Pose Synchronization of wheel robots

FL08 -11-1

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Outline

- Introduction, Preliminary
- Omnidirectional Wheel Robot
  - Attitude Coordination
  - Pose Synchronization
- Two-wheel Robot
  - Attitude Coordination
  - Pose Synchronization
    - » Approach, Previous Work, Research Progression
- Experiment
- Conclusion and Future Works

Introduction

Cooperative Control

A distributed control strategy that achieves specified tasks in multi-agent system

Fig. 1 School of Fishes(※)

Motivation

・Analysis of emergent and self-organized swarming behaviors in biological groups with distributed agent-to-agent interaction

・Interest in a group behavior of animals, formulation control of multi-vehicle systems and so on

Application

Mobile sensor networks, Robot networks, and many other Multi-agent systems

※ http://www.allposters.co.jp/-sp/-Posters_i1006775_.htm

Preliminary

- Graph : A set of connections (Edges) of between Objects (Vertice)

  Vertex (node) : Agent Edge : Information Flow

  -Directed Graph (Fig. 2) : the information flows from agent j to i

  -Undirected Graph (Fig. 3) : the information flows to both directions

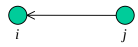

Fig. 2 Directed Graph

- Directed Graph

  -strongly connected (Fig. 4) :

  there is a directly path connecting any two distinct nodes

  -weakly connected (Fig. 2) :

  there is a path connecting any two distinct nodes ignoring the direction

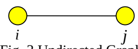

Fig. 3 Undirected Graph

- Undirected Graph

  -connected :

  there is a path between any two distinct nodes

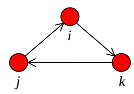

Fig. 4 Strongly Connected Graph

Preliminary

$G = (V, E)$: digraph

$V = \{v_1, \dots, v_n\}$: set of nodes (agents)

$E \subseteq V \times V$: set of edges (an edge of G : $e_{ij} = (v_i, v_j)$)

$N_i = \{v_j \in V : (v_j, v_i) \in E\}$: set of neighbors of node v_i

- Adjacency Matrix : $A = [a_{ij}] = \begin{cases} 1 & \text{if } v_j \in N_i \\ 0 & \text{otherwise} \end{cases}$

- Degree Matrix : $D = [d_{ij}] = \begin{cases} \sum_{j \neq i} a_{ij} & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

- Graph Laplacian : $L = [l_{ij}] = D - A$

$$\Rightarrow \quad L\mathbf{1} = 0, \quad \mathbf{1} = [1 \ \cdots \ 1]^T$$

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Omnidirectional Wheel Robot

Kinematic Model

$$\begin{bmatrix}\dot{x}_i\\ \dot{y}_i\\ \dot{\theta}_i\end{bmatrix} = \begin{bmatrix}\cos\theta_i & -\sin\theta_i & 0\\ \sin\theta_i & \cos\theta_i & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}v_{xi}\\ v_{yi}\\ \omega_i\end{bmatrix} \quad (1)$$

$x_i, y_i \in \Re$: position

$\theta_i \in \Re$: rotation angle

$v_{xi}, v_{yi} \in \Re$: body velocity

$\omega_i \in \Re$: body angular velocity

Control Input : v_{xi}, v_{yi}, ω_i

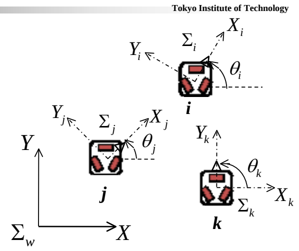

Fig. 5 Rigid Body Motion (Omni)

Σ_w : inertial coordinate frame

Σ_i : body-fixed coordinate frame

Attitude Coordination

Goal Attitude Coordination

A group of agents is said to Attitude Coordination, when all agents converge to the same orientation between the agents. Namely,

$$\lim_{t\to\infty}(\theta_i - \theta_j) = 0 \qquad \forall i, j$$

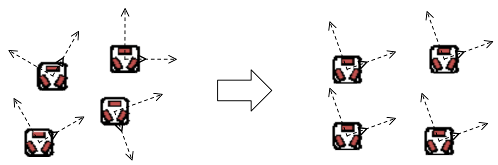
Fig. 6 Attitude Coordination (Omni)

Assumptions

Assumptions (A)

