Team Theory: An Extension to Predictive Control

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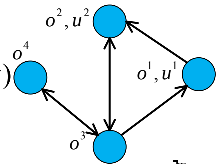
Outline

1. Introduction
2. State Feedback Predictive Control Law
3. Distributed Synthesis Procedure
4. Conclusions and Future Works

Discrete Time Linear Stochastic System

Distributed Control [1)]

$$\Sigma' \begin{cases} x(t+1) = Ax(t) + Bu(t) + Ww(t) \\ o(t) = Fx(t) + Gu(t) + v(t) \end{cases}$$



$u(t) = \left[u^1(t), u^2(t), \ldots, u^m(t)\right]^T$, $o(t) = \left[o^1(t), o^2(t), \ldots, o^p(t)\right]^T$,

$x(0) = x_0, t = 0,1,\ldots,T$. m control stations

$O_t = \{(\tau, k) \mid \tau = 0,1,\ldots,t; k = 1,2,\ldots,p\}$ p observation posts

$y^i(t) = \{o^k(\tau) \mid (\tau, k) \in Y_t^i \subset O_t\}$

observation data utilized by control station i

$$u^i(t) = \gamma^i(t, y^i(t))$$

Distributed Control Problem

The **common goal** for all control stations is as follows.

team

Problem 1 [1)]

$$\min_{\mathcal{U}} E\left\{\sum_{t=1}^{T} \left(x(t+1)^T Q x(t+1) + u(t)^T R u(t)\right)\right\}$$

$Q \geq 0$, $R > 0$; E: expectation

subject to

$$\mathcal{U} = \{u^i(t) = \gamma^i(t, y^i(t)) \mid t = 0,1,\ldots,T; i = 1,2,\ldots,m\}$$

Remark 1 [1)]

$Y_t^i = O_t$ ➡ classical information structure

$Y_t^i \neq O_t$ ➡ nonclassical information structure

LQG with Nonclassical Information Structure

Remark 2 [1)]

(A) If information structure is nonclassical, then optimal solution of LQG control problem $\gamma^{opt} \in S$ is not always an affine function ($\gamma^{opt}(y) \neq Ky + k$).

(B) If we restrict $\gamma \in S$ to an affine function and information structure is nonclassical, then LQG control problem is in general nonconvex optimization problem.

⬇

We will consider the special case.

Static Team Problem

Definition 1 [1)]

If no element of $\hat{u} = \{u^j(\tau) \mid j = 1,2,\ldots,m; \tau = 1,2,\ldots,t\}$ affects $y^i(t)$, then we call the **distributed control problem** the **static team problem**.

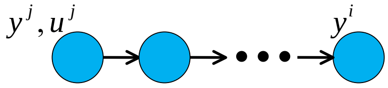
Theorem 1 [1)]

Static team problem is convex optimization problem and the only optimal solution is an affine function.

Partially Nested Information Structure

Definition 2 [1), 2)]

If what u^j affects y^i implies $y^j \subset y^i$ for all i, j, then the information structure is called **partially nested**.



Theorem 2 [1)]

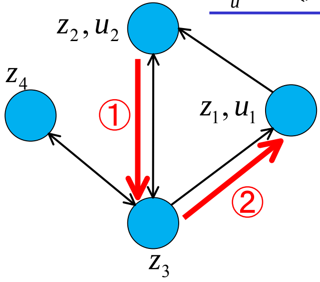
If the information structure is **partially nested**, then the **distributed control problem** is equivalent to the **static team problem**.

Distributed Control by Covariance Constraints

Problem 2 [6), 7)]

$$\min_u E\left\{\|z\|^2 + \|u\|^2\right\}$$

stationary variance



$$\begin{bmatrix} z_1(t+1) \\ z_2(t+1) \\ z_3(t+1) \\ z_4(t+1) \end{bmatrix} = \begin{bmatrix} \Phi_{11} & 0 & \Phi_{13} & 0 \\ \Phi_{21} & \Phi_{22} & \Phi_{23} & 0 \\ 0 & \Phi_{32} & \Phi_{33} & \Phi_{34} \\ 0 & 0 & \Phi_{43} & \Phi_{44} \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \end{bmatrix} + \begin{bmatrix} \Gamma_1 & 0 \\ 0 & \Gamma_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} + \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \\ w_4(t) \end{bmatrix}$$

$$u_1(t) = \mu_1(\bar{z}_1(t), \bar{z}_2(t-2), \bar{z}_3(t-1), \bar{z}_4(t-2))$$

$$u_2(t) = \mu_2(\bar{z}_1(t-1), \bar{z}_2(t), \bar{z}_3(t-1), \bar{z}_4(t-2))$$

$$\bar{z}_i(t) = \begin{bmatrix} z_i(t) \\ z_i(t-1) \\ z_i(t-2) \\ \vdots \end{bmatrix}$$

