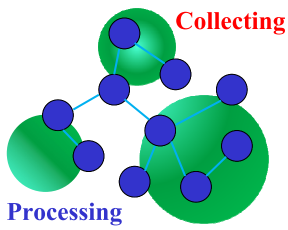
e. g.



$$I_1(t) = (z_1(t),\ z_2(t-1),\ z_3(t-2))$$
$$I_2(t) = (z_1(t-1),\ z_2(t),\ z_3(t-1))$$
$$I_3(t) = (z_1(t-2),\ z_2(t-1),\ z_3(t))$$

Distributed Control

- **Distributed Control**
  Spatio-temporally distributed collecting and processing of information
- **Advantages of Distributed Control**
  - **Scalability**
  - **Fault Tolerance**
  - **Flexibility**
  - **Time Complexity**
  - **Economic Efficiency**



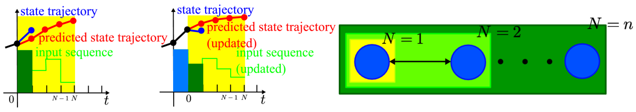
Past Researches of Distributed Control

- **Past Researches**
  - **From 1970s**
    - **Stability**,[21), 22)] **Optimality**[1)] and **Robustness**
  - **In recent years**
    - Involving **recent control theory** [5), 6)]
    - Considering **system structure** [7), 8), 9), 13), 15), 25), 26), 29), 30)]

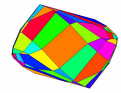
We focus on **Covariance Constraints**[7), 8), 13) 9), 15), 25), 26)] and **Decomposition approach** .

Predictive Control

- **Predictive Control**



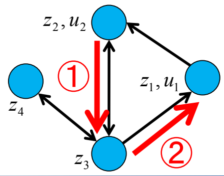
- **Advantages of Predictive Control**
  - **Considering Constraints**
    - **Energy Efficiency**
    - **Ecology**
    - **Security**
  - **Switching Control**



Objective

- **Objective**
  We propose **Distributed Predictive Control Laws** for **Linear Stochastic System** with **Mean** and **Covariance Constraints**.

Note: Mean and **Covariance Constraints** represent **State** and **Input Constraints**, **Power Constraint** and **Delay Structure**.



Outline

Output Feedback Control

- **Problem 1**

$$\min_{u(k)} \mathbf{Tr}\ P_{xx}V_{xx}(x(N)) + \sum_{k=0}^{N-1} \mathbf{Tr}\ QV(x(k),u(k))$$

subject to
$$x(k+1) = Ax(k) + Bu(k) + Fw(k)$$
$$y(k) = Cx(k) + v(k)$$
$$\mathcal{Y}(k) = (y(0), y(1), \dots, y(k))$$
$$u(k) = \mu_k(\mathcal{Y}(k))$$

$t \in \mathcal{Z}_+,\ x(t) \in \mathcal{R}^{n_x}$
$u(t) \in \mathcal{R}^{n_u},\ w(t) \in \mathcal{R}^{n_w},\ v(t) \in \mathcal{R}^{n_v}$ White Noise

$\mathbf{Tr}\ Q_r V(x(k),u(k)) \le \gamma_{kr}$ Covariance Constraints

$\mathbf{E} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in \mathcal{D} \subset \mathcal{R}^{n_x+n_u}$ Mean Constraints

$$\mathbf{Tr}\ QV(x(N), \mu'_N(x(N))) + \mathbf{Tr}\ P_{xx}V_{xx}(x(N+1)) \le \mathbf{Tr}\ P_{xx}V_{xx}(x(N))$$ for Stability

$x(N) \in \mathcal{O}_\infty$ for Feasibility

Centralized Kalman Filtering

- **Problem 2**

$$\min_{K(k)} \mathbf{Tr}\ P_{xx}V_{xx}(\tilde{x}(-N+1)) + \sum_{k=-N_n+1}^{0} \mathbf{Tr}\ Q_{xx}V_{xx}(\tilde{x}(k))$$

subject to
$$x(k+1) = Ax(k) + Fw(k)$$
$$y(k) = Cx(k) + v(k)$$
$$\mathcal{Y}(k) = (y(0), y(1), \dots, y(k))$$
$$\hat{x}(k) = A\hat{x}(k-1) + K(k)(y(k) - CA\hat{x}(k-1))$$
$$\tilde{x}(k) = x(k) - \hat{x}(k)$$ Kalman Gain

