Formation Control via Receding Horizon Control :
One-step Reachable Set and Stability Analysis

FL-08-10-1
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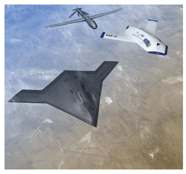
Introduction

- Multi-robot System
- Formation Control
- Collision Avoidance
- Application
  - UAVs
  - Satellite Orbit
  - Mobile Robots



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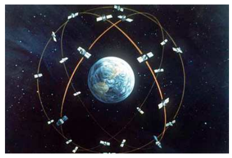

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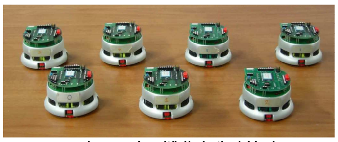
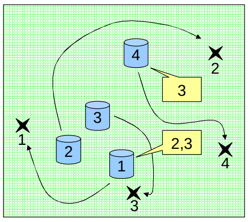


webuser.unicas.it/lai/robotica/video/

Considered Problem



- Formation Control Problem for multi-robot system
- Predefined Target
- Limited Detection Range (Spatially connected graph)
- Collision Avoidance

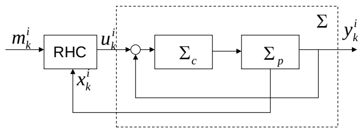
In This Seminar

- Last Seminar
  - Controller Analysis
  - Feasibility
  - Simulation
- This Seminar
  - One-step Reachable Set
  - Stability Analysis

Outline

- Introduction
- One-step Reachable Set
- Stability
- Simulation
- Conclusion

Problem Setting



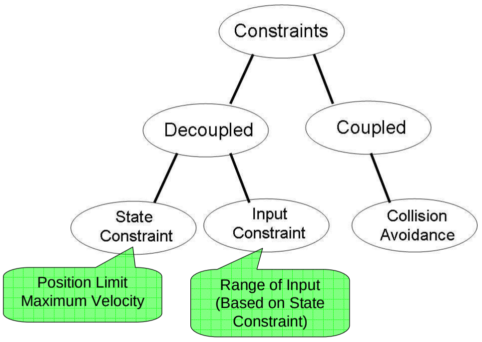
Consider the second ordered system with priori compensator

$$\Sigma: \begin{array}{l} x_{k+1}^i = Ax_k^i + Bu_k^i \\ y_k^i = Cx_k^i = [I \quad 0]x_k^i = p_k^i \end{array} \quad \begin{array}{l} x_k^i = [p_k^i \quad v_k^i]^T \in R^{n_p+n_v} \\ i \in \{1,\dots,N_v\} \\ k \in Z_+ := \{0,1,2,\dots\} \end{array}$$

- p_k^i : Position
- v_k^i : Velocity
- m_k^i : Target Position
- u_k^i : System Input

Assumption
- The matrix A is asymptotically stable
- Σ is an integral type servo system : $C(I-A)^{-1}B = I$

Constraints



Condition for Decoupled Constraints

Initial Condition for Decoupled Constraints

$$S_x = \{x_0 \in \mathrm{X} | x_k(x_0) \in \mathrm{X}, k \in Z_+\}$$

$$x_k(x_0) = (A + BC)^k x_0$$

where X is a state constraint.

$$u_k^i = p_k^i = Cx_k^i$$

Condition for Coupled Constraint

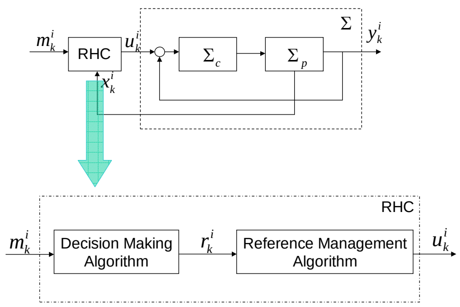
Safe region is a set of initial conditions that guarantee the collision avoidance when the agent try to STOP.

Decoupled Constraint is included !

$$S^{i,j} = \left\{ \begin{array}{l} (x_0^i, x_0^j) \,|\, (x_0^i, x_0^j) \in S_x \times S_x, \\ (x_k^i(x_0^i), x_k^j(x_0^j)) \in \mathrm{X}_{CA}, k \in Z_+ \end{array} \right\}$$

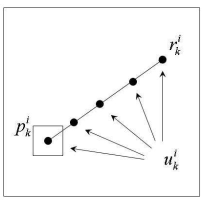
Collision Avoidance Constraint for all trajectories

Analysis of Controller



Modify the input so that the constraint fulfillment and collision avoidance are achieved.

