Stabilization control of wheeled pendulum

FL08-6-16
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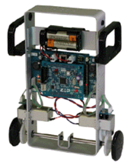
Introduction

Fujita.lab uses wheeled pendulum
e-nuvo WHEEL (ZMP INC.)

But its default controller has some problem…

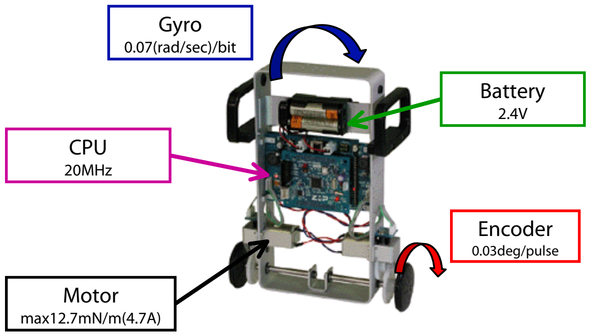
Build practical model
Apply another controller



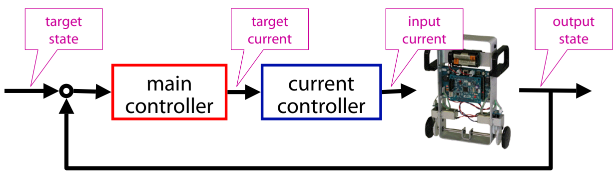
Outline

- Default Controller
- Applied Controller
- Frequency-shaped Controller
- Conclusion

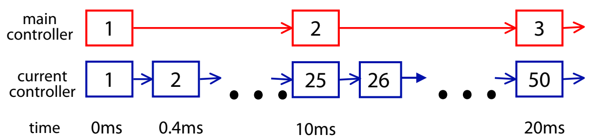
Hardware



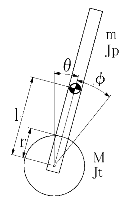
Software



Control-timing



Default Controller 1



wheel position

$$x_c = r(\theta + \phi)$$
$$y_c = 0$$

body position

$$x_p = x_c + l sin\theta$$
$$y_p = y_c + l cos\theta$$

- **m** : Mass of the body
- **M** : Mass of the cart
- **Jp** : Moment of inertia of the body
- **Jt** : Moment of inertia of the cart
- **Jm** : Moment of inertia of the motor rotor
- **l** : Length between the wheel axle and the gravity center of the body
- **r** : Radius of the wheel
- **c** : Friction of the wheel axle
- **Kt** : Torque constant of the motor
- **i** : Reduction ration of the gear

Lagrange equation of motion

$$\{(M+m)r^2 + mlr + Jt + iJm\}\ddot{\theta} + \{(M+m)r^2 + Jt + i^2 Jm\}\ddot{\phi} + c\dot{\phi} = au$$
$$\{(M+m)r^2 + 2mlr + ml^2 + Jp + Jt + Jm\}\ddot{\theta} - mgl\theta + \{(M+m)r^2 + mlr + Jt + iJm\}\ddot{\phi} = 0$$

equation of state

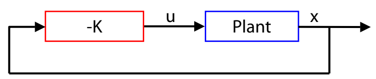
$$\frac{d}{dt}\begin{pmatrix}\theta \\ \phi \\ \dot{\theta} \\ \dot{\phi}\end{pmatrix} = A\begin{pmatrix}\theta \\ \phi \\ \dot{\theta} \\ \dot{\phi}\end{pmatrix} + Bu \qquad x = \begin{pmatrix}\theta \\ \phi \\ \dot{\theta} \\ \dot{\phi}\end{pmatrix}, A \in R^{4\times4}, B \in R^{4\times1}, u \in R$$

Default Controller 2

LQ controller

$$PA + A^TP - PBR^{-1}B^TP + Q = 0$$
$$K = R^{-1}B^TP$$

Q(weighting matrix) $\in R^{4\times4}$
R(weighting matrix) $\in R$



(Default value)

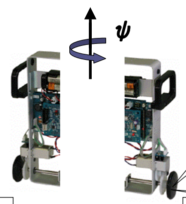
$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 15 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 10 \end{pmatrix}, R = 500$$

$$K = (-18.7127 \quad -0.1732 \quad -2.3976 \quad -0.1966)$$

Fault of Default Controller 1

Assume that there are two pendulums

$u = -Kx$ $\qquad$ $u = -Kx$



To control ψ $\qquad$ To control ψ

$u = -Kx - eps$ $\qquad$ $u = -Kx + eps$

$$eps = G1 \times (\psi_{ref} - \psi) + G2 \times (\dot{\psi}_{ref} - \dot{\psi})$$