$t \in \mathcal{Z}_+,\ x(t) \in \mathcal{R}^{n_x}$
$w(t) \in \mathcal{R}^{n_w},\ v(t) \in \mathcal{R}^{n_v}, Q_{xx} = H^T H \ge 0$ White Noise

$\mathbf{Tr}\ Q_r V(\tilde{x}(k)) \le \tilde{\gamma}_{kr}$ Covariance Constraints

$\mathbf{E}\ \tilde{x}(k) \in \tilde{\mathcal{D}} \subset \mathcal{R}^{n_x}$ Mean Constraints

$$\mathbf{Tr}\ P_{xx}V_{xx}(\tilde{x}(-N+1)) + \sum_{k=-N_n+1}^{0} \mathbf{Tr}\ Q_{xx}V_{xx}(\tilde{x}(k)) \ge \mathbf{Tr}\ P_{xx}V_{xx}(\tilde{x}(0))$$

for Stability

Output Feedback Control

- **Lemma 1(Output Feedback Control)**

  Problem 1 is reduced to **Problem 3**.

- **Problem 3**

$$\min_{u(k)} \mathbf{Tr}\ P_{x}V_{x}(\hat{x}(N)) + \sum_{k=0}^{N-1} \mathbf{Tr}\ QV(\hat{x}(k),u(k))$$

subject to $\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K(k)(y(k) - C\hat{x}(k))$

$\mathcal{I}(k) = (\hat{x}(0), \hat{x}(1),\dots,\hat{x}(k))$ Kalman Gain

$u(k) = \mu_k(\mathcal{I}(k))$

$\mathbf{Tr}\ Q_r V(\hat{x}(k),u(k)) \le \hat{\gamma}_{kr}$ Covariance Constraints

$\mathbf{E}\begin{bmatrix}\hat{x}(k)\\ u(k)\end{bmatrix} \in \hat{\mathcal{D}} \subset \mathcal{R}^{n_x+n_u}$ Mean Constraints

$\mathbf{Tr}\ QV(\hat{x}(N), \mu'_N(\hat{x}(N))) + \mathbf{Tr}\ P_{xx}V_{xx}(\hat{x}(N+1)) \le \mathbf{Tr}\ P_{xx}V_{xx}(\hat{x}(N))$

$\hat{x}(N) \in \hat{\mathcal{O}}_\infty$ for Feasibility for Stability

Decentralized Kalman Filtering

- **Problem 4**

$$\min_{K^i(k)} \mathbf{Tr}\ P_{xx}V_{xx}(\tilde{x}^i(-N_n+1)) + \sum_{k=-N_n+1}^{0} \mathbf{Tr}\ Q_{xx}V_{xx}(\tilde{x}^i(k))$$

subject to $x(k+1) = Ax(k) + Fw(k)$ $t \in \mathcal{Z}_+,\ x(t) \in \mathcal{R}^{n_x}$

$y(k) = Cx(k) + v(k)$ $w(t) \in \mathcal{R}^{n_w},\ v(t) \in \mathcal{R}^{n_y}, Q_{xx} = H^T H \ge 0$

$\mathcal{Y}^i(k) = (y^i(0), y^i(1),\dots, y^i(k))$ White Noise

$\hat{x}^i(k) = A\hat{x}^i(k-1) + K^i(k)(y^i(k) - CA\hat{x}^i(k-1))$

$\hat{x}^i(k+1 \mid k) = A\hat{x}^i(k)$ Kalman Gain

$\hat{x}^i(k) = \hat{x}^i(k \mid k)$ $y^i_j(k) = y_j(k)$ if $y_j(k) \in \mathcal{Y}^i(k)$

$\tilde{x}^i(k) = x(k) - \hat{x}^i(k)$ $y^i_j(k) = CA\hat{x}^i(k-1)$ if $y_j(k) \notin \mathcal{Y}^i(k)$

