Distributed Predictive Control of Linear Discrete Time Stochastic System with Mean and Covariance Constraints

FL08-16-2
Tatsuya Miyano

Outline

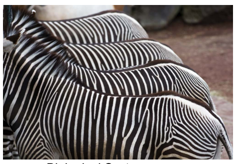
1. Introduction
2. State Feedback Control Law
3. Decomposition and Coordination
4. Output Feedback Control Law
5. Conclusions and Future Works

Background of Distributed Control

- **Background of Distributed Control**
  - **In the 1970s**
    - Mounting expectation and demand for control methodology for **Large Scale Systems**
  - **In recent years**
    - **Energy・Environmental Problems** and **Security**
    - **New Applications**

Power Plant
http://gazone.morrie.biz/

Transportation System
http://marukosugi.com/

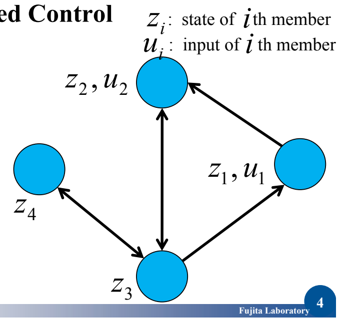


Biological System
http://www.yunphoto.net/

Distributed Control

- **Distributed Control**
  Spatio-Temporally distributed collecting and processing of information(**Information Structure**[2), 3)])
- **Advantages of Distributed Control**
  - **Time Complexity**
  - **Fault Tolerance**
  - **Expandability**
  - **Contractility**
  - **Flexibility**
  - **Economic Efficiency**

z_i: state of i th member
u_i: input of i th member



Past Researches of Distributed Control

- **Past Researches**
  - **From 1970s**
    - **Stability**,[20), 21)] **Optimality**[1)] and **Robustness**
  - **In recent years**
    - Involving **recent control theory** [4), 5)]
    - Considering **system structure** [6), 7), 8), 12), 23), 28), 29)]

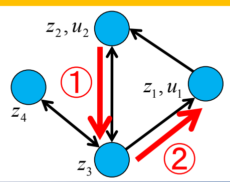
⇩

We focus on **Covariance Constraints**[6), 7), 12)] and **Decomposition Approach**[8), 14)].

Objective

- **Objective**
  We propose **Distributed Predictive Control Laws** for **Linear Discrete Time Stochastic System** with **Mean** and **Covariance Constraints**.

Note: Mean and **Covariance Constraints** represent **State** and **Input Constraints**, **Power Constraint** and **Communication Delay Structure**.



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Problem Statement

Problem 1

$$\min_{u(k)} \mathbf{Tr}\ P_{xx}V_{xx}(x(N)) + \sum_{k=0}^{N-1} \mathbf{Tr}\ QV(x(k),u(k))$$

$t \in \mathcal{Z}_+,\ x(t) \in \mathcal{R}^{n_x}$

$u(t) \in \mathcal{R}^{n_u},\ w(t) \in \mathcal{R}^{n_w}$ White Noise

subject to $x(k+1) = Ax(k) + Bu(k) + Fw(k)$

$\mathbf{Tr}\ Q_r V(x(k),u(k)) \le \gamma_{kr}$ Covariance Constraints

$\mathbf{E}\begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in \mathcal{D} \subset \mathcal{R}^{n_x+n_u}$ Mean Constraints

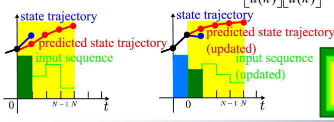
$\mathbf{Tr}\ QV(x(N), \mu'_N(x(N))) + \mathbf{Tr}\ P_{xx}V_{xx}(x(N+1)) \le \mathbf{Tr}\ P_{xx}V_{xx}(x(N))$ for Stability

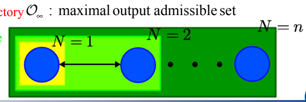
$x(N) \in \mathcal{O}_\infty$ for Feasibility

$V(x(k),u(k)) := \mathbf{E}\begin{bmatrix} x(k) \\ u(k) \end{bmatrix}\begin{bmatrix} x(k) \\ u(k) \end{bmatrix}^T$, $V_{xx}(x(k)) := \mathbf{E}\, x(k)x^T(k)$, $Q > 0$

Q_r : symmetric matrix, $r = 1,2,\ldots,m$

$\mathcal{O}_\infty$: maximal output admissible set





Example 1

Example 1

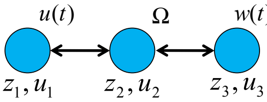
$$\begin{bmatrix} z_1(t+1) \\ z_2(t+1) \\ z_3(t+1) \end{bmatrix} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & 0 \\ \Xi_{21} & \Xi_{22} & \Xi_{23} \\ 0 & \Xi_{32} & \Xi_{33} \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \end{bmatrix} + \begin{bmatrix} \Upsilon_{11} & 0 & 0 \\ 0 & \Upsilon_{22} & 0 \\ 0 & 0 & \Upsilon_{33} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} + \begin{bmatrix} \Omega_{11} & 0 & 0 \\ 0 & \Omega_{22} & 0 \\ 0 & 0 & \Omega_{33} \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \end{bmatrix}$$