Distributed Control by Covariance Constraints

$$u_1(t) = \mu_1(\bar{z}_1(t), \bar{z}_2(t-2), \bar{z}_3(t-1), \bar{z}_4(t-2))$$

$$u_2(t) = \mu_2(\bar{z}_1(t-1), \bar{z}_2(t), \bar{z}_3(t-1), \bar{z}_4(t-2))$$

$\bar{z}(t-2)$ is replaced by $z(t-2)$.

$$u_1(t) = \bar{\mu}_1(z(t-2), w_1(t-2), w_1(t-1), w_3(t-2))$$

$$u_2(t) = \bar{\mu}_2(z(t-2), w_1(t-2), w_2(t-2), w_2(t-1), w_3(t-2))$$

$$z(t+1) = \Phi z(t) + \Gamma u(t) + w(t)$$

Time delay is at most 2-steps.

$$x(t+1) = Ax(t) + Bu(t) + Ww(t)$$

$$x(t) = \left[z(t)^T \quad w(t-1)^T \quad w(t-2)^T\right]^T$$

$$= \left[x_1(t) \quad x_2(t) \quad \dots \quad x_{12}(t)\right]^T$$

Distributed Control by Covariance Constraints

The **covariance constraints** are as follows.

$$\gamma \geq E\left(\|z\|^2 + \|u\|^2\right) = E\left(\|x\|^2 + \|u\|^2\right) - 8$$

$$0 = Ew_2(t-1)u_1(t) = Ex_6(t)u_1(t)$$

$$0 = Ew_2(t-2)u_1(t) = Ex_{10}(t)u_1(t)$$

$$0 = Ew_3(t-1)u_1(t) = Ex_7(t)u_1(t)$$

$$0 = Ew_4(t-1)u_1(t) = Ex_8(t)u_1(t)$$

$$0 = Ew_4(t-2)u_1(t) = Ex_{12}(t)u_1(t)$$

$$0 = Ew_1(t-1)u_2(t) = Ex_5(t)u_2(t)$$

$$0 = Ew_3(t-1)u_2(t) = Ex_7(t)u_2(t)$$

$$0 = Ew_4(t-1)u_2(t) = Ex_8(t)u_2(t)$$

$$0 = Ew_4(t-2)u_2(t) = Ex_{12}(t)u_2(t)$$

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Discrete Time Linear Stochastic System

We will consider the following system:

$$\Sigma \begin{cases} x(t+1) = Ax(t) + Bu(t) + w(t) \\ y(t) = Cx(t) + v(t) \end{cases}$$

white noise

where $t \in \mathcal{Z}_+,\ x(t) \in \mathcal{R}^{n_x},\ u(t) \in \mathcal{R}^{n_u},\ y(t) \in \mathcal{R}^{n_y}$

$$\mathbf{E}\, x(0) = \mathbf{E}\, w(t) = \mathbf{E}\, v(t) = 0$$

$$\mathbf{E}\, x(0)x^T(0) = I,\ \mathbf{E}\, w(t)x^T(0) = 0$$

$$\mathbf{E}\, w(j)w^T(k) = \mathbf{E}\, v(j)v^T(k) = \mathbf{E}\, y(j)v^T(k) = 0 \quad j \neq k$$

$$\mathbf{E}\, x(j)w^T(k) = 0 \quad j \leq k.$$

Assumption 1

The information structure is **partially nested**.

Problem 3[12)]

$$\min_{\mu_t} \ \mathbf{E}\, x^T(N) P_{xx} x(N) + \sum_{t=0}^{N-1} \mathbf{E} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}^T Q \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}$$

subject to

$$x(t+1) = Ax(t) + Bu(t) + w(t)$$

$$y(t) = Cx(t) + v(t)$$

$$\mathcal{X}(t) = (x(0), x(1), \ldots, x(t))$$

$$u(t) = \mu_t(\mathcal{X}(t)) \qquad Q_i \in \mathcal{S}^{n_x+n_u},\ Q > 0$$

symmetric matrix

$$\mathbf{E} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}^T Q_i \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \le \gamma_i$$

$$i = 1, 2, \ldots, m$$

covariance constraints

Remark 3

Problem 3 itself is not always a convex optimization problem. Because $Q > 0$ implies that objective function is convex, but $Q_i \in \mathcal{S}^{n_x+n_u}$ implies that constraints are not always convex.

