Vision-Based Cooperative Control (progress report) FL08_09_1

Naoto Kobayashi

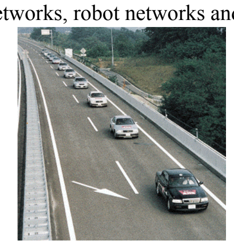
Outline

- **Introduction**
- Vision-based Cooperative Control
  - Problem Formulation
  - Analysis
  - Simulation
  - Experiment
- Conclusion / Future Works

Introduction

- **Cooperative Control**
  - Cooperative control is a distributed control strategy that achieves specified tasks in multi-agent systems.
  - It's been motivated by interests in group behavior of animals, formation control of multi-vehicle systems and so on.
  - It is hoped to be applied to sensor networks, robot networks and many other multi-agents systems.

School of Fish
http://www.yunphoto.net/



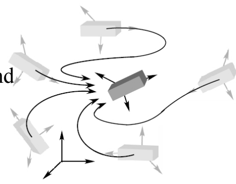
Automated Highway System
http://www.its.go.jp/ITS/

Introduction

- **Cooperative Control**
  - Consensus Problem
    : to reach an agreement regarding a certain quantity of interest that depends on the state of all agents.
  - Flocking Problem
    : to make all of agents' speeds be the same.
  - Coverage Problem
  - Formation Control Problem
    :
  - Pose Synchronization
    : to make all of the agents' positions and attitudes be the same.

in particular I address ...

leader following problem of pose synchronization

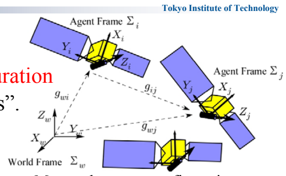


Pose synchronization

Introduction

- **Vision-based Cooperative Control**

Mounted camera (local camera) configuration
- Each agent can have its own "eyes".
→ autonomous agents system

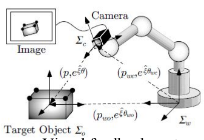


Mounted camera configuration

We have to get necessary information (neighbor's relative positions and attitudes) from camera images to achieve pose synchronization.

- **Visual Observer [4]**

- Visual observer that is proposed in [4] can estimate relative positions and attitudes between camera and target objects.



Visual feedback system with visual observer

Introduction

- **Last Seminar(FL08_01_1)**
  - Experiments (bird's eye camera configuration)
  - Simulations towards experiments(mounted camera configuration)

- **This Seminar (FL08_09_1)**
  - Simulations (mounted camera configuration - leader following -)
  - Experiments (mounted camera configuration - leader following -)
  - Convergence analysis (for easier problem)

Outline

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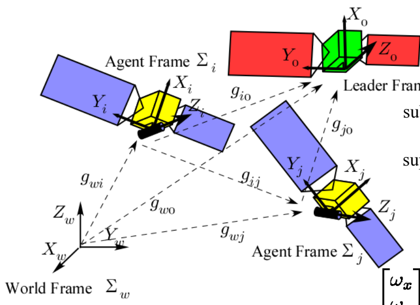
- **Vision-based Cooperative Control**
  - Problem Formulation
  - Analysis
  - Simulation
  - Experiment
- Conclusion / Future Works

Problem Formulation



p : position

$R\ (= e^{\hat{\xi}})$: attitude

v : linear velocity

ω : angular velocity

subscript ij : from frame i to frame j

superscript i : see from frame i

w : world frame

$i\ j$: agent frame

$\wedge$:

$$\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}^{\wedge} = \begin{bmatrix}0 & -\omega_z & \omega_y\\ \omega_z & 0 & -\omega_x\\ -\omega_y & \omega_x & 0\end{bmatrix}$$

$\vee$: inverse operator to $\wedge$

Problem Formulation

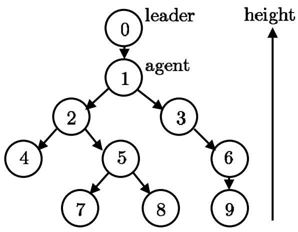
- Agents' kinematics

$$\begin{cases}\dot{p}^w_{wi} &= R_{wi}v^i_{wi}\\ \dot{R}_{wi} &= R_{wi}\hat{\omega}^i_{wi}\end{cases} \quad i = 0, 1, 2, \ldots, n$$

...(1)

- Information Graph
  - **Fixed** : topology of a graph does not change.
  - **Rooted tree** :

    one vertex has been designated the *root*, in which case the edges have a natural orientation, *towards* or *away* from the root.