Analysis of Controller (2)



$N_i(k)$: Neighbors of agent i at time k

- The input is defined as follow

$$u_k^i = K_k^i p_k^i + (1 - K_k^i) r_k^i, K_k^i \in [0,1]$$

To find the input u_k^i which is closest to the reference r_k^i

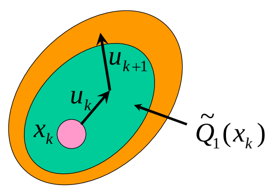
$$\min K_k^i + \sum_{j \in N_i(k)} \hat{K}_k^j$$

s.t. Model of system
Decoupled Constraint
Coupled Constraint

One-step Reachable Set

The one-step reachable set is the set of all possible next step position.

$$\tilde{Q}_1(x_k) := \left\{ p_{k+1} \in X_p \middle| \begin{array}{l} \forall u_k \in U, \\ p_{k+1} = C(Ax_k + Bu_k) \end{array} \right\}$$



Remark
The current position p_k need not to be inside the one-step reachable set

Stability Analysis

Lemma 1

$$u_k \in \tilde{Q}_1(x_k) \Leftrightarrow u_k = p_{k+1}$$

Proof

It's obvious from the definition of $\tilde{Q}_1(x_k)$,
if $u_k \in \tilde{Q}_1(x_k)$,then $u_k = C(Ax_k + Bu_k) = p_{k+1}$
and vice versa.

Stability Analysis

Lemma 2

Suppose that $N_i(k) = \phi, m^i \notin \tilde{Q}_1(x_k^i)$ and $r_k^i = m^i$
,then $K_k^i \neq 1$

Proof

From $u_k^i = K_k^i p_k^i + (1 - K_k^i) r_k^i,\ 0 \le K_k^i \le 1$
and $N_i(k) = \phi$, there is a path for agent to move
$\rightarrow u_k^i \neq p_k^i \rightarrow K_k^i \neq 1$

Stability Analysis

Lemma 3

Suppose that $r_k^i = m^i$, then $\lim_{k\to\infty} u_k^i = m^i$

Proof

From lemma 1,2 : $0 \le K_k^i < 1, u_k^i = p_{k+1}^i$

$$u_k^i = K_k^i p_k^i + (1 - K_k^i) m^i$$

$$u_k^i - m^i = K_k^i p_k^i - K_k^i m^i = K_k^i u_{k-1}^i - K_k^i m^i = K_k^i (u_{k-1}^i - m^i)$$

$$= \left(\prod_{t=1}^{k} K_t^i \right) (u_0^i - m^i)$$

$$\lim_{k\to\infty} (u_k^i - m^i) = 0 \rightarrow \lim_{k\to\infty} u_k^i = m^i$$

Stability Analysis

Lemma 4

Suppose that $r_k^i = m^i$, then $\lim_{k\to\infty} p_k^i = m^i$

Proof

From lemma 1,2 : $0 \le K_k^i < 1, u_k^i = p_{k+1}^i$

$$p_{k+1}^i = u_k^i = K_k^i p_k^i + (1 - K_k^i) m^i$$

$$p_{k+1}^i - m^i = K_k^i p_k^i - K_k^i m^i = K_k^i (p_k^i - m^i)$$

$$= \left(\prod_{t=0}^{k} K_t^i \right) (p_0^i - m^i)$$

$$\lim_{k\to\infty} (p_{k+1}^i - m^i) = 0 \rightarrow \lim_{k\to\infty} p_k^i = m^i$$

Stability Analysis

Theorem

Suppose that $N_i(k) = \phi$ and $r_k^i = m^i$, at the steady state the agent converges to the predefined configuration.

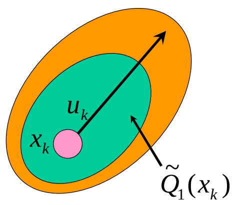
Proof

The proof is followed from lemma 3,4.

Remark

- Normally, we don't need to set the input inside the one-step reachable set because the next position always lies inside the one-step reachable set. (but the direction must be the same)
- Exception : When $m^i \in \tilde{Q}_1(x_k^i)$,the input must be

$$u_k^i = m^i$$



Simulation

$$x_{k+1}^i = \begin{bmatrix} 0.87 & 0 & 0.32 & 0 \\ 0 & 0.87 & 0 & 0.32 \\ -0.46 & 0 & 0.33 & 0 \\ 0 & -0.46 & 0 & 0.33 \end{bmatrix} x_k^i + \begin{bmatrix} 0.13 & 0 \\ 0 & 0.13 \\ 0.45 & 0 \\ 0 & 0.45 \end{bmatrix} u_k^i$$

$$y_k^i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x_k^i$$

$$d = 0.5 \quad d_s = 2.45$$

$$-10 \le p_k^i \le 10, -1 \le v_k^i \le 1$$

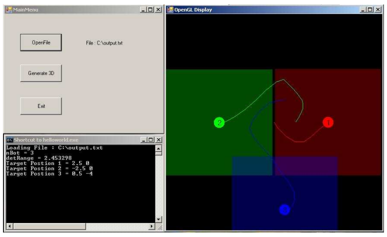
$$x_0^1 = [0 \quad 0 \quad 0 \quad 0]^T \quad m_0^1 = [2.5 \quad 0]^T$$

$$x_0^2 = [1 \quad 0 \quad 0 \quad 0]^T \quad m_0^2 = [-2.5 \quad 0]^T$$

$$x_0^3 = [0.5 \quad 1 \quad 0 \quad 0]^T \quad m_0^3 = [0.5 \quad -4]^T$$

Simulation (2)

- Visual C#
- OpenGL via Tao Library



Conclusion and Future Works

- Conclusion
  - Formation Control
  - One-step Reachable set
  - Stability
- Future Works
  - Robustness
  - Observability

Thank you