Introduction to Mechanism Design and Application to Pose Synchronization

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Outline

- Introduction, Preliminary
- Mechanism Design Problem
- Application to Pose Synchronization
- Conclusion and Future Work

Introduction

What is Mechanism Design ?

•In Japanese, this is called “制度設計”
•Developed in economics and game theory
⇨ A branch of economics or game theory

•The study of designing rules of a game or system to achieve a specific outcome, even though each agent may be self-interested.
•Mechanism design is done by setting up a structure in which agents have an incentive to behave according to the rules.
•The resulting mechanism is then said to implement the desired outcome.

Introduction

Design Objective

•To achieve basic outcomes such as truthfulness, individual rationality, budget balance and social welfare.
•To achieve fairness (minimizing variance between participants‘ utilities), maximizing the auction holder’s revenue, and Pareto efficiency.
•It is sometimes seen to consider mechanism design in order to resist harmful coalitions of players.

Design Difficulty

Often face trade-off problems between the objectives

Ex.) It is impossible to guarantee optimal results for all above four outcomes simultaneously in many situations, particularly in markets where buyers can also be sellers.

Introduction

Application

•Creation of markets, auctions and so on
•Design of matching algorithms
•Provision of public goods
•Optimal design of taxation schemes by governments

etc…

Triumph

•Most of triumphs have been yielded in economics.
•Recently, more and more triumphs are achieved in mathematics, computer science, electrotechnology and so on.
•Novel prize in economics in 2007

Aim of My Study

Mechanism Design —apply⇨ Control Design

Mechanism Design for Control

To design and impose individual objectives (Self-interested) and control rules (Rules) on each agent of a group in order to achieve the specified group objective (Outcome)

Aim of my coming study

To design individual objectives and control protocols which achieve cooperative tasks such as pose synchronization
⇨ consider mechanism design for cooperative control

Next, introduce mechanism design problem by using previous work

Preliminary

- Graph : A set of connections (Edges) of between Objects (Vertice)

  Vertex (node) : Agent Edge : Information Flow

  -Directed Graph (Fig. 1) : the information flows from agent j to i

  -Undirected Graph (Fig. 2) : the information flows to both directions

- Directed Graph

  -strongly connected (Fig. 3) :

  there is a directly path connecting any two distinct nodes

- Undirected Graph

  -connected :

  there is a path between any two distinct nodes

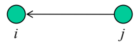

Fig. 1 Directed Graph

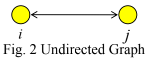

Fig. 2 Undirected Graph

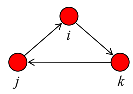

Fig. 3 Strongly Connected Graph

Preliminary

$G = (V, E)$: directed (or undirected) graph

$V = \{1, \ldots, n\}$: set of nodes (agents)

$E \subseteq V \times V$: set of edges (an edge of G : $e_{ij} = (i, j)$)

$N_i = \{ j \in V \mid (j, i) \in E \}$: set of neighbors of node i

(the set of nodes whose information node i can get)

Graph Laplacian : $L = [l_{ij}] = \begin{cases} |N_i| & i = j \\ -1 & j \in N_i \\ 0 & j \notin N_i \end{cases}$

$|N_i|$: the number of neighbors of agent i

Preliminary

Consensus

Convergence of certain values to a common value is called consensus

Consensus Problem

Determine a protocol which makes certain quantities of interest converge to a same value

Group Decision Value

The value obtained by achieving consensus

Outline

Mechanism Design Problem

Consensus Problem : D. Bauso et al. [6]

Dynamics

$\dot{x}_i(t) = u_i(t)$

x_i : agent i's state

u_i : agent i's input

Information graph : undirected and connected

Goal

Consensus:

Convergence of certain quantities of interest converge to a specified same value

desired group decision value

First, determine the following three elements for mechanism design

- A group Objective(outcome)
- Rules(individual objectives)
- Each agent's utilities(rules to determine next action)

Mechanism Design Problem

Consensus Problem : D. Bauso et al. [6]

Group Objective

$\lim_{t \to \infty} x_i(t) = f(x(0)) \quad \forall i$

$x(t) := [x_1(t) \ \cdots \ x_n(t)]^T$: states vector

$f(.) \in \Re$: agreement function (to get group decision value)