Rooted tree

Problem Formulation

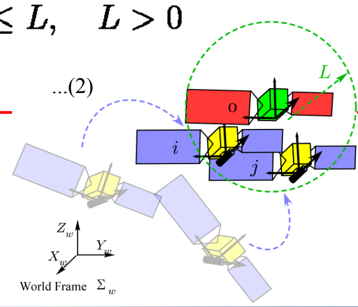
- Control Objective

Pose Synchronization in SE(3) (Leader following)

: Make all of the agents' positions converge to a certain region near the leader and make their attitudes be the same with the leader.

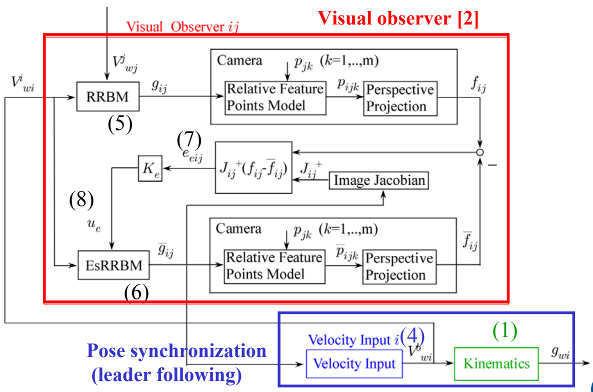
$$\begin{cases}\|p^w_{w0} - p^w_{wi}\| \le L, \quad L > 0\\ R_{w0} = R_{wi}\end{cases}$$

...(2)



Analysis

■ **Structure of Agent i with Visual Observer (Mounted camera)**



Pose synchronization (leader following)

Analysis

■ **1 Leader (standstill) and 1 follower**

- **Pose synchronization (leader following)**

Velocity Input

$$\begin{bmatrix}v^i_{wi}\\ \omega^i_{wi}\end{bmatrix} = \begin{bmatrix}k_v(\bar{p}^i_{ij} - d^i_{ij})\\ k_\omega \mathrm{sk}(\bar{R}_{ij})^{\vee}\end{bmatrix}$$

...(4)

* j : parent of agent i

$\bar{p}$: estimated position $\quad \bar{R}$: estimated attitude

$$d^i_{ij} := \frac{l}{\|\bar{p}^i_{ij}\|}\bar{p}^i_{ij}$$

: safety distance (for collision avoidance and guarantee of visibility of camera)

*$(h-1)l \le L$

h : height of the information graph

Analysis

■ **1 Leader (standstill) and 1 follower**

- **Pose synchronization (leader following)**

Difference between [1] and proposed velocity input (4)

[1] $\begin{bmatrix} v^i_{wi} \\ \omega^i_{wi} \end{bmatrix} = \sum_{j\in\mathcal{N}_i} \begin{bmatrix} p^i_{ij} \\ \mathrm{sk}(R_{ij})^\vee \end{bmatrix}$ ..(9)

- (9) consists of relative position / attitude, but the way to get the information is not showed in [1].
- strongly connected graph

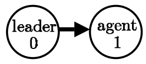
$\begin{bmatrix} v^i_{wi} \\ \omega^i_{wi} \end{bmatrix} = \begin{bmatrix} k_v(\bar{p}^i_{ij} - d^i_{ij}) \\ k_\omega \mathrm{sk}(\bar{R}_{ij})^\vee \end{bmatrix}$ ..(4)

- (4) consists of estimated relative position / attitude which can be obtained from visual observer.
- Safety distance d^j_{ji} are vectors on followers' body coordinates.
- rooted-tree graph (because a mounted camera can sense only forward direction)

Problem Formulation (easier problem)

■ **1 Leader (standstill) and 1 follower**

- Information graph



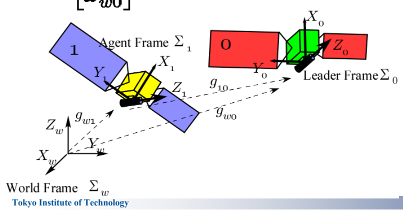
- Control objective

$$\begin{cases} \|p^w_{w0} - p^w_{w1}\| \le L, \quad L > 0 \\ R_{w0} = R_{w1} \end{cases} \quad ...(3)$$

