Study on Vision-Based Pose Synchronization
FL08_21_1

Naoto Kobayashi

Outline

- ■ Introduction
- ■ Visual Observer
  - • Modification of the Visual Observer-based Control System
- ■ Vision-based Pose Synchronization
  - • Proposal of Modified Control Laws
  - • Convergence Analysis
  - • Simulation / Experiment
- ■ Conclusion / Future Works

Cooperative Control

■ Cooperative Control

- • Cooperative control is a distributed control strategy that achieves specified tasks in multi-agent systems.
- • It's been motivated by interests in group behavior of animals, formation control of multi-vehicle systems and so on.
- • It is hoped to be applied to sensor networks, robot networks and many other multi-agents systems.

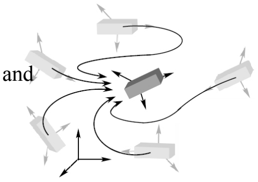
School of Fish
http://www.yunphoto.net/

Automated Highway System
http://www.its.go.jp/ITS/

Tasks of Cooperative Control

■ Tasks of Cooperative Control

- • Consensus Problem
  : to reach an agreement regarding a certain quantity of interest that depends on the state of all agents.
- • Flocking Problem
  : to make all of agents' speeds be the same.
- • Coverage Problem
- • Formation Control Problem
  :
- • Pose Synchronization [1]
  : to make all of the agents' positions and attitudes be the same.

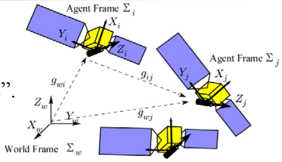


Pose synchronization

Vision-Based Cooperative Control

■ Vision-Based Cooperative Control

- • On-board camera configuration
  - Each agent can have its own "eyes".
  ⟶ autonomous agents system



On-board camera configuration

■ Researchers

- • R. Vidal et al. [2]
- • A. K. Das et al. [3]
- • F. Morbidi et al. [4]
- • N. Moshtaga et al. [5]

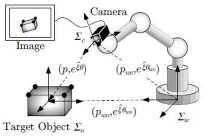
- • 2D
- • Nonholonomic constraint
- • Formation or attitude synchronization

We'll consider "Position and attitude (pose) synchronization" in "3D" space "Without nonholonomic constraint" in on-board camera configuration.

Visual Observer

■ Visual Observer [6]

- Visual observer proposed in [6] can estimate relative positions and attitudes between camera and target objects in 3D space.



Visual feedback system with visual observer

We'll combine pose synchronization control and visual observer.

Achievements

■ Achievements

- Modification of visual feedback system [7].
- Proposal of pose synchronization control law that only needs the information of relative pose between neighbors.
- Simulation
- Experiment.

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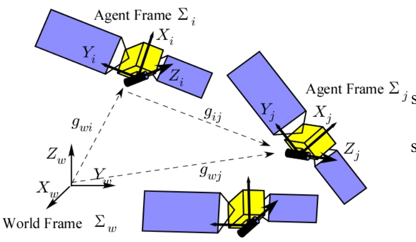
Multi-agent System with Cameras

■ Multi-agent System with Cameras



p : position

$e^{\hat{\xi}\theta_{wi}}$: attitude

$^{*}||\xi_{wi}|| = 1 \quad \theta_{wi} \in \mathcal{R}$

v : linear velocity

ω : angular velocity

subscript ij : from frame i to frame j

superscript i : see from frame i

w : world frame

$i \ j$: agent frame

$\wedge$:

$$\begin{bmatrix} \xi_x \\ \xi_y \\ \xi_z \end{bmatrix}^{\wedge} = \begin{bmatrix} 0 & -\xi_z & \xi_y \\ \xi_z & 0 & -\xi_x \\ -\xi_y & \xi_x & 0 \end{bmatrix}$$