A1 : $|\theta_i| < \frac{\pi}{2} \quad \forall i$

A2 : Information graph is fixed, strongly connected and balanced

Balanced :

the total number of edges entering a node and leaving the same node are equal for all nodes (Fig. 5)

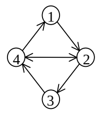

Fig. 7 Balanced Graph

Lemma

If a graph is unweighted and balanced,

$$\mathbf{1}^T L = 0, \quad \mathbf{1}^T = [1 \ \cdots \ 1]$$

Ex.) $L = \begin{bmatrix}1 & 0 & 0 & -1\\ -1 & 2 & 0 & -1\\ 0 & -1 & 1 & 0\\ 0 & -1 & -1 & 2\end{bmatrix}$

Proof of Lemma

Proof of Lemma

Let $\deg_{in}(v_i)$ represents the number of edges entering node i and $\deg_{out}(v_i)$ the number of edge leaving the same node.

Note that l_{ii} equals $\deg_{in}(v_i)$ and $\sum_{j,j\neq i} l_{ji}$ equals $\deg_{out}(v_i)$.

From the definition of L, if the graph is balanced, we have

$$\sum_j l_{ji} = \sum_{j,j\neq i}(l_{ji} + l_{ii}) = -\deg_{in}(v_i) + \deg_{out}(v_i) = 0.$$

Therefore, as the i column sum of L is the same as the ith element of the row vector $\mathbf{1}^T L$, one concludes that $\mathbf{1}^T L = 0$ if the graph is balanced.

□

Control Input and Theorem 1

Control Input

$$\omega_i = \sum_{j\in N_i}\sin(\theta_j - \theta_i) \qquad (2)$$

Theorem 1

Consider the system with n rigid bodies represented by (1). Under the assumptions (A), the control input (2) achieves attitude coordination. Namely,

$$\lim_{t\to\infty}(\theta_i - \theta_j) = 0 \qquad \forall i, j$$

Proof of Theorem 1

Proof

Define the potential function as following

$$V := \sum_{i=1}^{n}(1 - \cos\theta_i).$$

The derivative of this potential function along trajectories of the system (1) is given

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{n}\sin\theta_i \cdot \dot{\theta}_i\\ &= \sum_{i=1}^{n}\sum_{j\in N_i}\sin\theta_i \sin(\theta_j - \theta_i)\\ &= \sum_{i=1}^{n}\sum_{j\in N_i}\sin\theta_i(\sin\theta_j\cos\theta_i - \cos\theta_j\sin\theta_i)\\ &= \sum_{i=1}^{n}\sum_{j\in N_i}(-\cos\theta_j + \cos\theta_i\cos(\theta_j - \theta_i))\end{aligned}$$

Proof of Theorem 1

$$\dot V = \sum_{i=1}^{n}\sum_{j\in N_i}\left((1-\cos\theta_j)-(1-\cos\theta_i)-\cos\theta_i+\cos\theta_i\cos(\theta_j-\theta_i)\right)$$

$$= -\sum_{i=1}^{n}\sum_{j\in N_i}\underbrace{\cos\theta_i}_{>0}\underbrace{\left(1-\cos(\theta_j-\theta_i)\right)}_{\geq 0} \quad (\because (*))$$

$$\leq 0 \quad \left(\because |\theta_i| < \frac{\pi}{2}, \forall i\right) \quad (3)$$

$$(*)\quad \sum_{i=1}^{n}\sum_{j\in N_i}\left((1-\cos\theta_j)-(1-\cos\theta_i)\right) = -\mathbf{1}^T L \begin{bmatrix} 1-\cos\theta_1 \\ \vdots \\ 1-\cos\theta_n \end{bmatrix} = 0 \quad (\because \text{lemma1})$$

Therefore, V is negative semidefinite.

⇨ Use LaSalle's Invariance Principle

Proof of Theorem 1

Define the set $S := \left\{\theta_i \in \Re, \forall i \,|\, |\theta_i| < \frac{\pi}{2}, \dot V = 0\right\}$

From (A1) and (3),

$$\dot V = 0 \Rightarrow \theta_i = \theta_j, (j,i) \in E$$

Because of the strong connectivity of the graph,

$$S = \left\{\theta_i \in \Re, \forall i \,|\, |\theta_i| < \frac{\pi}{2}, \theta_i = \theta_j, \forall i, j\right\}$$

In addition,

$$\theta_i = \theta_j, \forall i, j \Rightarrow \omega_i = 0, \forall i$$

This implies that the set S is an invariant set.