$\mathbf{Tr}\ Q_r V(\tilde{x}(k)) \le \tilde{\gamma}_{kr}$ Covariance Constraints

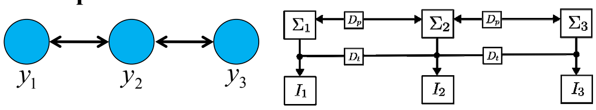
$\mathbf{E}\ \tilde{x}(k) \in \tilde{\mathcal{D}} \subset \mathcal{R}^{n_x}$ Mean Constraints

$\mathbf{Tr}\ P_{xx}V_{xx}(\tilde{x}(-N+1)) + \sum_{k=-N_n+1}^{0} \mathbf{Tr}\ Q_{xx}V_{xx}(\tilde{x}(k)) \ge \mathbf{Tr}\ P_{xx}V_{xx}(\tilde{x}(0))$

for Stability

An Example of Decentralized Kalman Filtering

- **Example 1**



$$\mathcal{Y}^1(t) = (\bar{y}_1(t), \bar{y}_2(t-1), \bar{y}_3(t-2))$$

$$\mathcal{Y}^2(t) = (\bar{y}_1(t-1), \bar{y}_2(t), \bar{y}_3(t-1))$$

$$\mathcal{Y}^3(t) = (\bar{y}_1(t-2), \bar{y}_2(t-1), \bar{y}_3(t))$$

$\bar{y}_j(t) := (y_j(t), y_j(t-1), \dots, y_j(0))$

$$K^i(t) = \begin{bmatrix} K^i_{11}(t) & K^i_{12}(t) & K^i_{13}(t) \\ K^i_{21}(t) & K^i_{22}(t) & K^i_{23}(t) \\ K^i_{31}(t) & K^i_{32}(t) & K^i_{33}(t) \end{bmatrix}$$

Distributed Kalman Filtering

- **Problem 5**

$$\min_{K^i(k)} \mathbf{Tr}\ P_{xx}V_{xx}(\tilde{x}^i(-N_h+1)) + \sum_{k=-N_h+1}^{0} \mathbf{Tr}\ Q_{xx}V_{xx}(\tilde{x}^i(k))$$ $N_h \le N_n$

subject to $x(k+1) = Ax(k) + Fw(k)$ $t \in \mathcal{Z}_+,\ x(t) \in \mathcal{R}^{n_x}$

$y(k) = Cx(k) + v(k)$ $w(t) \in \mathcal{R}^{n_w},\ v(t) \in \mathcal{R}^{n_y}, Q_{xx} = H^T H \ge 0$

$\mathcal{Y}^i(k) = (y^i(0), y^i(1),\dots, y^i(k))$ White Noise

$\hat{x}^i(k) = A\hat{x}^i(k-1) + K^i(k)(y^i(k) - CA\hat{x}^i(k-1))$

$\hat{x}^i(k+1 \mid k) = A\hat{x}^i(k)$ Kalman Gain

$\hat{x}^i(k) = \hat{x}^i(k \mid k)$ $y^i_j(k) = y_j(k)$ if $y_j(k) \in \mathcal{Y}^i(k)$

$\tilde{x}^i(k) = x(k) - \hat{x}^i(k)$ $y^i_j(k) = CA\hat{x}^i(k-1)$ if $y_j(k) \notin \mathcal{Y}^i(k)$

$\mathbf{Tr}\ Q_r V(\tilde{x}(k)) \le \tilde{\gamma}_{kr}$ Covariance Constraints

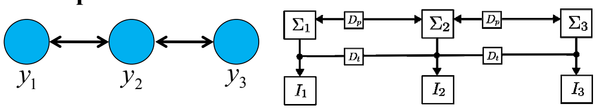
$\mathbf{E}\ \tilde{x}(k) \in \tilde{\mathcal{D}} \subset \mathcal{R}^{n_x}$ Mean Constraints

$\mathrm{Tr}\ P_{xx}V_{xx}(\tilde{x}(-N+1)) + \sum_{k=-N_h+1}^{0} \mathrm{Tr}\ Q_{xx}V_{xx}(\tilde{x}(k)) \ge \mathrm{Tr}\ P_{xx}V_{xx}(\tilde{x}(0))$

for Stability

An Example of Distributed Kalman Filtering

- **Example 2**



We assume that $N_h = 2$.