$\vee$: inverse operator to $\wedge$

- Agents' kinematics

$$\begin{cases} \dot{p}_{wi}^{w} &= e^{\hat{\xi}\theta_{wi}} v_{wi}^{i} \\ \dot{e}^{\hat{\xi}\theta_{wi}} &= e^{\hat{\xi}\theta_{wi}} \hat{\omega}_{wi}^{i} \end{cases} \quad i = 0, 1, 2, \ldots, n$$

...(1)

Visual Observer

■ Visual Observer (review) [6]

- Relative motion

$$V_{ij}^{j} = -\mathrm{Ad}_{(g_{ij}^{-1})} V_{wi}^{i} + V_{wj}^{j}$$

- Estimated relative motion

$$\bar{V}_{ij}^{j} = -\mathrm{Ad}_{(\bar{g}_{ij}^{-1})} V_{wi}^{i} + u_{eij}$$

- Estimation error

$$g_{eeij} := \bar{g}_{ij}^{-1} g_{ij} = \begin{bmatrix} e^{\hat{\xi}\theta_{eeij}} & p_{eeij} \\ 0 & 1 \end{bmatrix}$$

$$p_{eeij} := \overline{e}^{-\hat{\xi}\theta_{ij}} (p_{ij}^{i} - \bar{p}_{ij}^{i})$$
$$e^{\hat{\xi}\theta_{eeij}} := \overline{e}^{-\hat{\xi}\theta_{ij}} e^{\hat{\xi}\theta_{ij}}$$

- Estimation error vector

$$e_{eij} := \begin{bmatrix} p_{eeij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{eeij}})^{\vee} \end{bmatrix} = J^{\dagger}(\bar{g}_{ij})(f_{ij} - \bar{f}_{ij})$$

$J(\bar{g})$: image jacobian

f_{ij} : coordinate of feature points on the image plane

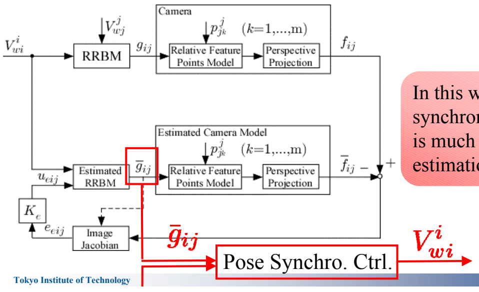
Visual Observer-based Control System

■ Visual Observer-based Control System (review) [6]

$$u_{eij} = K_{eij}\underline{e_{eij}} \quad \Longrightarrow$$

estimation error vector

- agent j doesn't move ($V_{wj}^{j} = 0$)
  estimation error vector converges to 0.
- agent j moves($V_{wj}^{j} \neq 0$)
  estimation error vector is L2-gain stable.



In this way, pose synchronization controller is much affected by estimation error...

Modification of Visual Observer

■ Derivation of the Quasi Relative Pose [7]

- Relative pose

$$g_{ij} = \bar{g}_{ij}\bar{g}_{ij}^{-1} g_{ij} = \underbrace{\bar{g}_{ij}}_{\text{estimated relative pose (known)}} \underbrace{g_{eeij}}_{\text{estimation error (we can calculate this from } e_{eij}\text{)}}$$

- Derivation of the estimation error

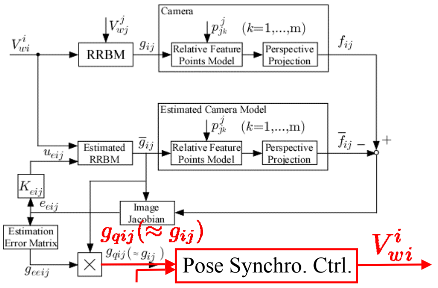
$$e_{eij} = \begin{bmatrix} p_{eeij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{eeij}})^{\vee} \end{bmatrix} \Longrightarrow g_{eeij} = \begin{bmatrix} e^{\hat{\xi}\theta_{eeij}} & p_{eeij} \\ 0 & 1 \end{bmatrix}$$

$$^{*} \ \xi\theta_{eeij} = \frac{\sin^{-1}||\mathrm{sk}(e^{\hat{\xi}\theta_{eeij}})^{\vee}||}{||\mathrm{sk}(e^{\hat{\xi}\theta_{eeij}})^{\vee}||}\mathrm{sk}(e^{\hat{\xi}\theta_{eeij}})^{\vee} \quad (-\tfrac{\pi}{2} \leq \theta_{eeij} \leq \tfrac{\pi}{2})$$

We can know the almost real relative pose (**quasi-relative pose**) if the estimation error is sufficiently small.

