Visual-Feedback Cooperative Control of Wheeled Robots

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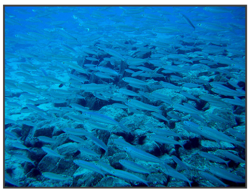
Cooperative Control

- Objective
  - Achieving specified tasks in multi-agent systems
- Motivation
  - Interests in group behavior of animals
  - Engineering applications (Sensor network etc.)



Cooperative Control

- Major problems
  - Consensus problem
    to reach an agreement regarding a certain quantity of interest that depends on the state of all agents
  - Flocking problem
    to make all of agents' speeds be the same
  - Synchronization problem
    to make all of the agents' attitudes or poses (attitudes and positions) be the same
  - Coverage problem
    to spread out agents as much as possible (efficiently) over a given area

And so on

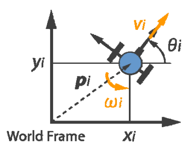
Outline

- Introduction
- Attitude Synchronization
  - Setup
  - Proposition
  - Information Acquisition Problem
- Visual Observer
- Simulation
- Summary

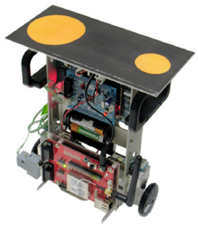
Kinematics of Wheeled Robots

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}$$

$$i = 1, 2, \ldots, n$$



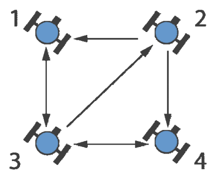
(x_i, y_i) : Position
θ_i : Rotation Angle
v_i, ω_i : Body Velocity
N_i : Set of Neighbors



Wheeled Robot
(Mobile Inverted Pendulum)

Information Graph Structure

- Information graph structure is expressed by the digraph, and there are some classes of them:
  - **Fixed**
    Topology of the graph never changes
  - **Strongly connected**
    The graph contains a directed path from *i* to *j* and a directed path from *j* to *i* for every pair of agents *u*,*v*



Attitude Synchronization

- Goal
  To achieve attitude synchronization of the system with *n* wheeled robots:

$$v_i = v_j,\ \lim_{t\to\infty} (\theta_i - \theta_j) = 0 \quad \forall i, j$$

- Proposition
  Under the follower assumptions, this control inputs achieve the attitude coordination:

Control Input:

$$\begin{cases} v_i = V(t) \\ \omega_i = k_i \sum_j \sin(\theta_j - \theta_i) \quad j \in N_i \end{cases}$$

Relative Attitude of neighbor

Assumptions:
- $|\theta_i(0)| \leq \frac{\pi}{2} \quad \forall i$
- Information graph is fixed and strongly connected

Information Acquisition

- Early in the study, the method of information acquisition is not studied sufficiently
- Recently, vision-based information acquisition have been studied intensively [1] [2] [3]

[1] Nima Moshtagh, A. Jadbabaie and K. Daniilidis ,
"Vision-Based, Distributed Control Laws for Motion Coordination of Nonholonomic Robots", 2008
[2] Nima Moshtagh, N. Michael, A. Jadbabaie and K. Daniilidis,
"Vision-Based, Distributed Formation Control and Flocking of Multiagent Systems", 2005
[3] René Vidal, O.Shakernia and S. Sastry,
"Omnidirectional Vision-based Formation Control", 2002

Information Acquisition

- Problems of the earlier studies
  - They rely on only the image-recognition techniques
  - They have not considered dynamical characteristics of the camera and the target sufficiently

- In this study, I design the visual observer to estimate the position and the attitude of the target object

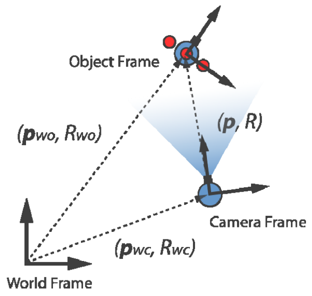
Outline

Formulation

- Relative position and attitude of the object from camera $(\boldsymbol{p}, R)$:

$$\boldsymbol{p} = R_{wc}^T(\boldsymbol{p}_{wo} - \boldsymbol{p}_{wc})$$
$$R = R_{wc}^T R_{wo}$$