$$\mathcal{Y}^1(t) = (\bar{y}_1(t), \bar{y}_2(t-1))$$

$$\mathcal{Y}^2(t) = (\bar{y}_1(t-1), \bar{y}_2(t), \bar{y}_3(t-1))$$

$$\mathcal{Y}^3(t) = (\bar{y}_2(t-1), \bar{y}_3(t))$$

$$K^1(t) = \begin{bmatrix} K^1_{11}(t) & K^1_{12}(t) & 0 \\ K^1_{21}(t) & K^1_{22}(t) & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad K^2(t) = \begin{bmatrix} K^2_{11}(t) & K^2_{12}(t) & K^2_{13}(t) \\ K^2_{21}(t) & K^2_{22}(t) & K^2_{23}(t) \\ K^2_{31}(t) & K^2_{32}(t) & K^2_{33}(t) \end{bmatrix} \quad K^3(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & K^3_{22}(t) & K^3_{23}(t) \\ 0 & K^3_{32}(t) & K^3_{33}(t) \end{bmatrix}$$

Outline

Centralized V. S. Decentralized

- **Performance V. S. Fault**

$$\min_{N_c} \; J_p(N_c) + J_f(N_c)$$

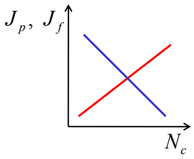
$$\text{subject to} \quad \frac{dJ_p}{dN_c} \le 0 \quad \frac{dJ_f}{dN_c} \ge 0$$

communication delay $\propto N_c$

N_c : number of computers

J_p : evaluation function of performance

J_f : evaluation function of fault



Decentralized V. S. Distributed

- **Performance V. S. Memory**

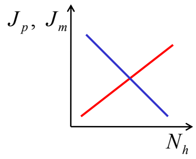
$$\min_{N_h} \; J_p(N_h) + J_m(N_h)$$

$$\text{subject to} \quad \frac{dJ_p}{dN_h} \le 0 \quad \frac{dJ_m}{dN_h} \ge 0$$

memory capacity $\propto N_h$

J_p : evaluation function of performance

J_m : evaluation function of memory

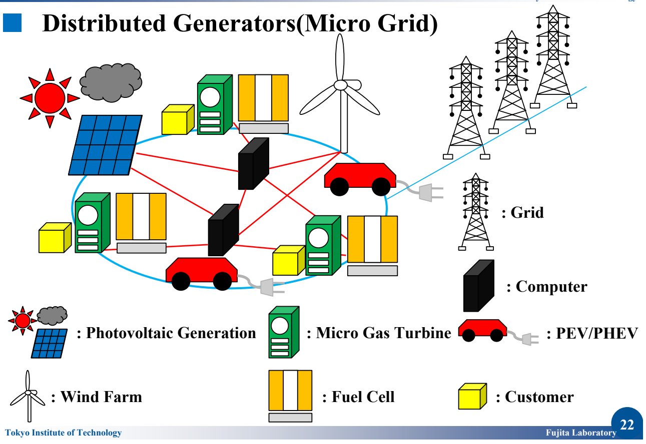


Outline

1. Introduction
2. Output Feedback Control Law
3. Optimal Decomposition
4. Distributed Generators(Micro Grid)
5. Conclusions and Future Works

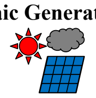
Distributed Generators(Micro Grid)

- **Distributed Generators(Micro Grid)**



Photovoltaic Generation and Micro Gas Turbines

- **Example 3**
  - **Problem Statement**
    - **Photovoltaic Generation**

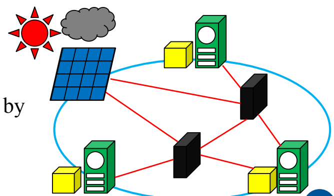
⇩ 

We **predict** the **Power** by **Weather Forecast**.

- **Micro Gas Turbine**

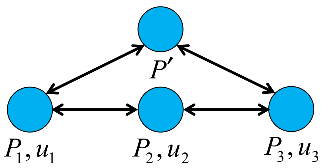
⇩ 

We **control** the **Total Power** by **Micro Gas Turbine**.