Modified Visual Observer-based Control System

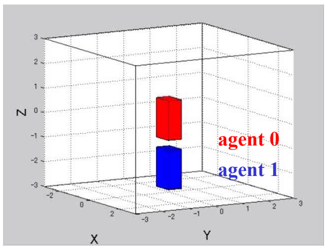
■ Modified Visual Observer-based Control System [7]



In this way, we can use almost real relative pose (**quasi-relative pose**) to construct the pose synchronization controller if the estimation error is sufficiently small.

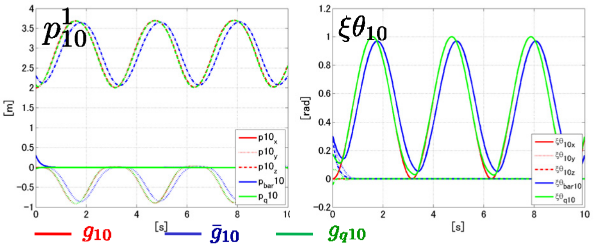
Quasi-relative Pose

■ Reasonability of the Quasi-relative Pose



Agent 1 estimates the relative pose with agent 0 by visual observer.

Quasi-relative Pose



We can confirm that quasi-relative pose is almost same as the real relative pose when estimation error is sufficiently small.

We regard the quasi-relative pose as real relative pose when we consider the pose synchronization control problem.

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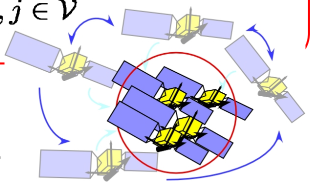
Pose Synchronization

■ Pose Synchronization

Pose Synchronization in SE(3)

: Multi-agent system is said to achieve the pose synchronization if the following condition is satisfied.

$$\begin{cases} ||p_{ij}^{i}|| \leq L, \quad L > 0 \\ e^{\hat{\xi}\theta_{wi}} = e^{\hat{\xi}\theta_{wj}} \end{cases} \quad \forall i, j \in \mathcal{V}$$

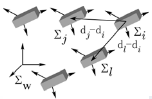


• Original definition [1]

$$\begin{cases} q_{wi}^{w} = q_{wj}^{w} \\ e^{\hat{\xi}\theta_{wi}} = e^{\hat{\xi}\theta_{wj}} \end{cases}$$

$$* \; q_{wi}^{w} = p_{wi}^{w} + \underline{d_i^{w}}$$

bias for formation construction

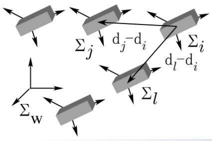


Control Laws

■ Original Pose Synchronization Control Laws [1]

As. A1 : There exists $e^{\hat{\xi}\theta_{\alpha}}$ such that $e^{-\hat{\xi}\theta_{wd}}e^{-\hat{\xi}\theta_{\alpha}}e^{\hat{\xi}\theta_{wi}} > 0 \quad \forall i$.
As. A2 : Graph is fixed and strongly connected.

$$V_{wi}^{i} = K_i \sum_{j\in\mathcal{N}_i} \left(w_{ij} \begin{bmatrix} e^{-\hat{\xi}\theta_{wi}} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} q_{wj}^{w} - q_{wi}^{w} \\ \mathrm{sk}(e^{-\hat{\xi}\theta_{wi}} e^{\hat{\xi}\theta_{wj}})^{\vee} \end{bmatrix} \right) + \boxed{\begin{bmatrix} e^{-\hat{\xi}\theta_{wi}} & 0 \\ 0 & e^{-\hat{\xi}\theta_{wi}} \end{bmatrix} \begin{bmatrix} e^{\hat{\xi}\theta_{wd}} v_{wd}^{d} \\ e^{\hat{\xi}\theta_{wd}} \omega_{wd}^{d} \end{bmatrix}}$$