$z(t+1)$ $\quad \Xi \quad$ $z(t)$ $\quad \Upsilon \quad$ $u(t)$ $\quad \Omega \quad$ $w(t)$

$I_1(t) = (z_1(t),\ z_2(t-1),\ z_3(t-2))$

$I_2(t) = (z_1(t-1),\ z_2(t),\ z_3(t-1))$

$I_3(t) = (z_1(t-2),\ z_2(t-1),\ z_3(t))$



I_i : information that i th member can obtain

w_i : disturbance of i th member

⇩ **Communication Delay** is reduced to **Delay of Disturbance**.

$I_1(t) = (z(t-2),\ w_1(t-1),\ w_1(t-2),\ w_2(t-2))$

$I_2(t) = (z(t-2),\ w_1(t-2),\ w_2(t-1),\ w_2(t-2),\ w_3(t-2))$

$I_3(t) = (z(t-2),\ w_2(t-2),\ w_3(t-1),\ w_3(t-2))$

Mean and Covariance Constraints

Communication Delay

$\mathbf{E}\ u_1(t)w_2(t-1) = 0$ Expectation

$\mathbf{E}\ u_1(t)w_3(t-1) = 0$

$\mathbf{E}\ u_1(t)w_3(t-2) = 0$

$\mathbf{E}\ u_2(t)w_1(t-1) = 0$

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$\mathbf{E}\ u_3(t)w_1(t-1) = 0$

$\mathbf{E}\ u_3(t)w_1(t-2) = 0$

$\mathbf{E}\ u_3(t)w_2(t-1) = 0$

Power Constraint

$\mathbf{E}\ z^T(t)Q_z z(t) + u^T(t)Q_u u(t) \le \gamma$

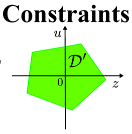
Note: An Extended State Realization

$$A := \begin{bmatrix} \Xi & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \end{bmatrix},\ B := \begin{bmatrix} \Upsilon \\ 0 \\ 0 \end{bmatrix},\ F := \begin{bmatrix} \Omega \\ I \\ 0 \end{bmatrix}$$

$$x(t) := \begin{bmatrix} z(t) \\ w(t-1) \\ w(t-2) \end{bmatrix}$$

State and Input Constraints

$$\mathrm{E}\begin{bmatrix} z(t) \\ u(t) \end{bmatrix} \in \mathcal{D}' \subset \mathcal{R}^{n_z+n_u}$$



Stability and Optimal Solution

Lemma 1(Stability)

If the following conditions are satisfied,

$\mathbf{Tr}\ QV(x(N), \mu'_N(x(N))) + \mathbf{Tr}\ P_{xx}V_{xx}(x(N+1)) \le \mathbf{Tr}\ P_{xx}V_{xx}(x(N))$

$x(N) \in \mathcal{O}_\infty$

then

$$\lim_{t\to\infty} V(x(t),u(t)) = 0.$$

Lemma 2(Optimal Solution)

Problem 1 is reduced to a **Convex Optimization Problem** involving an **LMI**.

Numerical Simulation

Simulation Setting

$$\min_u \sum_{k=0}^{1} \mathbf{E}\ 150(z_2 - r_{tgt})^2 + 150(z_1 - (z_2 - r_{tgt}))^2 + 150((z_2 - r_{tgt}) - z_3)^2 + + u_1^2 + u_2^2 + u_3^2$$

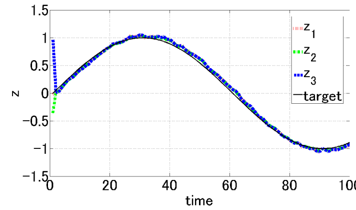
subject to

$$\sum_{k=0}^{1} \mathbf{E}\ 150(z_2 - r_{tgt})^2 + 150(z_1 - (z_2 - r_{tgt}))^2 + 150((z_2 - r_{tgt}) - z_3)^2 + u_1^2 + u_2^2 + u_3^2 \le 1$$

$\mathbf{E}\ u_i w_j = 0,\ {}^{\exists}i, j$

$\mathbf{E}\ |u_i| \le 1,\ i = 1, 2, 3$

$r_{tgt}(t) := \sin(2\pi t / 120)$



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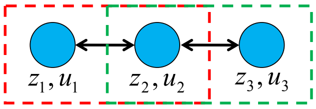
Example 2

- **Example 2**



Subsystem 1 **Subsystem 2**

We assume that $N = 2$ and we decompose only **Inputs** for simplicity.