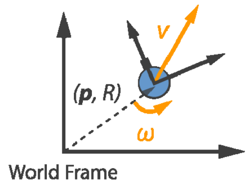
$$\boldsymbol{p} := \begin{bmatrix} x \\ y \end{bmatrix}$$
$$R := \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



Formulation

- Body Velocity

$$V := \begin{bmatrix} \boldsymbol{v} \\ \omega \end{bmatrix} = \begin{bmatrix} R^T\dot{\boldsymbol{p}} \\ (R^T\dot{R})^\vee \end{bmatrix}$$



$V_c := \begin{bmatrix} \boldsymbol{v}_c \\ \omega_c \end{bmatrix}$: Body velocity of camera

$V_o := \begin{bmatrix} \boldsymbol{v}_o \\ \omega_o \end{bmatrix}$: Body velocity of object

$V := \begin{bmatrix} \boldsymbol{v} \\ \omega \end{bmatrix}$: Relative body velocity of object from camera

■Wedge $\wedge$:

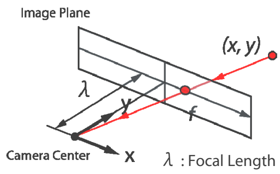
$$\hat{\omega} := \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} = W\omega$$
$$W := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} (= R_{\frac{\pi}{2}})$$

■Vee $\vee$: Inverse Operator to $\wedge$

Perspective Projection

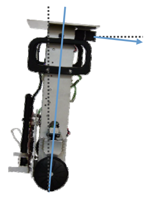
- Perspective projection is defined by the following equation:

$$f = \frac{\lambda}{y} x$$

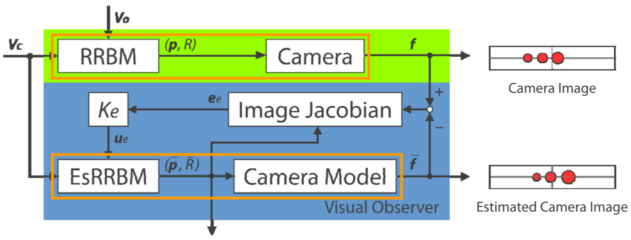


- The reason why the image plane is 1D:

  In the inverted pendulum or the vehicle, the vertical coordinate is affected by the uncontrolled motions



Visual Observer



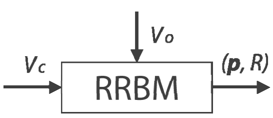
Block Diagram of Visual Observer

RRBM: Relative Rigid Body Motion
EsRRBM: Estimated Relative Rigid Body Motion
Image Jacobian: Matrix describing the relationship between the image information error and the estimation error

RRBM

- RRBM represents the relation between the body velocities of three coordinate frames
- It is calculated from temporal differentiation of $(\boldsymbol{p}, R)$:

$$V = \begin{bmatrix} R^T\dot{\boldsymbol{p}} \\ (R^T\dot{R})^\vee \end{bmatrix} = -\begin{bmatrix} R^T & R^T W\boldsymbol{p} \\ 0 & 1 \end{bmatrix} V_c + V_o$$



Passivity of RRBM

- Proposition 1

  If the target object is static, i.e., $V_o=0$,

$$\begin{bmatrix} R^T\dot{\boldsymbol{p}} \\ (R^T\dot{R})^\vee \end{bmatrix} = -\begin{bmatrix} R^T & R^T W\boldsymbol{p} \\ 0 & 1 \end{bmatrix} V_c$$