These terms are not composed of relative pose information.

$$* \; K_i = \begin{bmatrix} k_{pi} & 0 \\ 0 & k_{ei} \end{bmatrix} \otimes I_3 \qquad k_{pi} > 0, k_{ei} > 0$$

Control Laws

■ Modified Control Laws

As. B1 : There exists $e^{\hat{\xi}\theta_\alpha}$ such that $e^{-\hat{\xi}\theta_\alpha}e^{\hat{\xi}\theta_{wi}} > 0 \quad \forall i$.
As. B2 : Graph is fixed, undirected, connected and $w_{ij} = w_{ji} \; \forall (i,j) \in \mathcal{E}$.

$$V^i_{wi} = K_i \sum_{j\in\mathcal{N}_i} w_{ij} \begin{bmatrix} (1-\frac{d}{||p^i_{ij}||})p^i_{ij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee \end{bmatrix} \quad i \in (1,\cdots,n) \qquad \underline{d>0} \quad nd < L$$

desired distance

...(2)

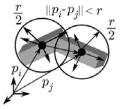
Proposition 1

Consider n agents represented by (1). Assume that As. B1, B2 are satisfied, then velocity input (2) achieves attitude synchronization and agents converge to the position that satisfies $\sum_{j\in\mathcal{N}_i} w_{ij}(1-\frac{d}{||p^i_{ij}||})p^i_{ij} = 0 \;\; \forall i$.

- Position synchronization might not be achieved.
- Collisions between neighbors might occur.
- To make agents follow desired trajectory, we have to consider leader following.

Control Laws

■ Collision Avoidance



- Collision

$$||p^i_{ij}|| \le r, \;\; r > 0$$

- Collision region

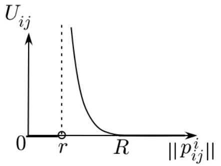
$$\Omega := \bigcup_{i\sim j} \Omega_{ij}, \quad \Omega_{ij} := \{p^W \; | p^W \in \mathcal{R}^{3n}, ||p^i_{ij}|| \le r\} \qquad {}^*p^W := [p^{wT}_{w1}, \cdots, p^{wT}_{wn}]^T$$

- Potential function for collision avoidance [1]

$$U_{ij} = \left(\min\left\{0, \frac{||p^i_{ij}||^2 - R^2}{||p^i_{ij}||^2 - r^2}\right\}\right)^2, \; j \in \mathcal{N}_i \quad r < R < d$$

$$\frac{\partial U_{ij}}{\partial p^i_{wi}} = \begin{cases} 0 & \text{if } R \le ||p^i_{ij}|| \\ s^i_{ij} & \text{if } r < ||p^i_{ij}|| < R \\ \text{not defined} & \text{if } ||p^i_{ij}|| = r \\ 0 & \text{if } ||p^i_{ij}|| < r \end{cases}$$

$$s^i_{ij} = 4\frac{(R^2-r^2)(||p^i_{ij}||^2 - R^2)}{(||p^i_{ij}||^2 - r^2)^3} p^i_{ij}$$



Control Laws

■ Modified Control Law with collision avoidance

As. B1 : There exists $e^{\hat{\xi}\theta_\alpha}$ such that $e^{-\hat{\xi}\theta_\alpha}e^{\hat{\xi}\theta_{wi}} > 0 \quad \forall i$.
As. B2 : Graph is fixed, undirected, connected and $w_{ij} = w_{ji} \; \forall (i,j) \in \mathcal{E}$.
As. B3 : $p^W(0) \notin \Omega$

$$V^i_{wi} = K_i \sum_{j\in\mathcal{N}_i} w_{ij} \left(\begin{bmatrix} (1-\frac{d}{||p^i_{ij}||})p^i_{ij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee \end{bmatrix} + \begin{bmatrix} \frac{\partial U_{ij}}{\partial p^i_{wi}} \\ 0 \end{bmatrix} \right) \quad i \in (1,\cdots,n)$$