System Model of Example 3

- **System Model**



P_i : power of i th micro gas turbine

P' : power of photovoltaic generation

$\Delta P'$: input of photovoltaic generation

u_i : input of i th micro gas turbine

w_i, w', v_i : white noise

$$\begin{bmatrix} P_1(t+1) \\ P_2(t+1) \\ P_3(t+1) \\ P'(t+1) \end{bmatrix} = \begin{bmatrix} a & b & 0 & d \\ b & a & b & 0 \\ 0 & b & a & d \\ d & 0 & d & c \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ P'(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \\ w'(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \Delta P'(t) \end{bmatrix}$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix}$$

Optimization Problem of Example 3

- **Kalman Filtering**

$$\min_{K^i} \sum_{k=-1}^{0} \mathbf{E}\ q_1\left(P_1(k) - \hat{P}_1^i(k)\right)^2 + q_2\left(P_2(k) - \hat{P}_2^i(k)\right)^2 + q_3\left(P_3(k) - \hat{P}_3^i(k)\right)^2$$

subject to

$$\mathbf{Tr}\ Q_r V_{PP}(P(k) - \hat{P}^i(k)) \leq \tilde{\gamma}_{kr}$$

$$\mathbf{E}\ P(k) - \hat{P}^i(k) \in \tilde{\mathcal{D}} \subset \mathcal{R}^{n_p+n_u}$$

- **Predictive Control**

$$\min_{u^i} \sum_{k=0}^{1} \mathbf{E}\ q_1\left(P_1^{ref}(k) - \hat{P}_1^i(k)\right)^2 + q_2\left(P_2^{ref}(k) - \hat{P}_2^i(k)\right)^2 + q_3\left(P_3^{ref}(k) - \hat{P}_3^i(k)\right)^2 + r_1 u_1^{i\,2}(k) + r_2 u_2^{i\,2}(k) + r_3 u_3^{i\,2}(k)$$

subject to

$$\mathbf{Tr}\ Q_r V(\hat{P}^i(k), u^i(k)) \leq \hat{\gamma}_{kr}$$

$$\mathbf{E}\begin{bmatrix}\hat{P}^i(k)\\ u^i(k)\end{bmatrix} \in \hat{\mathcal{D}} \subset \mathcal{R}^{n_p+n_u}$$

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5. Conclusions and Future Works

Conclusions and Future Works

- **Conclusions**
  - We have proposed **Distributed Predictive Control Laws**.
  - We have considered **Optimal Decomposition**.
  - We have introduced **Distributed Generators(Micro Grid)**.
- **Future Works**
  - Simulation of **Distributed Generators(Micro Grid)**
  - Precise evaluation of **Time Complexity** and **Fault Tolerance**
  - Considering **Spatially Inhomogeneous Disturbance** and **Optimal Decomposition**
  - Considering **Plug and Play Control**

Appendix

1. State Feedback Control Law
2. Numerical Simulation
3. Decomposition and Coordination

State Feedback Control

- **Problem 1**

$$\min_{u(k)} \mathbf{Tr}\ P_{xx}V_{xx}(x(N)) + \sum_{k=0}^{N-1} \mathbf{Tr}\ QV(x(k), u(k)) \qquad t \in \mathcal{Z}_+,\ x(t) \in \mathcal{R}^{n_x},\ u(t) \in \mathcal{R}^{n_u},\ w(t) \in \mathcal{R}^{n_w}$$

subject to $x(k+1) = Ax(k) + Bu(k) + Fw(k)$ — White Noise

$\mathbf{Tr}\ Q_r V(x(k), u(k)) \leq \gamma_{kr}$ — Covariance Constraints

$\mathbf{E}\begin{bmatrix}x(k)\\ u(k)\end{bmatrix} \in \mathcal{D} \subset \mathcal{R}^{n_x+n_u}$ — Mean Constraints

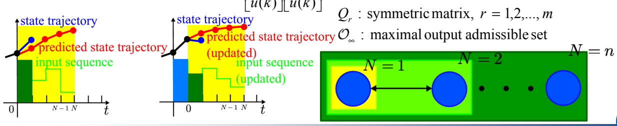
$\mathbf{Tr}\ QV(x(N), \mu_N'(x(N))) + \mathbf{Tr}\ P_{xx}V_{xx}(x(N+1)) \leq \mathbf{Tr}\ P_{xx}V_{xx}(x(N))$ — for Stability