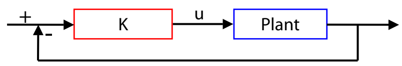
$G1, G2$: Gain
ψ_{ref} : Targetψ

LQ doesn't apply to ψ

Fault of Default Controller 2

Servo System

$$x_{ref} = \begin{pmatrix} 0 \\ \phi_{ref} \\ 0 \\ \dot{\phi}_{ref} \end{pmatrix} \qquad x = \begin{pmatrix} \theta \\ \phi \\ \dot{\theta} \\ \dot{\phi} \end{pmatrix}$$



$$\frac{d}{dt}\begin{pmatrix} \theta \\ \phi - \phi_{ref} \\ \dot{\theta} \\ \dot{\phi} - \dot{\phi}_{ref} \end{pmatrix} = (A - BK)\begin{pmatrix} \theta \\ \phi - \phi_{ref} \\ \dot{\theta} \\ \dot{\phi} - \dot{\phi}_{ref} \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} \theta \\ \phi \\ \dot{\theta} \\ \dot{\phi} \end{pmatrix} = (A - BK)\begin{pmatrix} \theta \\ \phi \\ \dot{\theta} \\ \dot{\phi} \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} \theta \\ \phi - \phi_{ref} \\ \dot{\theta} \\ \dot{\phi} - \dot{\phi}_{ref} \end{pmatrix} = (A - BK)\begin{pmatrix} \theta \\ \phi - \phi_{ref} \\ \dot{\theta} \\ \dot{\phi} - \dot{\phi}_{ref} \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} 0 \\ \phi_{ref} \\ 0 \\ \dot{\phi}_{ref} \end{pmatrix} = (A - BK)\begin{pmatrix} 0 \\ \phi_{ref} \\ 0 \\ \dot{\phi}_{ref} \end{pmatrix}$$

Contradict ϕ_{ref}

Outline

✓ Default Controller
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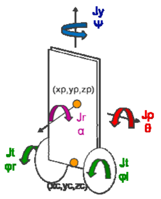
Equation of state



Speed of wheels' center

$$\dot{x}_c = \frac{r}{2}(\dot{\phi}_l + \dot{\phi}_r)cos\psi$$
$$\dot{y}_c = \frac{r}{2}(\dot{\phi}_l + \dot{\phi}_r)sin\psi$$
$$\dot{z}_c = 0$$

Jp : Moment of inertia of the body (pitch)
Jy : Moment of inertia of the body (yaw)
Jr : Moment of inertia of the body (roll)

Gravity center of the body

$$x_p = lsin\theta cos\psi + x_c$$
$$y_p = lsin\theta sin\psi + y_c$$
$$z_c = lcos\theta$$

Lagrange equation of motion

$$\begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}\begin{pmatrix} \ddot{\theta} \\ \ddot{\phi}_l \\ \ddot{\phi}_r \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \beta_{22} & 0 \\ 0 & 0 & \beta_{33} \end{pmatrix}\begin{pmatrix} \dot{\theta} \\ \dot{\phi}_l \\ \dot{\phi}_r \end{pmatrix} + \begin{pmatrix} \gamma_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \theta \\ \phi_l \\ \phi_r \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ \delta_{21} & 0 \\ 0 & \delta_{32} \end{pmatrix}\begin{pmatrix} u_l \\ u_r \end{pmatrix}$$

equation of state

$$\frac{d}{dt}\begin{pmatrix} \theta \\ \dot{\theta} \\ \dot{\phi}_l \\ \dot{\phi}_r \end{pmatrix} = A\begin{pmatrix} \theta \\ \dot{\theta} \\ \dot{\phi}_l \\ \dot{\phi}_r \end{pmatrix} + B\begin{pmatrix} u_l \\ u_r \end{pmatrix} \qquad x = \begin{pmatrix} \theta \\ \dot{\theta} \\ \dot{\phi}_l \\ \dot{\phi}_r \end{pmatrix}, A \in R^{4\times4}, B \in R^{4\times2}, u = \begin{pmatrix} u_l \\ u_r \end{pmatrix} \in R^2$$

Servo system

$$\begin{pmatrix} \dot{x} \\ \dot{\omega} \end{pmatrix} = \underbrace{\begin{pmatrix} A & 0 \\ -C & 0 \end{pmatrix}}_{A_d}\begin{pmatrix} x \\ \omega \end{pmatrix} + \underbrace{\begin{pmatrix} B \\ 0 \end{pmatrix}}_{B_d}u + \begin{pmatrix} 0 \\ I \end{pmatrix}r$$

$$y = (C \quad 0)\begin{pmatrix} x \\ \omega \end{pmatrix}$$

$$\tilde{\omega} = \omega - \omega_\infty$$
$$\tilde{x} = x - x_\infty$$

$$\begin{pmatrix} \dot{\tilde{x}} \\ \dot{\tilde{\omega}} \end{pmatrix} = A_d\begin{pmatrix} \tilde{x} \\ \tilde{\omega} \end{pmatrix} + B_d\tilde{u}$$

$$e = (-C \quad 0)\begin{pmatrix} \tilde{x} \\ \tilde{\omega} \end{pmatrix}$$

Evaluation function

$$Ja = \int_0^\infty \{e^TQ_1e + \tilde{\omega}^TQ_2\tilde{\omega} + \tilde{u}^TR\tilde{u}\}dt$$