...(3)

Proposition 2

Consider n agents represented by (1). Assume that As. B1, B2and B3 are satisfied, then velocity input (3) achieves attitude synchronization while collisions between neighbors are avoided. Moreover, agents converge to the position that satisfies $\sum_{j\in\mathcal{N}_i} w_{ij}\left(\left(1-\frac{d}{||p^i_{ij}||}\right)p^i_{ij} - \frac{\partial U_{ij}}{\partial p^i_{wi}}\right) = 0 \;\; \forall i$.

- Position synchronization might not be achieved.
- To make agents follow desired trajectory, we have to consider leader following.

Control Laws

■ Proof of Proposition 2

See [1] as to proof of attitude synchronization.
We'll only show proof of linear motion part.

Define the potential function regarding agent i's linear motion as

$$\varphi_i := \gamma_i + U_i$$

where $\gamma_i := \frac{1}{2}\sum_{j\in\mathcal{N}_i} w_{ij}(||p^i_{ij}|| - d)^2$ and $U_i := \sum_{j\in\mathcal{N}_i} w_{ij}U_{ij}$.

Fast, we show that linear velocity part of the input (3) is actually $-k_{pi}\left(\frac{\partial \varphi_i}{\partial p^i_{wi}}\right)^T$.

$$\begin{aligned} \frac{\partial U_i}{\partial p^i_{wi}} &= \frac{\partial}{\partial p^i_{wi}}(\gamma_i + U_i) \\ &= \frac{\partial}{\partial p^i_{wi}}\left(\frac{1}{2}\sum_{j\in\mathcal{N}_i} w_{ij}(||p^i_{ij}|| - d)^2 + \sum_{j\in\mathcal{N}_i} w_{ij}U_{ij}\right) \\ &= \sum_{j\in\mathcal{N}_i}\left(\frac{1}{2}w_{ij}\frac{\partial}{\partial p^i_{wi}}\left(||p^j_{wi} - p^i_{wi}||^2 - 2d||p^i_{wj} - p^i_{wi}|| + d^2\right) + w_{ij}\frac{\partial U_{ij}}{\partial p^i_{wi}}\right) \\ &= \cdots \end{aligned}$$

Control Laws

■ Proof of Proposition 2

$$\begin{aligned} \frac{\partial \varphi_i}{\partial p^i_{wi}} &= \sum_{j\in\mathcal{N}_i}\left(\frac{1}{2}w_{ij}\left(2(-p^i_{wj} + p^i_{wi})^T - 2d\frac{2(-p^i_{wj} + p^i_{wi})^T}{2||p^i_{wj} - p^i_{wi}||}\right) + w_{ij}\frac{\partial U_{ij}}{\partial p^i_{wi}}\right) \\ &= -\sum_{j\in\mathcal{N}_i} w_{ij}\left(\left(1-\frac{d}{||p^i_{ij}||}\right)p^{iT}_{ij} - \frac{\partial U_{ij}}{\partial p^i_{wi}}\right) \\ &= -\frac{1}{k_{pi}}v^{iT}_{wi} \end{aligned}$$

Now, define the potential function for the all agents' linear motion as

$$\varphi := \sum_{i=1}^n \varphi_i$$

Time derivative of φ yields

$$\begin{aligned} \dot{\varphi} &= \frac{\partial \varphi}{\partial p^I}\frac{dp^I}{dt} & {}^*p^I := [p^{1T}_{w1}, \cdots, p^{nT}_{wn}]^T \\ &= -\sum_{i=1}^n \frac{2}{k_{pi}}||v^i_{wi}||^2 & \because \text{graph is undirected} \\ &\le 0 \end{aligned}$$

Control Laws

■ Proof of Proposition 2

$\dot{\varphi} \le 0$ implies that collision between neighbors don't occur because of $p^W(0) \notin \Omega$ and $\lim_{||p^w_{ij}|| \to r+} \varphi = +\infty \;\; \forall (i,j) \in \mathcal{E}$.