$x(N) \in \mathcal{O}_\infty$ — for Feasibility

$V(x(k), u(k)) := \mathbf{E}\begin{bmatrix}x(k)\\ u(k)\end{bmatrix}\begin{bmatrix}x(k)\\ u(k)\end{bmatrix}^T$, $V_{xx}(x(k)) := \mathbf{E}\ x(k)x^T(k)$, $Q > 0$

Q_r : symmetric matrix, $r = 1, 2, \ldots, m$

$\mathcal{O}_\infty$: maximal output admissible set



Example 1

- **Example 1**

$$\underbrace{\begin{bmatrix}z_1(t+1)\\ z_2(t+1)\\ z_3(t+1)\end{bmatrix}}_{z(t+1)} = \underbrace{\begin{bmatrix}\Xi_{11} & \Xi_{12} & 0\\ \Xi_{21} & \Xi_{22} & \Xi_{23}\\ 0 & \Xi_{32} & \Xi_{33}\end{bmatrix}}_{\Xi}\underbrace{\begin{bmatrix}z_1(t)\\ z_2(t)\\ z_3(t)\end{bmatrix}}_{z(t)} + \underbrace{\begin{bmatrix}\Upsilon_{11} & 0 & 0\\ 0 & \Upsilon_{22} & 0\\ 0 & 0 & \Upsilon_{33}\end{bmatrix}}_{\Upsilon}\underbrace{\begin{bmatrix}u_1(t)\\ u_2(t)\\ u_3(t)\end{bmatrix}}_{u(t)} + \underbrace{\begin{bmatrix}\Omega_{11} & 0 & 0\\ 0 & \Omega_{22} & 0\\ 0 & 0 & \Omega_{33}\end{bmatrix}}_{\Omega}\underbrace{\begin{bmatrix}w_1(t)\\ w_2(t)\\ w_3(t)\end{bmatrix}}_{w(t)}$$

$I_1(t) = (z_1(t),\ z_2(t-1),\ z_3(t-2))$

$I_2(t) = (z_1(t-1),\ z_2(t),\ z_3(t-1))$

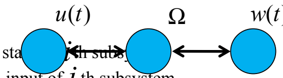
$I_3(t) = (z_1(t-2),\ z_2(t-1),\ z_3(t))$

z_i : state of i th subsystem

u_i : input of i th subsystem

I_i : information that i th member can obtain

w_i : disturbance of i th member



⇩ **Communication Delay** is reduced to **Delay of Disturbance**.

$I_1(t) = (z(t-2),\ w_1(t-1),\ w_1(t-2),\ w_2(t-2))$

$I_2(t) = (z(t-2),\ w_1(t-2),\ w_2(t-1),\ w_2(t-2),\ w_3(t-2))$

$I_3(t) = (z(t-2),\ w_2(t-2),\ w_3(t-1),\ w_3(t-2))$

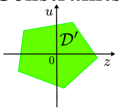
Mean and Covariance Constraints

■ **Communication Delay**

$\mathbf{E}\ u_1(t)w_2(t-1) = 0$
$\mathbf{E}\ u_1(t)w_3(t-1) = 0$
$\mathbf{E}\ u_1(t)w_3(t-2) = 0$
$\mathbf{E}\ u_2(t)w_1(t-1) = 0$
$\mathbf{E}\ u_2(t)w_3(t-1) = 0$
$\mathbf{E}\ u_3(t)w_1(t-1) = 0$
$\mathbf{E}\ u_3(t)w_1(t-2) = 0$
$\mathbf{E}\ u_3(t)w_2(t-1) = 0$

■ **State and Input Constraints**

$$\mathrm{E}\begin{bmatrix} z(t) \\ u(t) \end{bmatrix} \in \mathcal{D}' \subset \mathcal{R}^{n_z+n_u}$$