⇨ Attitude coordination is achieved. □

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Pose Synchronization

Goal Pose Synchronization

A group of agents is said to pose synchronize when all agents converge to the same position and orientation between the agents. Namely,

$$\begin{cases} \lim_{t\to\infty}(x_i - y_j) = 0 \\ \lim_{t\to\infty}(y_i - y_j) = 0 \quad \forall i, j \\ \lim_{t\to\infty}(\theta_i - \theta_j) = 0 \end{cases}$$

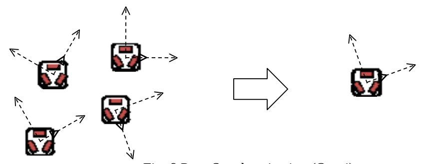

Fig. 8 Pose Synchronization (Omni)

Control Input and Theorem 2

Control Input

$$\begin{bmatrix} v_{xi} \\ v_{yi} \\ \omega_i \end{bmatrix} = \sum_{j\in N_i}\begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_j - x_i \\ y_j - y_i \\ \sin(\theta_j - \theta_i) \end{bmatrix} \quad (4)$$

Theorem 2

Consider the system with n rigid bodies represented by (1). Under the assumptions (A), the control input (4) achieves pose synchronization. Namely,

$$\begin{cases} \lim_{t\to\infty}(x_i - y_j) = 0 \\ \lim_{t\to\infty}(y_i - y_j) = 0 \quad \forall i, j \\ \lim_{t\to\infty}(\theta_i - \theta_j) = 0 \end{cases}$$

Proof of Theorem 2

Proof

For the convenience of notation, let $p_i = [x_i \quad y_i]^T$.

Define the potential function as following

$$\bar V := \sum_{i=1}^{n}\left(\frac{1}{2}\|p_i\|^2 + (1-\cos\theta_i)\right).$$

The derivative of this potential function along trajectories of the system (1) is given

$$\dot{\bar V} = \sum_{i=1}^{n}\left(p_i^T \dot p_i + \sin\theta_i \cdot \dot\theta_i\right)$$

$$= \sum_{i=1}^{n}\sum_{j\in N_i}\left(p_i^T(p_j - p_i) + \sin\theta_i \sin(\theta_j - \theta_i)\right)$$

$$= \sum_{i=1}^{n}\sum_{j\in N_i}\left(-\frac{1}{2}\|p_i\|^2 + \frac{1}{2}\|p_j\|^2 - \frac{1}{2}\|p_i - p_j\|^2 - \cos\theta_j + \cos\theta_i\cos(\theta_j - \theta_i)\right)$$

Proof of Theorem 2

$$\dot{V} = \sum_{i=1}^{n}\sum_{j\in N_i}\left(-\frac{1}{2}\underbrace{\|p_i - p_j\|^2}_{\ge 0} - \underbrace{\cos\theta_i}_{>0}\underbrace{\left(1-\cos(\theta_j-\theta_i)\right)}_{\ge 0}\right) \quad (\because (**))$$

$$\le 0 \quad \left(\because |\theta_i| < \frac{\pi}{2} \quad \forall i\right) \quad (5)$$

$$(**)\ \sum_{i=1}^{n}\sum_{j\in N_i}\left(\frac{1}{2}\|p_j\|^2 - \frac{1}{2}\|p_i\|^2 + (1-\cos\theta_j) - (1-\cos\theta_i)\right) = -\mathbf{1}^T L \begin{bmatrix} \frac{1}{2}\|p_1\|^2 + (1-\cos\theta_1) \\ \vdots \\ \frac{1}{2}\|p_n\|^2 + (1-\cos\theta_n) \end{bmatrix}$$

$$= 0 \quad (\because \text{lemma1})$$

Therefore, $\bar{V}$ is negative semidefinite.