Theorem 3[12)]

Problem 3 is reduced to a generalized eigenvalue problem[19)].

convex optimization problem involving LMI

Remark 4(trace)

$$\mathbf{E} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}^T Q \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} = \mathbf{Tr}\, QV(x(t), u(t))$$

$$V(x(t), u(t)) = \begin{bmatrix} V_{xx}(x(t)) & V_{xu}(x(t), u(t)) \\ V_{xu}^T(x(t), u(t)) & V_{uu}(u(t)) \end{bmatrix} := \mathbf{E} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}^T$$

We propose the following predictive control problem.

Problem 4

prediction time

$$\min_{\mu_k} \ \mathbf{Tr}\, P_{xx} V_{xx}(x(t+N \mid t)) + \sum_{k=0}^{N-1} \mathbf{Tr}\, QV(x(t+k \mid t), u(t+k \mid t))$$

current time

subject to

$$x(t+1 \mid t) = Ax(t \mid t) + Bu(t \mid t) + w(t \mid t)$$

$$y(t \mid t) = Cx(t \mid t) + v(t \mid t)$$

symmetric matrix

$$\mathcal{X}(t+k \mid t) = (x(t \mid t), x(t+1 \mid t), \ldots, x(t+k \mid t))$$

$$u(t+k \mid t) = \mu_k(\mathcal{X}(t+k \mid t)) \qquad Q_i \in \mathcal{S}^{n_x+n_u},\ Q > 0$$

$$\mathbf{Tr}\, Q_i V(x(t+k \mid t), u(t+k \mid t)) \le \gamma_i \quad i = 1, 2, \ldots, m$$

covariance constraints

polyhedron

$$\mathbf{E} \begin{bmatrix} x(t+k \mid t) \\ u(t+k \mid t) \\ y(t+k \mid t) \end{bmatrix} \in \mathcal{D}, \qquad 0 \in \mathcal{D}$$

mean constraints

Remark 5

Problem 4 itself is not always a convex optimization problem. Because $Q > 0$ implies that objective function is convex, but $Q_i \in \mathcal{S}^{n_x+n_u}$ implies that constraints are not always convex.

Lemma 1(stability)

If the following conditions are satisfied, then the closed loop system is asymptotically stable.

$$\mathbf{Tr}\, QV(x(t+N), \mu_N'(x(t+N))) + \mathbf{Tr}\, P_{xx} V_{xx}(x(t+N+1)) \le$$

$$\le \mathbf{Tr}\, P_{xx} V_{xx}(x(t+N))$$

stabilizable input

$$x(t+N) \in \mathcal{O}_\infty$$

maximal output admissible set

Proof of Lemma 1

Let $J(x(t), \mu_k^*, N)$ be a Lyapunov function. $J(x(t), \mu_k^*, N)$ is defined as follows.

$$J(x(t), \mu_k^*, N) := \min_{\mu_k} \ \mathbf{Tr}\, P_{xx} V_{xx}(x(t+N)) + \sum_{k=0}^{N-1} \mathbf{Tr}\, QV(x(t+k), u(t+k))$$

Then, we obtain from the conditions

$$J(x(t+1), \mu_k^*, N) = \mathbf{Tr}\, P_{xx} V_{xx}(x(t+N+1)) + \min_{\mu_k} \sum_{k=0}^{N-1} \mathbf{Tr}\, QV(x(t+k+1), u(t+k+1))$$

$$\le \mathbf{Tr}\, P_{xx} V_{xx}(x(t+N+1)) + \min_{\mu_k} \sum_{k=1}^{N-1} \mathbf{Tr}\, QV(x(t+k), u(t+k)) +$$

$$+ \mathbf{Tr}\, QV(x(t+N), \mu_N'(x(t+N)))$$

$$J(x(t+1), \mu_k^*, N) - J(x(t), \mu_k^*, N)$$

$$\le \mathbf{Tr}\, QV(x(t+N), \mu_N'(x(t+N))) + \mathbf{Tr}\, P_{xx} V_{xx}(x(t+N+1)) -$$

$$- \mathbf{Tr}\, P_{xx} V_{xx}(x(t+N)) - \min_{\mu_k} \mathbf{Tr}\, QV(x(t), u(t)) \le -\min_{\mu_k} \mathbf{Tr}\, QV(x(t), u(t)) \le 0,$$

$$\min_{\mu_k} \mathbf{Tr}\, QV(x(t), u(t)) = 0 \Leftrightarrow \min_{\mu_k} V(x(t), u(t)) = 0.$$

□

Problem 5

We rewrite **Problem 4** as **Problem 5**.