  Then, the following inequality holds for the above equation where γ_c is a positive scalar

$$\int_0^T (V_c)^T \nu_c d\tau \geq -\gamma_c^2 \qquad \nu_c := -\begin{bmatrix} \boldsymbol{p} \\ e_R(R) \end{bmatrix}$$

$$\mathrm{sk}(R) := \frac{1}{2}(R - R^T) = \begin{bmatrix} 0 & -\sin\theta \\ \sin\theta & 0 \end{bmatrix}$$
$$e_R(R) := \mathrm{sk}(R)^\vee = \sin\theta$$

Passivity of RRBM

- Proof

Consider the positive definite function

$$V_r = \frac{1}{2}\|\boldsymbol{p}\|^2 + \phi(R)$$

where ϕ is the error function of the rotation matrix:

$$\phi(I) = 0$$
$$\dot\phi(R) = e_R^T(R)(\dot R R^T)^\vee = e_R^T(R)(R^T\dot R)^\vee$$

Temporal differentiation of V:

$$\dot V_r = \boldsymbol{p}^T\dot{\boldsymbol{p}} + e_R^T(R)(R^T\dot R)^\vee$$
$$= \boldsymbol{p}RR^T\dot{\boldsymbol{p}} + e_R^T(R)(R^T\dot R)^\vee$$

Passivity of RRBM

$$\dot V_r = [\boldsymbol{p}^T\ e_R^T(R)]\begin{bmatrix} R & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix} R^T\dot{\boldsymbol{p}}\\ (R^T\dot R)^\vee\end{bmatrix}$$
$$= -\,[\boldsymbol{p}^T\ e_R^T(R)]\begin{bmatrix} R & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix} R^T & R^T W\boldsymbol{p}\\ 0 & 1\end{bmatrix} V_c$$
$$= -\,[\boldsymbol{p}^T\ e_R^T(R)]\begin{bmatrix} I & W\boldsymbol{p}\\ 0 & 1\end{bmatrix} V_c$$
$$= -\,[\boldsymbol{p}^T\ e_R^T(R)]\, V_c = \nu_c^T V_c = (V_c)^T\nu_c$$

$$\Rightarrow \int_0^T (V_c)^T\nu_c d\tau = \int_0^T \dot V_r d\tau = V_r(T) - V_r(0)$$
$$\geq -V_r(0) \geq -\gamma_c^2 \qquad \square$$

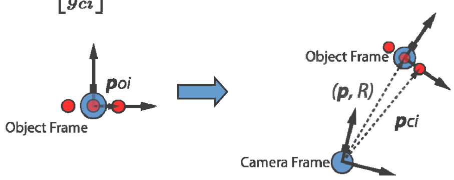
Camera Model

- Assume that the target has *m* feature points
  - The position of feature point *i* in the object frame:

$$\boldsymbol{p}_{oi} = \begin{bmatrix} x_{oi}\\ y_{oi}\end{bmatrix} \qquad i = 1,2,\ldots,m$$

  - The relative position of feature point *i* in the camera frame:

$$\boldsymbol{p}_{ci} = \begin{bmatrix} x_{ci}\\ y_{ci}\end{bmatrix} = R\boldsymbol{p}_{oi} + \boldsymbol{p} \qquad i = 1,2,\ldots,m$$

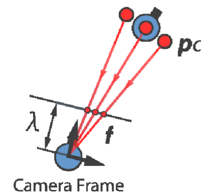


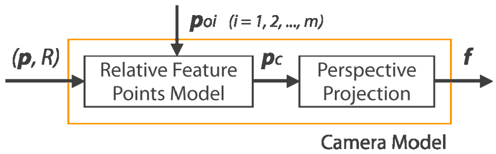
Camera Model

- Perspective projection of $\boldsymbol{p}_{ci}$:

$$f_i = \frac{\lambda}{y_{ci}} x_{ci}$$

$$\boldsymbol{p}_c := \begin{bmatrix} \boldsymbol{p}_{c1}\\ \vdots\\ \boldsymbol{p}_{cm}\end{bmatrix} \quad \boldsymbol{f} := \begin{bmatrix} f_1\\ \vdots\\ f_m\end{bmatrix}$$