In [6], consider the case $f(x(t))$ is time invariant

Rule

Prepare the following individual objective function

$J_i(x_i(t), x^{(i)}(t), u_i(t)) = \lim_{T \to \infty} \int_0^T \left(F(x_i(t), x^{(i)}(t)) + \rho u_i^2(t) \right) dt$

$x^{(i)} := [x_1^{(i)} \ \cdots \ x_n^{(i)}]^T$

$x_j^{(i)} = \begin{cases} x_j & j \in N_i \\ 0 & otherwise \end{cases}$

$\rho > 0$

$F : \Re \times \Re^n \to \Re$ is a nonnegative penalty function that measures the deviation of x_i from neighbors' states (the same function for each agent)

A protocol is said optimal if each u_i optimizes the corresponding individual objective

Mechanism Design Problem

Consensus Problem : D. Bauso et al. [6]

Utility

Each agent chooses u_i to minimize its own individual objective function J_i

Mechanism Design Problem [6]

For any agreement function $f(.)$, determine a penalty function $F(.)$ such that there exists an optimal consensus protocol $u(.)$ with respect to $f(x(0))$ for any initial state $x(0)$.

Group objective : to converge all agents states to a desired value $f(x(0))$

Rule : individual objective function J_i

Utility : to minimize each individual objective function

Mechanism Design Problem

Consensus Problem : D. Bauso et al. [6]

Solving the problem is a difficult task.

⇨ translate it into a sequence of more tractable problems

Receding Horizon Control Scheme

$$J_i\left(x_i(t), x^{(i)}(t), u_i(t)\right) = \lim_{T\to\infty}\int_0^T \left(F\left(x_i(t), x^{(i)}(t)\right) + \rho u_i^2(t)\right)dt$$

⇨ $$\hat{J}_i\left(x_i(t_k), x^{(i)}(t_k), \hat{u}_i(\tau, t_k)\right) = \lim_{T\to\infty}\int_{t_k}^T \left(F\left(\hat{x}_i(\tau, t_k), x^{(i)}(\tau, t_k)\right) + \rho \hat{u}_i^2(\tau, t_k)\right)dt \cdots (1)$$

$$\text{subject to}\ \begin{cases} \dot{\hat{x}}_i(\tau,t_k) = \hat{u}_i(\tau,t_k), & \dot{\hat{x}}_j(\tau,t_k) = \hat{u}_j(\tau,t_k) := 0 \quad \forall j \in N_i, \\ \hat{x}_i(t_k,t_k) = x_i(t_k), & \hat{x}_j(t_k,t_k) = x_j(t_k) \quad \forall j \in N_i \end{cases}$$

naive assumption

initial states of each step

t_k : discrete time, $\delta = t_{k+1} - t_k$: one-step action horizon, t_0 : initial time $\Rightarrow t_k = t_0 + \delta k,\ k = 0,1,\ldots$

$\hat{x}_i(\tau, t_k),\ \hat{x}^{(i)}(\tau, t_k)$: predicted state of agent i and of his neighbors respectively $\tau \geq t_k$

⇨ find $\hat{u}_i^*(\tau, t_k) = \arg\min \hat{J}_i\left(x_i(t_k), x^{(i)}(t_k), \hat{u}_i(\tau, t_k)\right)$

- take the limit for $\delta \to 0$ to find an approximate solution to the mechanism design problem of the given receding horizon solution

Mechanism Design Problem

Consensus Problem : D. Bauso et al. [6]

Theorem 1 [6]

Consider an agent i with first-order dynamics, at times $t_k = 0,1,\ldots$, assign it an objective function as (1) whose penalty function is

$$F(\hat{x}_i(\tau,t_k)) = \rho\left(\frac{1}{\mathrm{dg}/\mathrm{d}x_i}\sum_{j\in N_i}\left(\theta\left(x_j(t_k)\right) - \theta\left(\hat{x}_i(\tau,t_k)\right)\right)\right)^2,$$

Where $g(.) : \Re \to \Re$ is increasing, $\theta(.) : \Re \to \Re$ is concave, and $1/\mathrm{d}g(y)/\mathrm{d}y$ is convex. Then the following control policy is the unique optimal solution to the mechanism design problem of receding horizon control scheme ver.,

$$\hat{u}_i^*(\tau,t_k) = u_i(x_i(\tau)) = \frac{1}{\mathrm{dg}/\mathrm{d}x_i}\sum_{j\in N_i}\left(\theta\left(x_j(t_k)\right) - \theta\left(\hat{x}_i(\tau,t_k)\right)\right).$$

$g(.)$: significant function for determining the group decision value $\sum_i g(x_i(t)) = \text{const.} \to f(.) : \text{const.}$

Proof : ommited.