The largest invariance set that satisfies $\dot{\varphi} = 0$ here is

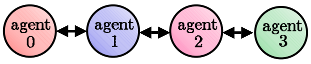
$$\begin{aligned} E &= \{p \; | v^i_{wi} = 0 \;\; \forall i\} \\ &= \{p \mid \sum_{j\in\mathcal{N}_i} w_{ij}\left((1-\frac{d}{||p^i_{ij}||})p^i_{ij} - \frac{\partial U_{ij}}{\partial p^i_{wi}}\right) = 0 \;\; \forall i\} \end{aligned}$$

Now, by LaSalle's theorem we can show that agents' position converges to the set E . ■

Simulation (control input (2))

■ Problem Formulation

- Graph



$w_{01} = w_{10} = 0.4$ $w_{12} = w_{21} = 0.6$ $w_{23} = w_{32} = 0.1$

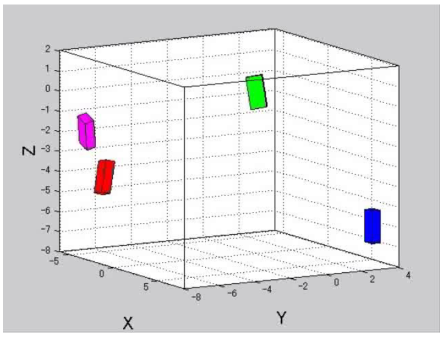
- Control input (2)

$$V_{wi}^i = K_i \sum_{j \in \mathcal{N}_i} w_{ij} \begin{bmatrix} (1 - \frac{d}{\|p_{ij}^i\|}) p_{ij}^i \\ \mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee \end{bmatrix}$$

$d = 3.0[\mathrm{m}]$
$L = 12.1[\mathrm{m}]$
$K_i = I_6 \ \forall i$

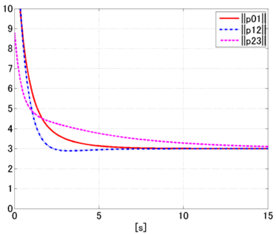
Simulation (control input (2))

■ Simulation Result

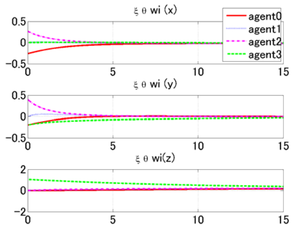


Simulation (control input (2))

■ Simulation Result



Distance between neighbors

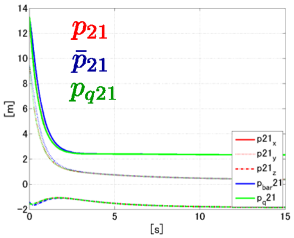


Attitude

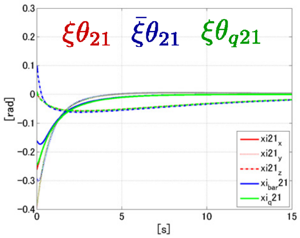
- Attitude synchronization is achieved.
- Agents converge to the position that satisfies $\|p_{ij}^i\| = d$.

Simulation (control input (2))

■ Simulation Result



Relative position



Relative attitude

- Quasi-relative pose is almost equals to (real) relative pose when estimation error is sufficiently small.
- Estimation error soon be small enough.

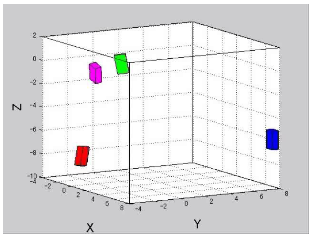
Simulation (control input (3))