■ **Power Constraint**

$$\mathbf{E}\ z^T(t)Q_z z(t) + u^T(t)Q_u u(t) \le \gamma$$

Note: An Extended State Realization

$$A := \begin{bmatrix} \Xi & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \end{bmatrix},\ B := \begin{bmatrix} \Upsilon \\ 0 \\ 0 \end{bmatrix},\ F := \begin{bmatrix} \Omega \\ I \\ 0 \end{bmatrix}$$

$$x(t) := \begin{bmatrix} z(t) \\ w(t-1) \\ w(t-2) \end{bmatrix}$$

Stability and Optimal Solution

■ **Lemma 1(Stability)**

If the following conditions are satisfied,

$$\mathbf{Tr}\ QV(x(N), \mu_N'(x(N))) + \mathbf{Tr}\ P_{xx}V_{xx}(x(N+1)) \le \mathbf{Tr}\ P_{xx}V_{xx}(x(N))$$

$$x(N) \in \mathcal{O}_\infty$$

then

$$\lim_{t\to\infty} V(x(t), u(t)) = 0.$$

■ **Lemma 2(Optimal Solution)**

Problem 1 is reduced to a **Convex Optimization Problem** involving an **LMI**.

Numerical Simulation

■ **Simulation Setting**

$$\min_u \sum_{k=0}^{1} \mathbf{E}\ 150(z_2 - r_{tgt})^2 + 150(z_1 - (z_2 - r_{tgt}))^2 + 150((z_2 - r_{tgt}) - z_3)^2 + u_1^2 + u_2^2 + u_3^2$$

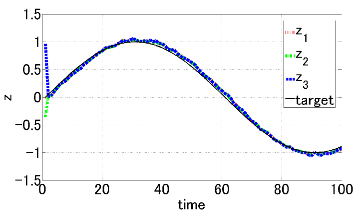
subject to

$$\sum_{k=0}^{1} \mathbf{E}\ 150(z_2 - r_{tgt})^2 + 150(z_1 - (z_2 - r_{tgt}))^2 + 150((z_2 - r_{tgt}) - z_3)^2 + u_1^2 + u_2^2 + u_3^2 \le 1$$

$\mathbf{E}\ u_i w_j = 0,\ {}^{\exists}i, j$

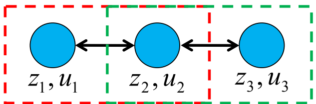
$\mathbf{E}\ |u_i| \le 1,\ i = 1, 2, 3$

$r_{tgt}(t) := \sin(2\pi t / 120)$



Example 2

■ **Example 2**



Subsystem 1 Subsystem 2

We assume that $N = 2$ and we decompose only **Inputs** for simplicity.

Input Sequence:

$$U(0) = [u_1(0)\ \ u_2(0)\ \ u_3(0)\ \ u_1(1)\ \ u_2(1)\ \ u_3(1)]^T$$

$$U_1(0) = [u_1(0)\ \ u_2^1(0)\ \ u_1(1)\ \ u_2^1(1)]^T\ \ U_2(0) = [u_2^2(0)\ \ u_3(0)\ \ u_2^2(1)\ \ u_3(1)]^T$$

Lagrange Multipliers

We use **Lagrange Multipliers**.

$$U(0) = [u_1(0)\ \ u_2(0)\ \ u_3(0)\ \ u_1(1)\ \ u_2(1)\ \ u_3(1)]^T$$

$$U_1(0) = [u_1(0)\ \ u_2^1(0)\ \ u_1(1)\ \ u_2^1(1)]^T\ \ U_2(0) = [u_2^2(0)\ \ u_3(0)\ \ u_2^2(1)\ \ u_3(1)]^T$$

Note:

$$\lambda_{c1}(u_2^1(0) - u_2^2(0)) = 0,\quad \lambda_{c2}(u_2^1(1) - u_2^2(1)) = 0$$

$$\lambda_{c1} \ge 0,\ \lambda_{c2} \ge 0$$

Decomposition and Coordination

■ **Lemma 3(Decomposition and Coordination)**

Problem 1 is solved by computing **Partial Optimization Problems** of each subsystem and updating Lagrange Multipliers(by **Gradient Method**) .

Note: Lemma 3 shows that **Problem 1** is solved by **Distributed Information** and **Distributed Computing**.