- Use naive assumption : $\hat{x}^{(i)}(\tau,t_k) = x^{(i)}(t_k),\ \forall \tau \geq t_k$ (the states of neighbors are constant over the planning horizon)
- Use the Pontryagin minimum principle for determining an optimal control protocol

Mechanism Design Problem

Consensus Problem : D. Bauso et al. [6]

Examples:

Condition
- Information graph is undirected and connected
- u_i minimizes J_i

assign ⇨ $F(x_i, x^{(i)})$	calculate ⇨ $u_i(x_i, x^{(i)})$	$f(x(0))$	Group decision value
$\left(\sum_{j\in N_i}(x_j - x_i)\right)^2$	$\sum_{j\in N_i}(x_j - x_i)$	$\frac{1}{n}\sum_i x_i(0)$	Arithmetic Mean
$\left(x_i\sum_{j\in N_i}(x_j - x_i)\right)^2$	$x_i\sum_{j\in N_i}(x_j - x_i)$	$\sqrt[n]{\prod_i x_i(0)}$	Geometric Mean
$\left(x_i^2\sum_{j\in N_i}(x_j - x_i)\right)^2$	$-x_i^2\sum_{j\in N_i}(x_j - x_i)$	$\frac{n}{\sum_i \frac{1}{x_i(0)}}$	Harmonic Mean
$\left(\frac{1}{2x_i}\sum_{j\in N_i}(x_j - x_i)\right)^2$	$\frac{1}{2x_i}\sum_{j\in N_i}(x_j - x_i)$	$\sqrt{\frac{1}{n}\sum_i x_i^2(0)}$	Mean of Order 2

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Previous Work

Pose Synchronization [5,11]

Kinematic Model

$$\begin{bmatrix}\dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i\end{bmatrix} = \begin{bmatrix}\cos\theta_i & -\sin\theta_i & 0 \\ \sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}v_{xi} \\ v_{yi} \\ \omega_i\end{bmatrix} \quad i = 1,\ldots,n \quad \cdots (2)$$

$x_i, y_i \in \Re$: position

$\theta_i \in \Re$: rotation angle

$v_{xi}, v_{yi} \in \Re$: body velocity

$\omega_i \in \Re$: body angular velocity

Control Input : v_{xi}, v_{yi}, ω_i

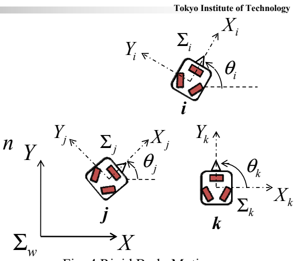


Fig. 4 Rigid Body Motion

Σ_w : inertial coordinate frame

Σ_i : body-fixed coordinate frame

Previous Work

Pose Synchronization

A group of agents is said to pose synchronize when all agents converge to the same position and orientation between the agents.

Assumptions

A1 : $|\theta_i(t)| < \frac{\pi}{2} \quad \forall i, t$

A2 : Information graph is fixed, directed and strongly connected

A3 : Information graph is fixed, undirected and connected

Theorem 2

Consider the system with n rigid bodies represented by (2). Under the assumptions A1 and A2, the following control input achieves pose synchronization.

$$\begin{bmatrix} v_{xi} \\ v_{yi} \\ \omega_i \end{bmatrix} = \sum_{j\in N_i} \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_j - x_i \\ y_j - y_i \\ \sin(\theta_j - \theta_i) \end{bmatrix} \quad \cdots(3)$$

Application to Pose Synchronization

Consider to apply mechanism design problem to pose synchronization problem

Group Objective : desired group decision value

⇒ Need to know the convergence value of pose synchronization

Group decision value : $\alpha_x, \alpha_y, \alpha_\theta$

Eq. (2), (3) ⇒

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \sum_{j\in N_i}(x_j - x_i) \\ \sum_{j\in N_i}(y_j - y_i) \\ \sum_{j\in N_i}\sin(\theta_j - \theta_i) \end{bmatrix} \quad \cdots(4)$$

As for x_i, y_i, we can interpret (4) as the consensus problem considered in [1].