■ Simulation Result

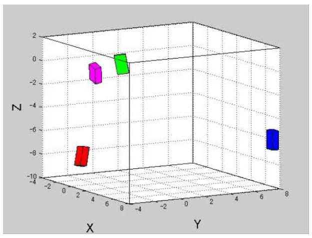
- Control input (3)

$$V_{wi}^i = K_i \sum_{j \in \mathcal{N}_i} w_{ij} \left(\begin{bmatrix} (1 - \frac{d}{\|p_{ij}^i\|}) p_{ij}^i \\ \mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee \end{bmatrix} + \begin{bmatrix} \frac{\partial U_{ij}}{\partial p_{wi}^i} \\ 0 \end{bmatrix} \right)$$

$R = 2.0[\mathrm{m}]$ $d = 3.0[\mathrm{m}]$
$r = 1.6[\mathrm{m}]$ $L = 12.1[\mathrm{m}]$



Control input (2)

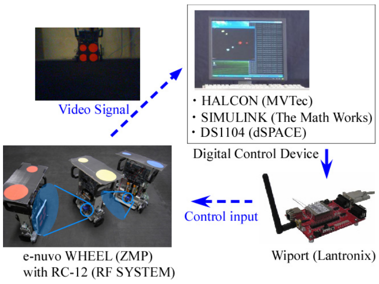


Control input (3)

Collisions between neighbors are avoided with control input(3).

Experiment

■ Experimental Environment



- HALCON (MVTec)
- SIMULINK (The Math Works)
- DS1104 (dSPACE)

- Experiment is performed on 2D plane.
- A local controller due to Astolfi[As] is embedded so that it tracks to the position and orientation given by $T_s V_{wi}^i$, where we choose $T_s = 1[\mathrm{s}]$.

Experiment

■ Problem Formulation

- Graph

: Leader following

Though the graph used in this experiment doesn’t satisfies As. B2,
we can confirm rough property of the input (2) .

- Velocity Input (2)

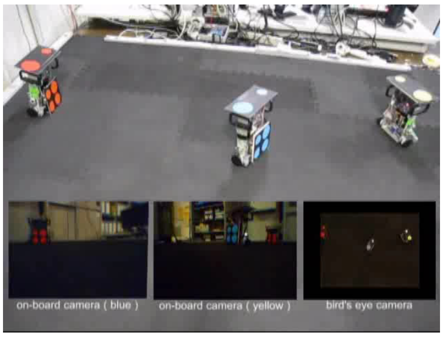
$$V^i_{wi} = K_i \sum_{j\in\mathcal{N}_i} w_{ij} \begin{bmatrix} (1-\frac{d}{||p^i_{ij}||})p^i_{ij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee \end{bmatrix}$$

$$V^0_{w0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$k_{pi} = 1 \quad k_{ei} = 1 \quad w_{ij} = 1 \quad d = 0.40[\mathrm{m}] \qquad i = 1,2$$

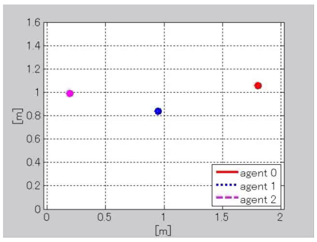
Experiment

■ Experimental Result

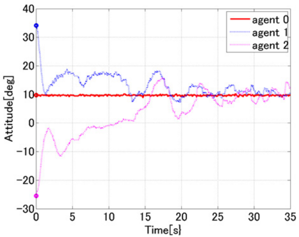


Experiment

■ Experimental Result



Position

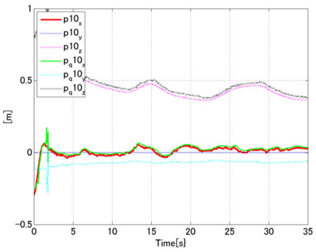


Attitude

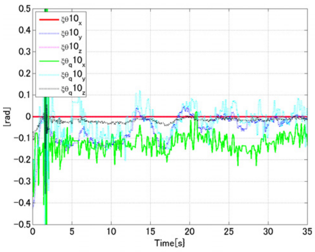
- **Attitude synchronization is achieved.**
- **Agents converge to the position that satisfies $||p^i_{ij}|| = d$.**