⇨ Use LaSalle's Invariance Principle

Proof of Theorem 2

Define the set $\bar{S} := \left\{(x_i, y_i, \theta_i) \in \Re^3, \forall i \,|\, |\theta_i| < \frac{\pi}{2}, \dot{V} = 0\right\}$

From (A1) and (5),

$$\dot{V} = 0 \Rightarrow x_i = x_j, y_i = y_j, \theta_i = \theta_j, (j,i) \in E$$

Because of the strong connectivity of the graph,

$$\bar{S} = \left\{(x_i, y_i, \theta_i) \in \Re^3, \forall i \,|\, |\theta_i| < \frac{\pi}{2}, x_i = x_j, y_i = y_j, \theta_i = \theta_j, \forall i, j\right\}$$

In addition,

$$x_i = x_j, y_i = y_j, \theta_i = \theta_j, \forall i, j \Rightarrow v_{xi} = 0, v_{yi} = 0, \omega_i = 0, \forall i$$

This implies that the set $\bar{S}$ is an invariant set.

⇨ Pose Synchronization is achieved. □

Expansion of Pose Synchronization

Goal Desired Group Behavior

Specifying the desired group behavior after pose synchronization is achieved

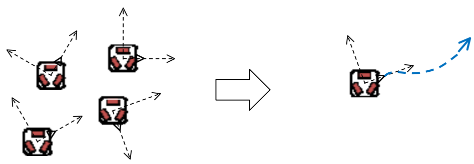

Fig. 9 Pose Synchronization with Desired Behavior

⇨ Modify the body velocity input (4)

Control Input

Control Input

$$\begin{bmatrix} v_{xi} \\ v_{yi} \\ \omega_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} v_{cx} \\ v_{cy} \\ \omega_c \end{bmatrix} + \sum_{j\in N_i}\begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_j - x_i \\ y_j - y_i \\ \sin(\theta_j - \theta_i) \end{bmatrix} \quad (6)$$

$v_{cx}, v_{cy} \in \Re$: desired velocity

$\omega_c = \dot{\theta}_c \in \Re$: desired angular velocity

Theorem 3

Theorem 3

Consider the system with n rigid bodies represented by (1) and supposed that v_{cx}, v_{cy} and ω_c represent desired group trajectories. Under the assumptions that $|\theta_i - \theta_c| < \frac{\pi}{2} \quad \forall i$ and the interconnection graph is fixed, strongly connected and balanced, the control input (6) achieves pose synchronization and desired group behavior after the synchronization.

Proof

Omit. (Please refer the resume.)

Proof uses the potential function following

$$\hat{V} := \sum_{i=1}^{n}\left(\frac{1}{2}\|\bar{p}_i\|^2 + (1-\cos\bar{\theta}_i)\right), \quad \bar{p}_i := p_i - \int_0^t v_c\,dt, \quad \bar{\theta}_i = \theta_i - \theta_c.$$

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Two-wheel Robot

Kinematic Model

$$\begin{bmatrix}\dot{x}_i\\ \dot{y}_i\\ \dot{\theta}_i\end{bmatrix}=\begin{bmatrix}\cos\theta_i & 0\\ \sin\theta_i & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v_i\\ \omega_i\end{bmatrix} \quad (7)$$

$x_i, y_i \in \Re$: position

$\theta_i \in \Re$: rotation angle

$v_i \in \Re$: velocity

$\omega_i \in \Re$: angular velocity

Control Input : v_i, ω_i

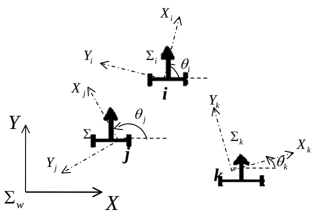

Fig. 10 Rigid Body Motion (Two-wheel)

Σ_w : inertial coordinate frame

Σ_i : body-fixed coordinate frame

Attitude Coordination

Corollary

Consider the system with n rigid bodies represented by (7). Under the assumptions (A), the control input (2) achieves attitude coordination. Namely,

$$\lim_{t\to\infty}(\theta_i-\theta_j)=0 \qquad \forall i, j$$

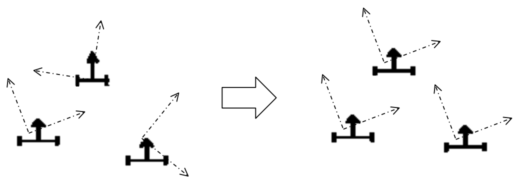

Fig. 11 Attitude Coordination (Two-wheel)