Problem 5

$$\min_{U} \ \mathbf{Tr}\, P_{xx} V_{xx}(x(t+N\mid t)) + \mathbf{E}\left\{ \begin{bmatrix} x^T(t\mid t) & W^T \end{bmatrix} Y \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix} + U^T \Phi U + 2U^T \Psi^T \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix} \right\}$$

subject to

$$\mathbf{E}\left\{ \begin{bmatrix} x^T(t\mid t) & W^T \end{bmatrix} Y_{ki} \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix} + U^T \Phi_{ki} U + 2U^T \Psi_{ki}^T \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix} \right\} \le \gamma_i$$

$$i = 1,2,\ldots,m \quad k = 0,1,\ldots,N-1 \qquad \cdots(1)$$

$$\mathbf{E}\left\{ \Theta_{xW} \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix} + \Theta_U U \right\} \le \Gamma \qquad \cdots(2)$$

$$U := \begin{bmatrix} u(t\mid t) \\ u(t+1\mid t) \\ \vdots \\ u(t+N-1\mid t) \end{bmatrix}, \quad W := \begin{bmatrix} w(t\mid t) \\ w(t+1\mid t) \\ \vdots \\ w(t+N-1\mid t) \end{bmatrix}$$

Optimality Conditions of Problem 5

$$\mathbf{E}\left\{ 2\Phi U + 2\Psi^T \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix} \right\} + \Theta_U^T \lambda^l + 2\sum_{k=0}^{N-1}\sum_{i=1}^{m} \lambda_{ki}^q \left(\mathbf{E}\left\{ \Phi_{ki} U + \Psi_{ki}^T \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix} \right\} \right) = 0 \qquad \cdots(3)$$

$$\lambda_{ki}^q \left(\mathbf{E}\left\{ \begin{bmatrix} x^T(t\mid t) & W^T \end{bmatrix} Y_{ki} \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix} + U^T \Phi_{ki} U + 2U^T \Psi_{ki}^T \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix} \right\} - \gamma_i \right) = 0 \qquad \cdots(4)$$

$$\lambda_j^l \left(\mathbf{E}\left\{ \Theta_{xW}^j \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix} + \Theta_U^j U \right\} - \Gamma^j \right) = 0 \qquad \cdots(5)$$

$$\Theta_{xW} := \begin{bmatrix} \Theta_{xW}^{1\,T} & \Theta_{xW}^{2\,T} & \cdots & \Theta_{xW}^{r\,T} \end{bmatrix}^T, \quad \Theta_U := \begin{bmatrix} \Theta_U^{1\,T} & \Theta_U^{2\,T} & \cdots & \Theta_U^{r\,T} \end{bmatrix}^T$$

$$\Gamma := \begin{bmatrix} \Gamma^{1T} & \Gamma^{2T} & \cdots & \Gamma^{rT} \end{bmatrix}^T, \quad \lambda_{ki}^q \ge 0, \quad \lambda_j^l \ge 0, \quad \lambda^l := \begin{bmatrix} \lambda_1^l & \lambda_2^l & \cdots & \lambda_r^l \end{bmatrix}^T$$

$$i = 1,2,\ldots,m \quad k = 0,1,\ldots,N-1 \quad j = 1,2,\ldots,r$$

$$\mathbf{E}\left\{ \begin{bmatrix} x^T(t\mid t) & W^T \end{bmatrix} Y_{ki} \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix} + U^T \Phi_{ki} U + 2U^T \Psi_{ki}^T \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix} \right\} \le \gamma_i$$

$$\mathbf{E}\left\{ \Theta_{xW} \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix} + \Theta_U U \right\} \le \Gamma, \quad \hat{\Phi} := \Phi + \sum_{k=0}^{N-1}\sum_{i=1}^{m} \tilde{\lambda}_{ki}^q \tilde{\Phi}_{ki} \ge 0, \quad \hat{\Psi} := \Psi + \sum_{k=0}^{N-1}\sum_{i=1}^{m} \tilde{\lambda}_{ki}^q \tilde{\Psi}_{ki} \qquad \cdots(6)$$

※ "~" means that corresponding constraints are active.