Experiment

■ Experimental Result



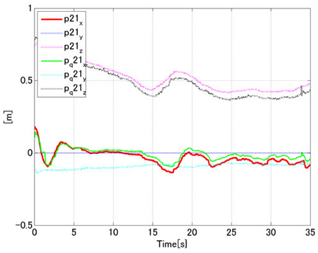
p^1_{10}, p^1_{q10}



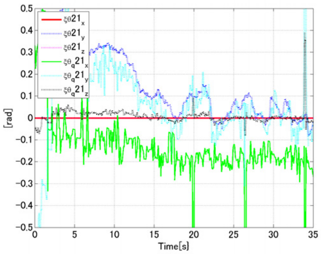
θ_{10}, θ_{q10}

Experiment

■ Experimental Result



p^2_{21}, p^2_{q21}



θ_{21}, θ_{q21}

- **There remains a little error between real relative pose value and quasi-value.**
- **It is due to calibration error and distortion of camera image.**

Outline

- Introduction
- Visual Observer
  - Modification of the Visual Observer-based Control System
- Vision-based Pose Synchronization
  - Proposal of Modified Control Laws
  - Convergence Analysis
  - Simulation / Experiment
- **Conclusion / Future Works**

Conclusion / Future Works

■ **Conclusion**

- Modification of the visual observer-based control system
- Proposal of modified control laws
- Convergence analysis
- Simulation / experiment

■ **Future Works**

- Master thesis
- Visibility maintenance (formation control)

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Appendix

■ **Comparison between my study and [5]**

	my study	[5]
kinematics	$\begin{cases} \dot{p}^w_{wi} &= e^{\hat{\xi}\theta_{wi}} v^i_{wi} \\ \dot{e}^{\hat{\xi}\theta_{wi}} &= e^{\hat{\xi}\theta_{wi}} \hat{\omega}^i_{wi} \end{cases}$	$\dot{x}_i = \cos\theta_i$ $\dot{y}_i = \sin\theta_i$ $\dot{\theta}_i = \omega_i$
original control law	$V^i_{wi} = K_i \sum_{j\in\mathcal{N}_i} \left(w_{ij} \begin{bmatrix} e^{-\hat{\xi}\theta_{wi}} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} q^w_{wj} - q^w_{wi} \\ \mathrm{sk}(e^{-\hat{\xi}\theta_{wi}} e^{\hat{\xi}\theta_{wi}})^\vee \end{bmatrix} \right) + \begin{bmatrix} e^{-\hat{\xi}\theta_{wi}} & 0 \\ 0 & e^{-\hat{\xi}\theta_{wi}} \end{bmatrix} \begin{bmatrix} e^{\hat{\xi}\theta_{wd}} v^d_{wd} \\ e^{\hat{\xi}\theta_{wd}} \omega^d_{wd} \end{bmatrix}$ • position and attitude • 3D space	$\omega_i = \kappa \sum_{j\in\mathcal{N}_i} \sin(\theta_i - \theta_j)$ $\kappa < 0$ • attitude only • planer space
necessary values for vision-based controller	• pixels of 4 feature point	• bearing angle, optical flow and time-to-collision (which can be gotten by image processing)

Appendix

■ **Comparison between my study and [5]**

	my study	[5]
vision-based control law	$V^i_{wi} = K_i \sum_{j\in\mathcal{N}_i} w_{ij} \begin{bmatrix} (1 - \frac{d}{\lVert p^i_{qij} \rVert}) p^i_{qij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{qij}})^\vee \end{bmatrix}$ •we don’t have to consider vision-based part when we consider cooperative control part when estimation error is sufficiently small. • position and attitude • 3D space	$\omega_i = \frac{-\kappa \sum_{j\in\mathcal{N}_i} (\frac{1}{\tau_{ij}} \sin\beta_{ij} + \dot{\beta}_{ij} \cos\beta_{ij})}{1 + \kappa \sum_{j\in\mathcal{N}_i} \cos\beta_{ij}}$ $\kappa < 0$ • using exactly measurable values • equals to original control law •attitude only • planer space
	• (estimated) quasi-relative pose values can be used to many application (collision avoidance, coverage, etc.) .	