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Pose Synchronization

Goal Pose Synchronization

The goal is to achieve pose synchronization of two-wheel robots represented by model (7). Namely,

$$\begin{cases}\lim_{t\to\infty}(x_i-x_j)=0\\ \lim_{t\to\infty}(y_i-y_j)=0 \qquad \forall i, j\\ \lim_{t\to\infty}(\theta_i-\theta_j)=0\end{cases}$$

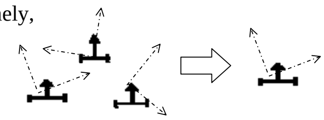

Fig. 12 Pose Synchronization (Two-wheel)

This problem is very difficult, because model (7) has the constraint as following

$$\dot{x}_i\sin\theta_i-\dot{y}_i\cos\theta_i=0$$

Henceforth, introduce my research progression.

Approach

Control of two-wheel robots' positions and attitudes : Astolfi [6]

Using this methods, we can control them toward desired ones.

Control Input

$$\begin{cases}v_i=k_\rho\rho_i\\ \omega_i=k_\alpha\alpha_i+k_\phi\phi_i\end{cases} \quad (8) \quad \left(-\frac{\pi}{2}<\alpha_i\le\frac{\pi}{2}\right)$$

k_ρ, k_α, k_ϕ : gain

$D_i(x_{di}, y_{di}, \theta_{di})$: agent i's target states

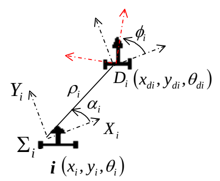

Fig. 13 Robot Kinematics (Two-wheel)

This system has the unique equilibrium point $(\rho_i, \alpha_i, \phi_i)=(0, 0, 0)$.
In my approach, use this system's target states.

Approach

One Approach

Control Input (4) ⇨ Target Position and Attitude

(A) Calculate input (4) from neighbors' states.

(B) Suppose that the two wheel robot moves freely with control input (4) for a constant time, then determine the target position and attitude after the constant time.

(C) Calculate ρ_i, α_i, ϕ_i and take control input (8).

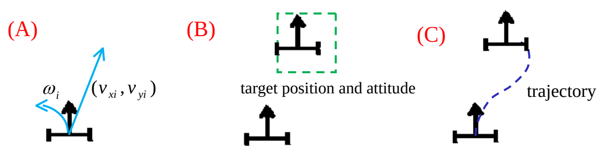

Fig. 14 One approach

Previous Work

A. Kwok and S. Martinez [8] (ACC, 2008.)

"Coverage Control with Unicycles via Hybrid Modeling"

- Use Astolfi's approach for control unicycles' positions
- Use hybrid system (many hybrid automata, flow state, jump state)
- Use 'Hybrid LaSalle's Invariance Principle' for analysis
- Consider only agents' positions
- Astolfi's approach makes ε -neighborhood of the target point

Flow State

- Forward motion mode
- Rotation mode
- Resting mode

Jump State

- Switching direction of travel
- Forward motion to rotation
- Rotation to forward motion
- Forward motion or rotation to resting
- resting to forward motion

Research Progression

Control Input (4)

$$\begin{bmatrix} v_{xi}\\ v_{yi}\\ \omega_i\end{bmatrix} = \sum_{j\in N_i}\begin{bmatrix}\cos\theta_i & \sin\theta_i & 0\\ -\sin\theta_i & \cos\theta_i & 0\\ 0&0&1\end{bmatrix}\begin{bmatrix}x_j-x_i\\ y_j-y_i\\ \sin(\theta_j-\theta_i)\end{bmatrix}$$

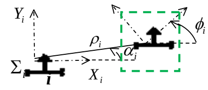

Fig. 15 Looks of analysis

Target position and attitude body-fixed coordinate frame(after 1 [s])