- Ass. 1

Leader 0 is standstill.

$$\begin{bmatrix} v^0_{w0} \\ \omega^0_{w0} \end{bmatrix} = 0$$



Analysis

■ **1 Leader (standstill) and 1 follower**

- **Pose synchronization (leader following)**

Velocity Input

$$\begin{bmatrix} v^1_{w1} \\ \omega^1_{w1} \end{bmatrix} = \begin{bmatrix} k_v(\bar{p}^1_{10} - d^1_{10}) \\ k_\omega \mathrm{sk}(\bar{R}_{10})^\vee \end{bmatrix} \quad ...(4)$$

* $\bar{p}$: estimated position $\bar{R}$: estimated attitude

$d^1_{10} := \dfrac{l}{\|\bar{p}^1_{10}\|}\bar{p}^1_{10}$: safety distance (for collision avoidance and guarantee of visibility of camera)

* $l \le L$

Analysis

■ **1 Leader (standstill) and 1 follower**

- **Visual observer [2]**

- Relative Rigid Body Motion (RRBM) between leader 0 and agent 1 (with camera)

$$\begin{bmatrix} v^0_{10} \\ \omega^0_{10} \end{bmatrix} = \begin{bmatrix} R^T_{10}\dot{p}^1_{10} \\ (R^T_{10}\dot{R}_{10})^\vee \end{bmatrix} = -\begin{bmatrix} R^T_{10} & -R^T_{10}\hat{p}^1_{10} \\ 0 & R^T_{10} \end{bmatrix} \begin{bmatrix} v^1_{w1} \\ \omega^1_{w1} \end{bmatrix} + \begin{bmatrix} v^0_{w0} \\ \omega^0_{w0} \end{bmatrix} \quad ...(5)$$

- Estimated Relative Rigid Body Motion (EsRRBM)

* $\begin{bmatrix} v^0_{10} \\ \omega^0_{w0} \end{bmatrix} = 0$

$$\begin{bmatrix} \bar{v}^0_{10} \\ \bar{\omega}^0_{10} \end{bmatrix} = \begin{bmatrix} \bar{R}^T_{10}\dot{\bar{p}}^1_{10} \\ (\bar{R}^T_{10}\dot{\bar{R}}_{10})^\vee \end{bmatrix} = -\begin{bmatrix} \bar{R}^T_{10} & -\bar{R}^T_{10}\hat{\bar{p}}^1_{10} \\ 0 & \bar{R}^T_{10} \end{bmatrix} \begin{bmatrix} v^1_{w1} \\ \omega^1_{w1} \end{bmatrix} + u_e \quad ...(6)$$

Analysis

■ **1 Leader (standstill) and 1 follower**

- **Visual observer [2]**

- Estimated error

$$e_e := \begin{bmatrix} p_{ee} \\ \mathrm{sk}(R_{ee})^\vee \end{bmatrix} := \begin{bmatrix} \bar{R}^T_{10}(p^1_{10} - \bar{p}^1_{10}) \\ \mathrm{sk}(\bar{R}^T_{10}T_{10})^\vee \end{bmatrix} \quad ...(7)$$

- Estimated error system

$$\begin{bmatrix} R^T_{ee}\dot{p}_{ee} \\ (R^T_{ee}\dot{R}_{ee})^\vee \end{bmatrix} = -\begin{bmatrix} R^T_{ee} & -R^T_{ee}\hat{p}_{ee} \\ 0 & R^T_{ee} \end{bmatrix} u_e + \begin{bmatrix} v^0_{w0} \\ \omega^0_{w0} \end{bmatrix}$$

* $\begin{bmatrix} v^0_{10} \\ \omega^0_{w0} \end{bmatrix} = 0$

Input to observer

$$u_e = k_e e_e \quad ...(8)$$

➡ Estimation error will converge to 0.