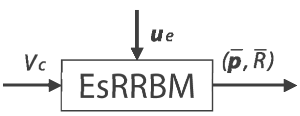


Estimated RRBM

- RRBM in the observer
- We choose estimates $(\bar{\boldsymbol{p}}, \bar R)$ and $\bar V$ of the RRBM, respectively as

$$\bar V = \begin{bmatrix} \bar R^T\dot{\bar{\boldsymbol{p}}}\\ (\bar R^T\dot{\bar R})^\vee\end{bmatrix} = -\begin{bmatrix} \bar R^T & \bar R^T W\bar{\boldsymbol{p}}\\ 0 & 1\end{bmatrix} V_c + \boldsymbol{u}_e$$

- The new input $\boldsymbol{u}_e$ is to be determined in order to drive the estimated values to their actual values



Passivity of Estimated RRBM

- Proposition 2

If there is not input, i.e., $\boldsymbol{u}_e$=0,

$$\bar V = \begin{bmatrix} \bar R^T\dot{\bar{\boldsymbol{p}}}\\ (\bar R^T\dot{\bar R})^\vee\end{bmatrix} = -\begin{bmatrix} \bar R^T & \bar R^T W\bar{\boldsymbol{p}}\\ 0 & 1\end{bmatrix} V_c$$

Then, the following inequality holds for the above equation where γ_e is a positive scalar

$$\int_0^T (V_c)^T\nu_{e1} d\tau \geq -\gamma_e^2 \qquad \nu_{e1} := -\begin{bmatrix} \bar{\boldsymbol{p}}\\ e_R(\bar R)\end{bmatrix}$$

Passivity of Estimated RRBM

- Proof

Consider the positive definite function

$$V_{\bar{r}} = \frac{1}{2}\|\bar{p}\|^2 + \phi(\bar{R})$$

Temporal differentiation of V:

$$\dot{V}_{\bar{r}} = \bar{p}^T\dot{\bar{p}} + e_R^T(\bar{R})(\bar{R}^T\dot{\bar{R}})^\vee$$
$$= \ldots = (V_c)^T\nu_{e1}$$

$$\Rightarrow \boxed{\int_0^T (V_c)^T\nu_{e1}d\tau} = \int_0^T \dot{V}_{\bar{r}}d\tau = V_{\bar{r}}(T) - V_{\bar{r}}(0)$$
$$\geq -V_{\bar{r}}(0) \boxed{\geq -\gamma_e^2} \quad \square$$

Passivity of Estimated RRBM

- Proposition 3

If the camera is static, i.e., V_c=0,

$$\bar{V} = \begin{bmatrix} \bar{R}^T\dot{\bar{p}} \\ (\bar{R}^T\dot{\bar{R}})^\vee \end{bmatrix} = u_e$$

Then, the following inequality holds for the above equation where γ_e is a positive scalar

$$\int_0^T u_e^T\nu_{e2}d\tau \geq -\gamma_e^2 \qquad \nu_{e2} := \begin{bmatrix} \bar{R}^T\bar{p} \\ e_R(\bar{R}) \end{bmatrix}$$

Passivity of Estimated RRBM

- Proof

Consider the positive definite function

$$V_{\bar{r}} = \frac{1}{2}\|\bar{p}\|^2 + \phi(\bar{R})$$

Temporal differentiation of V:

$$\dot{V}_{\bar{r}} = \bar{p}^T\dot{\bar{p}} + e_R^T(\bar{R})(\bar{R}^T\dot{\bar{R}})^\vee$$
$$= \ldots = u_e^T\nu_{e2}$$

$$\Rightarrow \boxed{\int_0^T u_e^T\nu_{e2}d\tau} = \int_0^T \dot{V}_{\bar{r}}d\tau = V_{\bar{r}}(T) - V_{\bar{r}}(0)$$
$$\geq -V_{\bar{r}}(0) \boxed{\geq -\gamma_e^2} \quad \square$$