Optimal Solutions of Problem 5

We will consider **4 patterns**.

i) **condition (1)** and **condition (2)** are **inactive**

$$U := -\Phi^{-1}\Psi^T \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix}$$

ii) **condition (1)** is **active**, but **condition (2)** is **inactive**

We can obtain an optimal solution by solving a generalized eigenvalue problem[19] from **Theorem 3**.[12]

iii) **condition (1)** is **inactive**, but **condition (2)** is **active**

We can obtain an optimal solution explicitly[17].

iv) **condition (1)** and **condition (2)** are **active**

We can obtain an optimal solution by solving a following problem.

Problem 6

Lemma 2

If **condition (1)** and **condition (2)** are **active**, then **Problem 5** is reduced to **Problem 6**.

Problem 6

$$\max_{s_1, s_2, \tilde{\lambda}_j^l, \tilde{\lambda}_{ki}^q} \ s_1$$

subject to

$$\mathbf{E}\begin{bmatrix} -s_1 + s_2 + \sum_{k=0}^{N-1}\sum_{i=1}^{m} \tilde{\lambda}_{ki}^q \left(\begin{bmatrix} x^T(t\mid t) & W^T \end{bmatrix} \tilde{Y}_{ki} \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix} - \tilde{\gamma}_i \right) & \hat{\Psi}^T \begin{bmatrix} x(t\mid t) \\ W \end{bmatrix} \\ \begin{bmatrix} x^T(t\mid t) & W^T \end{bmatrix} \hat{\Psi} & \hat{\Phi} \end{bmatrix} \ge 0$$

$$\mathbf{E}\begin{bmatrix} -2s_2 & \tilde{\Theta}_U^T \tilde{\lambda}^l \\ \tilde{\lambda}^{lT} \tilde{\Theta}_U & \hat{\Phi} \end{bmatrix} \ge 0$$

$$s_1 \in \mathcal{R}, \quad s_2 \in \mathcal{R}, \quad \tilde{\lambda}_j^l > 0, \quad \tilde{\lambda}_{ki}^q > 0, \quad \hat{\Phi} \ge 0$$

Implementation Methodologies

We will consider two approaches for implementation. First approach is Model Predictive Control approach(MPC approach). And second approach is Multi-Parametric Quadratic Programming approach(mp-QP approach).

Algorithm 1(MPC approach)

Initial Condition: $t := 0$

Step 1: Compute **Problem 5**.

Step 2: Input $u(t) := \mu_0(x(t), W)$.

Step 3: Let $t := t+1$ and go to **Step 1**.

How to compute?

⇩

future work

mp-QP Approach

Algorithm 2(mp-QP approach)

Initial Condition: $t := 0$

Step 1: Look up an optimal solution in a table.

Step 2: Input $u(t) := \mu_0(x(t), W)$.

Step 3: Let $t := t+1$ and go to **Step 1**.

How to get a table and search an optimal solution?

⇩

future work

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Information and Computing

	information	computing	
①	global	global	centralized control
②	global	local	distributed computing
③	local	global	centralized control
④	local	local	distributed control

The type of control laws that we proposed in the previous section was ① or ③ as shown above. Those are **not** *distributed control laws* **but** *centralized control laws*.

future work

We propose control laws those type is ② or ④.

Overlapping Decomposition

We will consider the following system and LQ control problem.

$$\Sigma'' : \begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$\Sigma''_1 : \begin{bmatrix} x_1(t+1) \\ x_2(t+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u_1(t)$$

$$\Sigma''_2 : \begin{bmatrix} x_2(t+1) \\ x_3(t+1) \end{bmatrix} = \begin{bmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u_2(t)$$

$$\tilde{\Sigma}'' : \begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_2(t+1) \\ x_3(t+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & A_{13} \\ A_{21} & A_{22} & 0 & A_{23} \\ A_{21} & 0 & A_{22} & A_{23} \\ A_{31} & 0 & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

Overlapping Decomposition

Theorem 4 [20)]

If K_1 and K_2 are chosen to stabilize and optimize the decoupled subsystems as shown below, then *the original system is stabilized*, **but** *this is a suboptimal control law for the original system*.

$$u_1(t) = K_1 \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \quad u_2(t) = K_2 \begin{bmatrix} x_2(t) \\ x_3(t) \end{bmatrix}$$

future work

We will consider optimality and relation with covariance and mean constraints.

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Conclusions and Future Works

Conclusions

- We have proposed a predictive control problem with covariance constraints and showed optimal solutions.
- We have discussed distributed synthesis procedure.

Future Works

- Considering non-convexity of **Problem 4**
- Considering implementation methodologies
- An extension to output feedback control law
- Considering information structures more precisely
- Considering decomposition methodologies

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