Input Sequence:

$$U(0) = [u_1(0)\ \ u_2(0)\ \ u_3(0)\ \ u_1(1)\ \ u_2(1)\ \ u_3(1)]^T$$

$$U_1(0) = [u_1(0)\ \ u_2^1(0)\ \ u_1(1)\ \ u_2^1(1)]^T \quad U_2(0) = [u_2^2(0)\ \ u_3(0)\ \ u_2^2(1)\ \ u_3(1)]^T$$

Lagrange Multipliers

We use **Lagrange Multipliers**.

$$U(0) = [u_1(0)\ \ u_2(0)\ \ u_3(0)\ \ u_1(1)\ \ u_2(1)\ \ u_3(1)]^T$$

$$U_1(0) = [u_1(0)\ \ u_2^1(0)\ \ u_1(1)\ \ u_2^1(1)]^T \quad U_2(0) = [u_2^2(0)\ \ u_3(0)\ \ u_2^2(1)\ \ u_3(1)]^T$$

Note:

$$\lambda_{c1}(u_2^1(0) - u_2^2(0)) = 0, \quad \lambda_{c2}(u_2^1(1) - u_2^2(1)) = 0$$

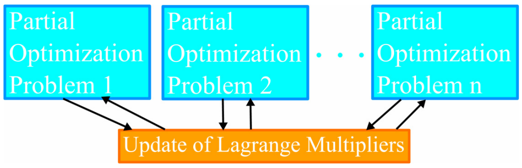
$$\lambda_{c1} \geq 0,\ \lambda_{c2} \geq 0$$

Decomposition and Coordination

- **Lemma 3(Decomposition and Coordination)**

Problem 1 is solved by computing **Partial Optimization Problems** of each subsystem and updating Lagrange Multipliers(by **Gradient Method**) .

Note: Lemma 3 shows that **Problem 1** is solved by **Distributed Information** and **Distributed Computing**.



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Problem Statement

- **Problem 2**

$$\min_{u(k)} \mathbf{Tr}\ P_{xx}V_{xx}(x(N)) + \sum_{k=0}^{N-1} \mathbf{Tr}\ QV(x(k), u(k))$$

subject to $x(k+1) = Ax(k) + Bu(k) + Fw(k)$ $\quad t \in \mathcal{Z}_+,\ x(t) \in \mathcal{R}^{n_x}$

$y(k) = Cx(k) + Gv(k)$ $\quad u(t) \in \mathcal{R}^{n_u},\ w(t) \in \mathcal{R}^{n_w},\ v(t) \in \mathcal{R}^{n_v}$

$\mathcal{Y}(k) = (y(0), y(1), \ldots, y(k))$ $\quad$ White Noise

$u(k) = \mu_k(\mathcal{Y}(k))$

$\mathbf{Tr}\ Q_r V(x(k), u(k)) \leq \gamma_{kr}$ $\quad$ Covariance Constraints

$\mathbf{E}\begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in \mathcal{D} \subset \mathcal{R}^{n_x+n_u}$ $\quad$ Mean Constraints

$\mathbf{Tr}\ QV(x(N), \mu_N'(x(N))) + \mathbf{Tr}\ P_{xx}V_{xx}(x(N+1)) \leq \mathbf{Tr}\ P_{xx}V_{xx}(x(N))$ $\quad$ for Stability

$x(N) \in \mathcal{O}_\infty$ $\quad$ for Feasibility

Lemma 4

- **Lemma 4(Optimal Solution)**

  Problem 2 is reduced to **Problem 3**.

- **Problem 3**

$$\min_{u(k)} \mathrm{Tr}\ P_{xx}V_{xx}(\hat{x}(N)) + \sum_{k=0}^{N-1} \mathrm{Tr}\ QV(\hat{x}(k), u(k))$$

subject to $\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + FK(k)(y(k) - C\hat{x}(k))$

$\mathcal{I}(k) = (\hat{x}(0), \hat{x}(1), \ldots, \hat{x}(k))$ Kalman Gain

$u(k) = \mu_k(\mathcal{I}(k))$

$\mathbf{Tr}\ Q_r V(\hat{x}(k), u(k)) \le \hat{\gamma}_{kr}$ Covariance Constraints

$\mathbf{E}\begin{bmatrix}\hat{x}(k)\\ u(k)\end{bmatrix} \in \hat{\mathcal{D}} \subset \mathcal{R}^{n_x+n_u}$ Mean Constraints

$\mathbf{Tr}\ QV(\hat{x}(N), \mu'_N(\hat{x}(N))) + \mathbf{Tr}\ P_{xx}V_{xx}(\hat{x}(N+1)) \le \mathbf{Tr}\ P_{xx}V_{xx}(\hat{x}(N))$

$\hat{x}(N) \in \hat{\mathcal{O}}_\infty$ for Feasibility for Stability

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Conclusions and Future Works

- **Conclusions**
  - We have proposed **Distributed Predictive Control Laws**.
- **Future Works**
  - Considering extended **Distributed Filtering**
  - Extensions to **Non-linear** and **Infinite Dimensional Systems**
  - Precise evaluation of **Time Complexity** and **Fault Tolerance**
  - Considering **Spatially Inhomogeneous Disturbance** and **Optimal Decomposition**

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