Estimated Camera Model

- Similarly to camera model, modeling the estimated camera model in the observer:

$$\bar{p}_{ci} = \begin{bmatrix} \bar{x}_{ci} \\ \bar{y}_{ci} \end{bmatrix} = \bar{R}p_{oi} + \bar{p} \qquad \bar{f}_i = \frac{\lambda}{\bar{y}_{ci}}\bar{x}_{ci}$$

$$\bar{p}_c := \begin{bmatrix} \bar{p}_{c1} \\ \vdots \\ \bar{p}_{cm} \end{bmatrix} \qquad \bar{f} := \begin{bmatrix} \bar{f}_1 \\ \vdots \\ \bar{f}_m \end{bmatrix}$$

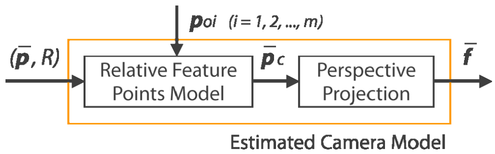


Image Jacobian

- Image Jacobian represents the relationship between the image information error and the estimation error
- The vector of the estimation error is given by

$$e_e := \begin{bmatrix} p_{ee} \\ e_R(R_{ee}) \end{bmatrix} \qquad \begin{aligned} p_{ee} &:= \bar{R}^T(p - \bar{p}) \\ R_{ee} &:= \bar{R}^T R \end{aligned}$$

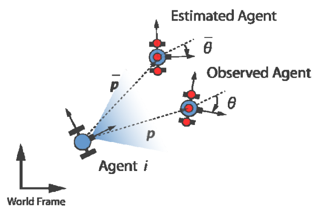


Image Jacobian

- Relation between the actual feature point *i* and estimated one is expressed as

$$(p_{ci} - \bar{p}_{ci}) = \bar{R}\,[I \quad Wp_{oi}]\, e_e$$

- Using a first-order Taylor expansion approximation, the relation between the actual image information and the estimated one is expressed as

$$f_i - \bar{f}_i = \begin{bmatrix} \frac{\lambda}{\bar{y}_{ci}} & -\frac{\lambda\bar{x}_{ci}}{\bar{y}_{ci}^2} \end{bmatrix} (p_{ci} - \bar{p}_{ci})$$
$$= \begin{bmatrix} \frac{\lambda}{\bar{y}_{ci}} & -\frac{\lambda\bar{x}_{ci}}{\bar{y}_{ci}^2} \end{bmatrix} \bar{R}\,[I\ Wp_{oi}]\, e_e = J_i e_e$$

$$\boxed{J_i := \begin{bmatrix} \frac{\lambda}{\bar{y}_{ci}} & -\frac{\lambda\bar{x}_{ci}}{\bar{y}_{ci}^2} \end{bmatrix} \bar{R}\,[I\ Wp_{oi}]}$$

Image Jacobian

- Hence, the relation between all the feature points on the image and the estimation error is expressed as

$$\boldsymbol{f} - \bar{\boldsymbol{f}} = J\boldsymbol{e}_e \qquad J := \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_m \end{bmatrix}$$

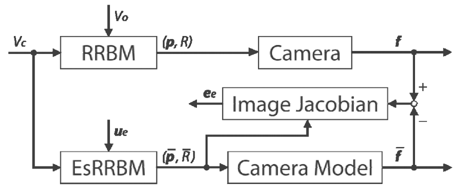
- If J is full column rank, the estimation error can be uniquely defined by the image feature vector :

$$\boldsymbol{e}_e = J^{+}(\boldsymbol{f} - \bar{\boldsymbol{f}}) = (J^T J)^{-1} J^T (\boldsymbol{f} - \bar{\boldsymbol{f}})$$