Analysis

■ **1 Leader (standstill) and 1 follower**

Assume that there are 1 leader at standstill and l follower (1) mounting camera. Then with some assumption on the velocity gain k_v and estimation gain k_e , velocity input (4) and estimation input (8) achieve (at least) position synchronization (defined as upper equation of (3)).

*attitude synchronization is under consideration.

Proof :

Consider the potential function

$$V := \frac{1}{2}\|p_{ee}\|^2 + \frac{1}{2}\|\bar{p}^1_{10} - d^1_{10}\|^2$$

Derivative of this potential function is

$$\dot{V} = p^T_{ee}\dot{p}_{ee} + (\bar{p}^{1T}_{10} - d^{1T}_{10})(\dot{\bar{p}}^1_{10} - \dot{d}^1_{10})$$
$$= \ldots$$

Analysis

■ **1 Leader (standstill) and 1 follower**

Proof :

$$\dot V = p_{ee}^T \dot p_{ee} + (\bar p_{10}^{1T} - d_{10}^{1T})(\dot{\bar p}_{10}^1 - \dot d_{10}^1)$$

$$= \dots$$

$$= -k_e ||(p_{10}^1 - d_{10}^1) - \frac{1}{2}(\frac{3||\bar p_{10}^1|| - l}{||\bar p_{10}^1||} I + \frac{l \bar p_{10}^1 \bar p_{10}^{1T}}{||\bar p_{10}^1||^3})(\bar p_{10}^1 - d_{10}^1)||^2$$

$$- \frac{1}{4||\bar p_{10}^1||^4}(\bar p_{10}^{1T} - d_{10}^{1T})\{||\bar p_{10}^1||^2\{(4k_v - k_e)||\bar p_{10}^1||^2$$

$$-(4k_v - 2k_e)l||\bar p_{10}^1|| - k_e l^2\}I$$

$$+\{(4k_v - 2k_e)||\bar p_{10}^1|| + k_e l\} l \bar p_{10}^1 \bar p_{10}^{1T}\}(\bar p_{10}^1 - d_{10}^1)$$

If $k_e < 2k_v$, then ← ?? This assumption seems unnatural...

$\dot V \le 0$

mistake on calculation?

I have to consider the meaning of this assumption

Analysis

■ **1 Leader (standstill) and 1 follower**

Proof :

If $k_e < 2k_v$, then

$\dot V \le 0$

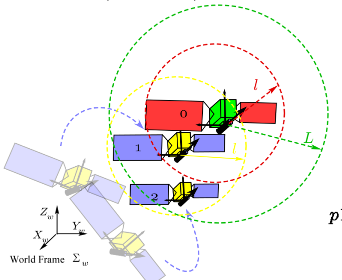
The maximum invariant set E that satisfies $\dot V = 0$ is

$E = \{(p_{10}^1, \bar p_{10}^1) \mid p_{10}^1 = \bar p_{10}^1 = d_{10}^1\}$

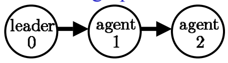
Then Lasalle's Invariance Principle implies that position synchronization (defined as upper equation of (3)) is achieved.

Simulation

■ **1 Leader (standstill) and 2 follower**



• Information graph



• Initial condition

$p0(0) = \begin{bmatrix}0\\0\\0\end{bmatrix}$ $\xi 0(0) = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$p1(0) = \begin{bmatrix}5\\5\\-5\end{bmatrix}$ $\xi 1(0) = \begin{bmatrix}0\\-\pi/8\\\pi/3\end{bmatrix}$

$p2(0) = \begin{bmatrix}8\\3\\-8\end{bmatrix}$ $\xi 2(0) = \begin{bmatrix}\pi/6\\\pi/8\\0\end{bmatrix}$

• Control objective

$$\begin{cases} ||p_{w0}^w - p_{wi}^w|| < L \\ R_{w0} = R_{wi} \end{cases}$$

$L = 3.0$
$i = 1, 2$

$0 < ||e_e 10(0)|| < 0.25$
$0 < ||e_e 21(0)|| < 0.25$

Simulation

■ **1 Leader (standstill) and 2 follower**

Velocity input

$$\begin{bmatrix} v_{w1}^1 \\ \omega_{w1}^1 \end{bmatrix} = \begin{bmatrix} k_v(\bar p_{10}^1 - d_{10}^1) \\ k_\omega \mathrm{sk}(\bar R_{10})^\vee \end{bmatrix} \qquad d_{10}^1 = \frac{l}{||\bar p_{10}^1||}\bar p_{10}^1$$