$$\begin{bmatrix} x_{di}\\ y_{di}\\ \theta_{di}\end{bmatrix} = \begin{bmatrix}\cos\theta_i & \sin\theta_i & 0\\ -\sin\theta_i & \cos\theta_i & 0\\ 0&0&1\end{bmatrix}\begin{bmatrix}\sum_{j\in N_i}(x_j-x_i)\\ \sum_{j\in N_i}(y_j-y_i)\\ \sum_{j\in N_i}\sin(\theta_j-\theta_i)\end{bmatrix} \Rightarrow \begin{bmatrix}\rho_i\\ \alpha_i\\ \phi_i\end{bmatrix} = \begin{bmatrix}\left\| \sum_{j\in N_i}(x_j-x_i) \quad \sum_{j\in N_i}(y_j-y_i) \right\| \\ \tan^{-1}\dfrac{\left(-\sin\theta_i\sum_{j\in N_i}(x_j-x_i)+\cos\theta_i\sum_{j\in N_i}(y_j-y_i)\right)}{\left(\cos\theta_i\sum_{j\in N_i}(x_j-x_i)+\sin\theta_i\sum_{j\in N_i}(y_j-y_i)\right)} \\ \sum_{j\in N_i}\sin(\theta_j-\theta_i)\end{bmatrix}$$

Control Input (8)

$$\Rightarrow \begin{cases} v_i = k_\rho \rho_i \\ \omega_i = k_\alpha \alpha_i + \cancel{k_\phi \phi_i} \end{cases}$$

I challenge to synchronize only positions first. I'm considering and calculating an appropriate potential function･･･(in progress).

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Experiment Environment

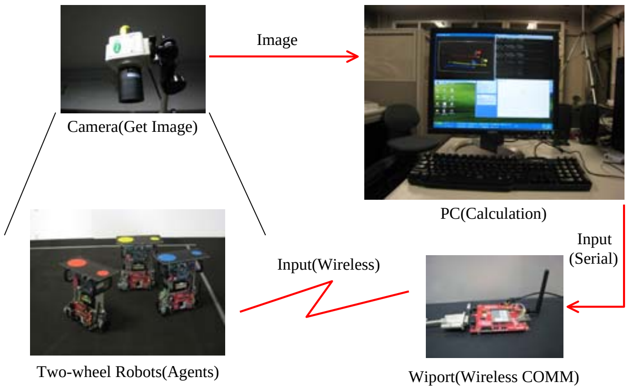


Fig. 16 Experiment environment

Experiment

For collision avoidance, set up the virtual positions and synchronize them.

Gains: $k_\rho = 0.05, k_\alpha = 0.4, k_\phi = -0.075$

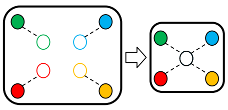
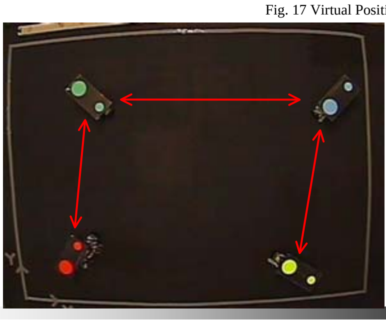


Fig. 17 Virtual Position Synchronization



Experimental Result

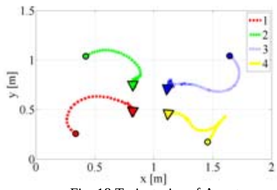


Fig. 18 Trajectories of Agents

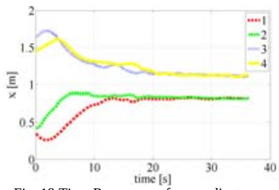


Fig. 19 Time Responses of *x*-coordinates

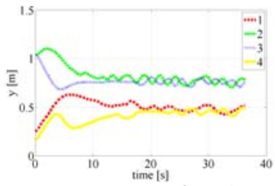


Fig. 20 Time Responses of *y*-coordinates

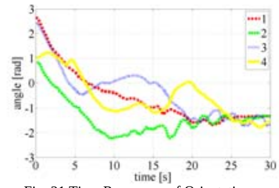


Fig. 21 Time Responses of Orientations

Outline

- Introduction , Preliminary
- Omnidirectional Wheel Robot
  - Attitude Coordination
  - Pose Synchronization
- Two-wheel Robot
  - Attitude Coordination
  - Pose Synchronization
    - » Approach, Previous Work, Research Progression
- Experiment
- Conclusion and Future Works

Conclusion and Future Works

Conclusion

- Achieve pose synchronization of omnidirectional wheel robots
- Achieve attitude coordination of two-wheel robots
- Show the experiment of pose synchronization of two-wheel robots

Future Works

- Analyze pose synchronization of two-wheel robots
- Accomplish graduation thesis

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