Estimation Error System

- The estimation error system will be derived in the same way as RRBM, and it is calculated from temporal differentiation of $(\boldsymbol{p}_{ee}, R_{ee})$:

$$V_{ee} = \begin{bmatrix} R_{ee}^T \dot{\boldsymbol{p}}_{ee} \\ (R_{ee}^T \dot{R}_{ee})^{\vee} \end{bmatrix} = - \begin{bmatrix} R_{ee}^T & R_{ee}^T W \boldsymbol{p}_{ee} \\ 0 & 1 \end{bmatrix} \boldsymbol{u}_e + V_o$$



Passivity of Estimation Error System

- Proposition 4

If the target object is static, i.e., V_o=0,

$$V_{ee} = \begin{bmatrix} R_{ee}^T \dot{\boldsymbol{p}}_{ee} \\ (R_{ee}^T \dot{R}_{ee})^{\vee} \end{bmatrix} = - \begin{bmatrix} R_{ee}^T & R_{ee}^T W \boldsymbol{p}_{ee} \\ 0 & 1 \end{bmatrix} \boldsymbol{u}_e$$

Then, the following inequality holds for the above equation where γ is a positive scalar

$$\int_0^T \boldsymbol{u}_e^T (-\boldsymbol{e}_e) d\tau \geq -\gamma^2$$

Passivity of Estimation Error System

- Proof

Consider the positive definite function

$$V_e = \frac{1}{2} \|\boldsymbol{p}_{ee}\|^2 + \phi(R_{ee})$$

Temporal differentiation of V:

$$\dot{V}_e = \boldsymbol{p}_{ee}^T \dot{\boldsymbol{p}}_{ee} + e_R^T(R_{ee})(R_{ee}^T \dot{R}_{ee})^{\vee}$$
$$= \ldots = \boldsymbol{u}_e^T (-\boldsymbol{e}_e)$$

$$\Rightarrow \int_0^T \boldsymbol{u}_e^T (-\boldsymbol{e}_e) d\tau = \int_0^T \dot{V}_e d\tau = V_e(T) - V_e(0)$$
$$\geq -V_e(0) \geq -\gamma^2 \qquad \square$$

Estimation Error Feedback

- Based on the above passivity property of the visual observer, we consider the following control law

$$\boldsymbol{u}_e = K_e \boldsymbol{e}_e$$
$$K_e := \mathrm{diag}\{k_{e1}, k_{e2}, k_{e3}\} > 0 \quad \text{(Positive Definite)}$$

- Proposition 5

If V_o=0, the equilibrium point $\boldsymbol{e}_e$=0 for the visual observer is asymptotic stable

Estimation Error Feedback

- Proof

Consider the positive definite function

$$V_e = \frac{1}{2} \|\boldsymbol{p}_{ee}\|^2 + \phi(R_{ee})$$

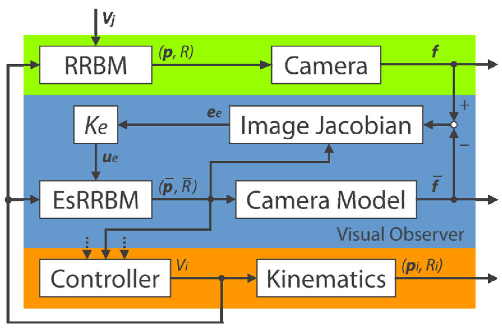
Temporal differentiation of V:

$$\dot{V}_e = \boldsymbol{p}_{ee}^T \dot{\boldsymbol{p}}_{ee} + e_R^T(R_{ee})(R_{ee}^T \dot{R}_{ee})^{\vee}$$
$$= \ldots = \boldsymbol{u}_e^T (-\boldsymbol{e}_e)$$

$$\Rightarrow \dot{V}_e = (K_e \boldsymbol{e}_e)^T (-\boldsymbol{e}_e) = -\boldsymbol{e}_e^T K_e \boldsymbol{e}_e < 0$$

$$(\boldsymbol{e}_e \neq 0) \qquad \square$$

Visual-Feedback Cooperative Control System



Block Diagram of Agent *i*

Outline

- Introduction
- Attitude Synchronization
- Visual Observer
- Simulation
  - Setup
  - Result
- Summary