$$\begin{bmatrix} v_{w2}^2 \\ \omega_{w2}^2 \end{bmatrix} = \begin{bmatrix} k_v(\bar p_{21}^2 - d_{21}^1) \\ k_\omega \mathrm{sk}(\bar R_{21})^\vee \end{bmatrix} \qquad d_{21}^2 = \frac{l}{||\bar p_{21}^2||}\bar p_{21}^2$$

$$\frac{L}{2} > l = 1.0$$

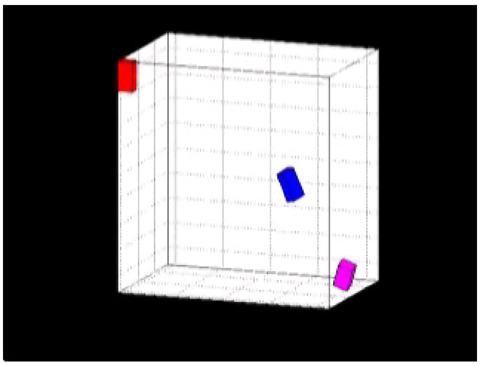
$* \begin{bmatrix} v_{w0}^0 \\ \omega_{w0}^0 \end{bmatrix} = 0$ leader is standstill

Input to observer

$$u_e = k_e e_e$$

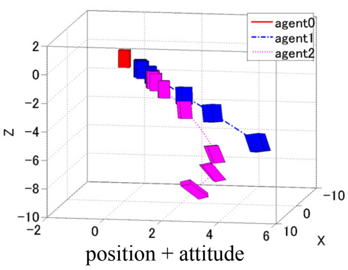
Simulation

■ **1 Leader (standstill) and 2 follower**

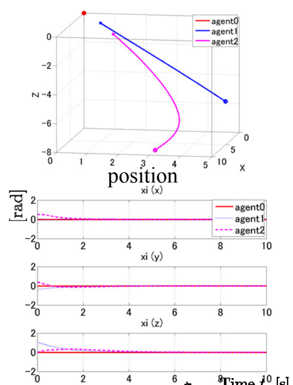


Simulation

■ **1 Leader (standstill) and 2 follower**



position + attitude

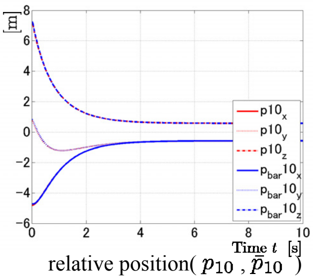


position

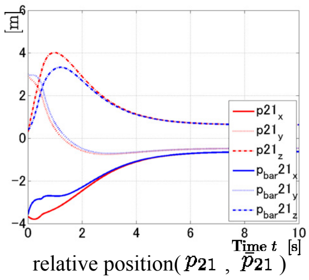
attitude (ξ)

Simulation

■ 1 Leader (standstill) and 2 follower



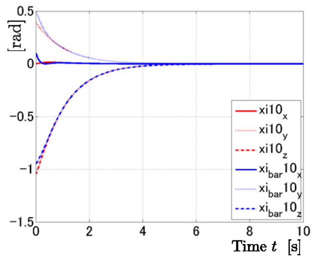
relative position(p_{10} , $\bar{p}_{10}$)



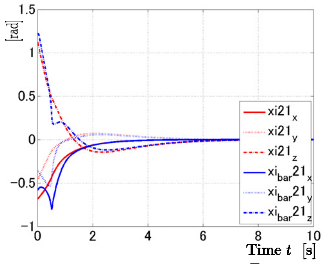
relative position(p_{21} , $\bar{p}_{21}$)

Simulation

■ 1 Leader (standstill) and 2 follower



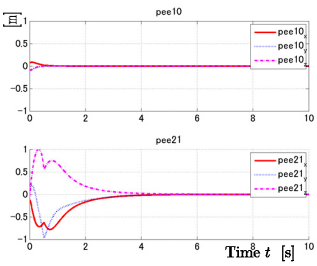
relative attitude (ξ_{10} , $\bar{\xi}_{10}$)