Previous Work

Consensus Problem : R.Olfati-Saber et al. [1]

Information graph is undirected and connected

$$(\alpha_x \quad \alpha_y) = \left(\frac{1}{n}\sum_i x_i(0) \quad \frac{1}{n}\sum_i y_i(0) \right)$$

Group decision value is the average of all agents' initial positions.

Information graph is directed and strongly connected

$$(\alpha_x \quad \alpha_y) = \left(\frac{\sum_i \gamma_i x_i(0)}{\sum_i \gamma_i} \quad \frac{\sum_i \gamma_i y_i(0)}{\sum_i \gamma_i} \right)$$

Group decision value is the weighted average of all agents' initial positions.

$\gamma := [\gamma_1 \ \cdots \ \gamma_n]^T, \forall \gamma_i > 0$: a left eigenvectors of L corresponding to eigenvalue 0 whose all elements are positive [4]

Application to Pose Synchronization

In the case that an information graph is undirected and connected, we can say that the control inputs about positions in Theorem 2 is one of the solutions of mechanism design problem corresponding to

$$\begin{cases} F_1(x_i, x^{(i)}) = \left(\sum_{j\in N_i}(x_j - x_i)\right)^2 \\ F_2(y_i, y^{(i)}) = \left(\sum_{j\in N_i}(y_j - y_i)\right)^2 \end{cases}$$

where each agent's utility is that each input minimizes F_1 and F_2.

$x^{(i)} \in \Re^n \quad \left(x_j^{(i)} = \begin{cases} x_j & j \in N_i \\ 0 & otherwise \end{cases} \right)$

$y^{(i)} \in \Re^n \quad \left(y_j^{(i)} = \begin{cases} y_j & j \in N_i \\ 0 & otherwise \end{cases} \right)$

Lastly, analyze the group decision value about attitudes for preliminary to future work

Group Decision Value about Attitudes

- Undirected Information Graph

Theorem 3

Assume all the conditions except for replacing A2 with A3 in theorem 2 hold. Then, the control input (3) achieves pose synchronization and the group decision value about attitudes will be the average of all agents' initial attitudes. Namely,

$$\lim_{t\to\infty}\theta_i(t) = \frac{1}{n}\sum_i \theta_i \quad \forall i$$

Proof (one about convergence to a same value is omitted) :

Undirected graph ⇒ $\sum_i \dot{\theta}_i(t) = \sum_i \sum_{j\in N_i} \sin(\theta_j(t) - \theta_i(t)) = 0 \quad \forall t$

⇒ $\sum_i \theta_i(t)$ is an invariant quantity

⇒ $\lim_{t\to\infty} \sum_i \theta_i(t) = \sum_i \alpha_\theta = \sum_i \theta_i(0)$

⇒ $\alpha_\theta = \frac{1}{n}\sum_i \theta_i(0)$ □

α_θ : group decision value about attitudes

Group Decision Value about Positions

Simulation

Initial States

$\theta_1(0) = \frac{\pi}{3}, \quad \theta_2(0) = \frac{\pi}{3}$

$\theta_3(0) = 0, \quad \theta_4(0) = \frac{\pi}{3}$

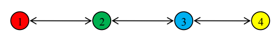
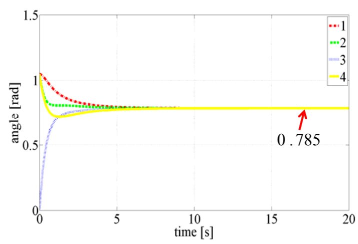


Fig. 5 Graph Structure (undirected, connected)

Group Decision Value

$$\alpha_\theta = \frac{\frac{\pi}{3} + \frac{\pi}{3} + 0 + \frac{\pi}{3}}{4} = \frac{\pi}{4} \approx 0.785 \text{ [rad]}$$



Group Decision Value about Attitudes

This results can be applied to the attitude coordination problem with two-wheeled mobile robots [11].

Analysis of a group decision value about attitudes for directed information graphs is in progress…

I've not achieved the individual objective function about attitudes. It is difficult task because there exists sinusoidal term in (3).

Mechanism design problem with directed graphs is future work...

Outline

- Introduction, Preliminary
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Conclusion and Future Work

Conclusion

- Introduce mechanism design
- Try to apply mechanism design to pose synchronization
- Analyze a group decision value about attitudes

Future Work

- Study of mechanism design, game theory and potential game
- Analysis of the group decision value about attitude under weakly conditions
- Experiment of robust control with a wheeled inverted pendulum

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