Setup

- Assume 3 agents on the field
  - Agent 0: It has 3 feature points

    Control Input:

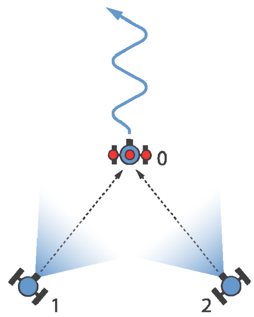
    $$\begin{cases} v_0 = V \\ \omega_0 = \cos At \end{cases}$$

  - Agent 1, 2: They have camera

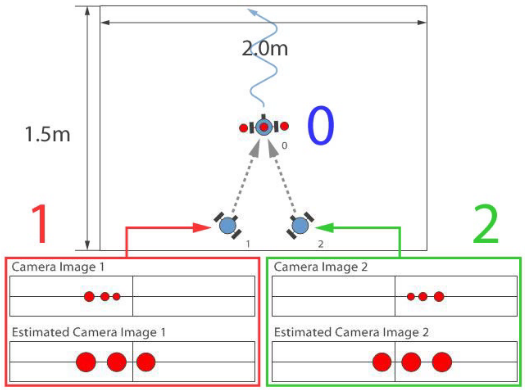
    Control Input:

    $$\begin{cases} v_i = V \\ \omega_i = \sin \bar{\theta}_{i0} \end{cases} \qquad i = 1, 2$$

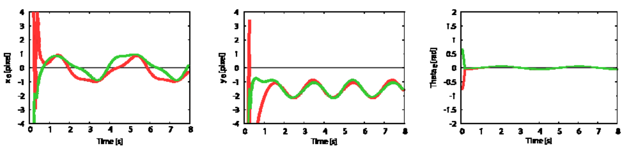
    $\bar{\theta}_{i0}$: Estimated relative attitude of agent 0 from agent *i*

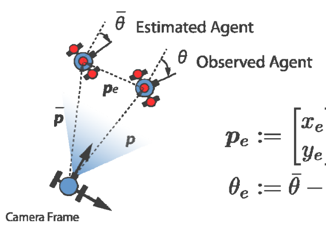


Simulation Result



Simulation Result



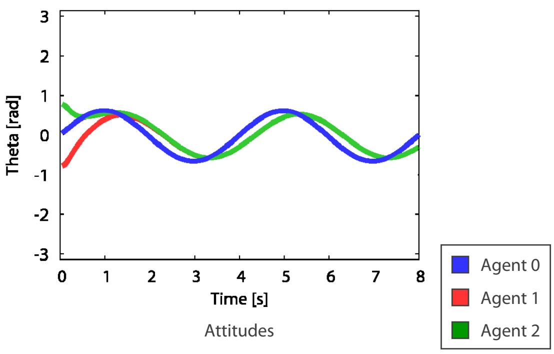


$$p_e := \begin{bmatrix} x_e \\ y_e \end{bmatrix} = \bar{p} - p$$

$$\theta_e := \bar{\theta} - \theta$$

Agent 1
Agent 2

Simulation Result



Attitudes

Outline

- Introduction
- Attitude Synchronization
- Visual Observer
- Simulation
- Summary

Summary

- Attitude synchronization problem
  - Achievement of attitude synchronization requires neighbors' information
- Visual observer
  - By the use of visual observer, the position and the attitude of the observed agent can be obtained
  - The estimation error system has passivity
- Simulation
  - We confirmed the effectiveness of visual observer
  - Attitude synchronization is achieved by use of the estimates of visual observer

Future Works

- Verification experiment
- Proof of convergence