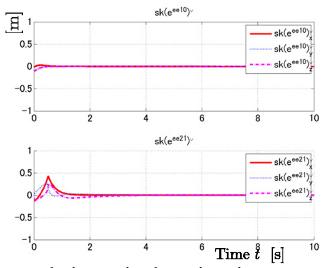
relative attitude(ξ_{21} , $\bar{\xi}_{21}$)

Simulation

■ 1 Leader (standstill) and 2 follower



relative position estimation error (p_{ee})



relative attitude estimation error ($\mathrm{sk}(R_{ee})^{\vee}$)

Simulation

■ 1 Leader (standstill) and 2 follower

Estimation errors e_e converge to 0.

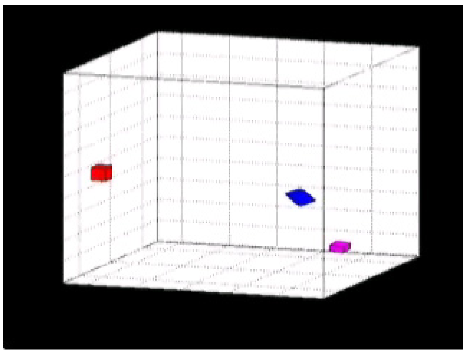
2 followers converge to { a certain region near the leader. / the same attitude with a leader.

$$\begin{cases} \|p^w_{w0} - p^w_{wi}\| < L \\ R_{w0} = R_{wi} \end{cases} \quad L = 3.0,\ i = 1, 2 \qquad \text{...(2)}$$

Pose synchronization is achieved.

Simulation

■ 1 Leader (moving) and 2 follower

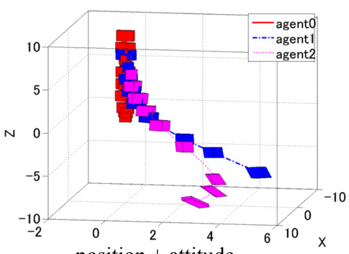


Simulation

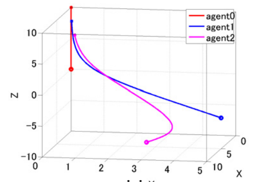
■ 1 Leader (moving) and 2 follower

$$\begin{bmatrix} v^0_{w0} \\ \omega^0_{w0} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0.5 \end{bmatrix}^T$$

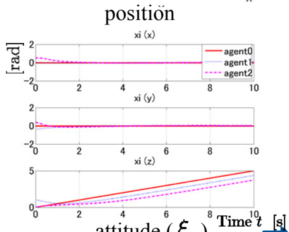
leader is moving



position + attitude



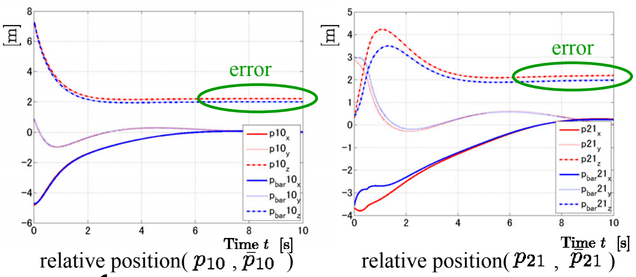
position



attitude (ξ)

Simulation

■ 1 Leader (moving) and 2 follower



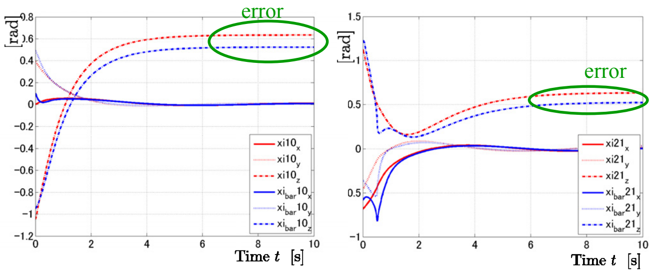
relative position(p_{10} , $\bar{p}_{10}$)   relative position(p_{21} , $\bar{p}_{21}$)

{ • estimation error
  • synchronization error   ⟶ L_2 stable ?

∗ Bigger L or smaller l than the case with standstill leader is needed.

Simulation

■ 1 Leader (moving) and 2 follower

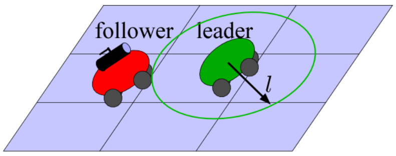


relative attitude (ξ_{10} , $\bar{\xi}_{10}$)   relative attitude(ξ_{21} , $\bar{\xi}_{21}$)

{ • estimation error
  • synchronization error   ⟶ L_2 stable ?

Experiment

■ 1 Leader and 1 follower



Velocity input

$$\begin{bmatrix} v_{w1}^1 \\ \omega_{w1}^1 \end{bmatrix} = \begin{bmatrix} \bar{p}_{10}^1 - d_{10}^1 \\ \mathrm{sk}(\bar{R}_{10})^\vee \end{bmatrix} \qquad d_{10}^1 := \frac{l}{||\bar{p}_{10}^1||}\bar{p}_{10}^1$$

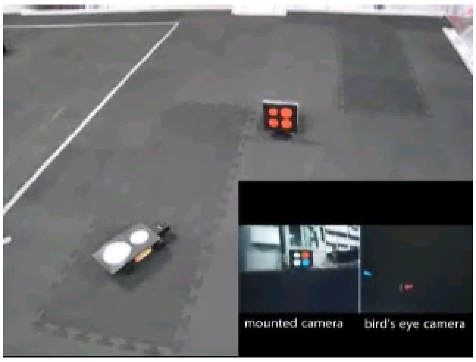
$$l = 0.35[\mathrm{m}]$$

Input to observer

$$u_e = k_e e_e$$

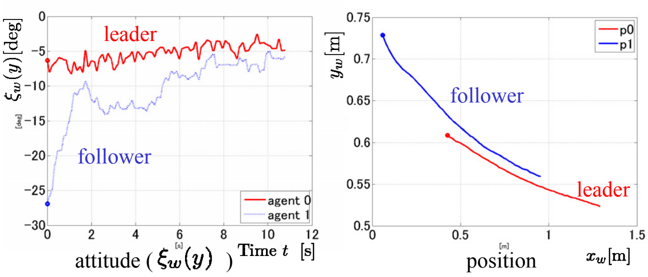
Experiment

■ 1 Leader and 1 follower



Experiment

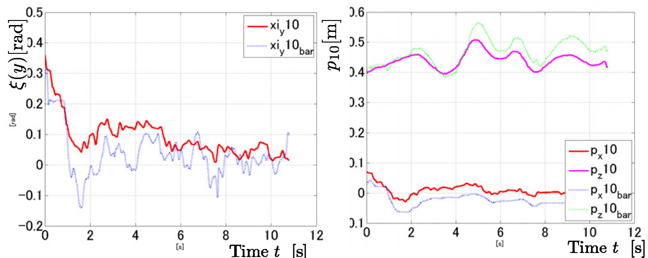
■ 1 Leader and 1 follower



attitude ($\xi_w(y)$)   position

Experiment

■ 1 Leader and 1 follower



relative attitude($\xi(y)$)   relative position (p_{10})

{ • estimation error
  • synchronization error
  ⟶ 0

{ • estimation error
  • synchronization error
  ⟶ L_2 stable ?

Experiment

■ **1 Leader and 1 follower**

A follower converge to { a certain region near the leader. / the same attitude with a leader.

$$\begin{cases} \|p_{w0}^{w} - p_{wi}^{w}\| \leq L, \quad L > 0 \\ R_{w0} = R_{wi} \end{cases} \quad \forall i \qquad \text{...(2)}$$

Pose synchronization is achieved.

*Generally, bigger L or smaller l than the case with standstill leader is needed.

Outline

■ Introduction
■ Vision-based Cooperative Control
- Problem Formulation
- Analysis
- Simulation
- Experiment

■ **Conclusion / Future Works**

Conclusion / Future Works

■ **Conclusion**
- Problem formulation (vision-based leader following of pose synchronization)
- Convergence analysis (for easier problem)
- Simulations
- Experiments (1 leader and 1 follower)

■ **Future Works**
- Convergence analysis (moving leader, multi-follower)
- Experiments (1 leader and 2 follower)

References

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