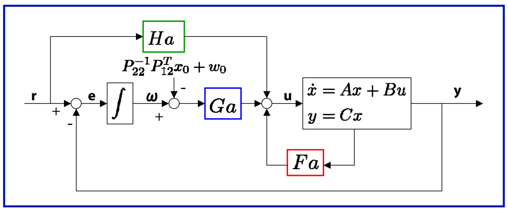
$Q_1 \in R^{2\times2}$
$Q_2 \in R^{2\times2}$
$R \in R^{2\times2}$

Riccati equation

$$A_d^TP + PA_d - PB_dR^{-1}B_d^TP + \begin{pmatrix} C^TQ_1C & 0 \\ 0 & Q_2 \end{pmatrix} = 0$$

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{pmatrix}$$

Servo System



$$u = F_a x + G_a\omega + H_a r - G_a P_{22}^{-1}P_{12}^{T}x_0 - G_a\omega_0$$

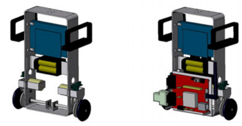
$$F_a = -R^{-1}B^{T}P_{11}$$

$$G_a = -R^{-1}B^{T}P_{12}$$

$$H_a = (-F_a + G_a P_{22}^{-1}P_{12}^{T} \quad I)\begin{pmatrix} A & B \\ -C & 0 \end{pmatrix}^{-1}\begin{pmatrix} 0 \\ I \end{pmatrix}$$

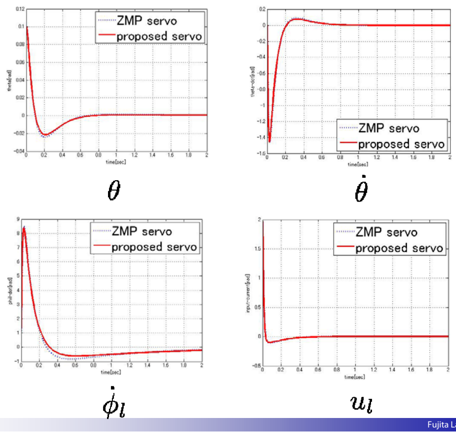
Parameter identification

			ZMP	CAD
Mass of the body	m	[kg]	0.6113	0.8817
Mass of the cart	M	[kg]	0.0363	0.0354
Moment of inertia of the body (pitch)	Jp	[kg・m^2]	2.276E-03	3.587E-03
Moment of inertia of the body (yaw)	Jy	[kg・m^2]		1.855E-03
Moment of inertia of the body (roll)	Jr	[kg・m^2]		4.898E-03
Moment of inertia of the cart	Jt	[kg・m^2]	4.694E-06	4.634E-06
Length between the wheel axle and the gravity center of the body	l	[m]	98.6E-03	79.1E-03



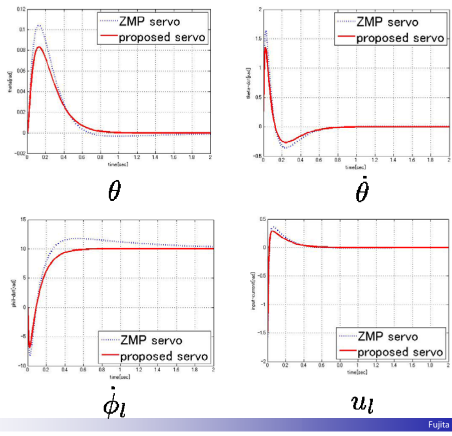
Simulation 1

$$x_0 = \begin{pmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad r = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$



Simulation 2

$$x_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad r = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$$



Outline

- ✓ Default Controller
- ✓ Applied Controller
- Frequency-shaped Controller
- Conclusion

Frequency-shaped LQ controller

LQ controller…

Weight matrix is constant

- •Weak in phase-lag
- •Weak in uncertainness

Frequency-shaped LQ controller

Evaluation function

Evaluation function

$$J=\frac{1}{2\pi}\int_0^{\infty}\{\hat{e}^TQ_1\hat{e}+\hat{\tilde{\omega}}^TQ_2\hat{\tilde{\omega}}+\hat{\tilde{u}}^TR\hat{\tilde{u}}\}dt$$

$\hat{e},\hat{\tilde{\omega}},\hat{\tilde{u}}$ is Fourier Transform of $e,\tilde{\omega},\tilde{u}$

Q_1, Q_2 : constant
$R = D^TD$
$\hat{\tilde{v}} = D\hat{\tilde{u}}$

$$J=\frac{1}{2\pi}\int_0^{\infty}\{\hat{e}^TQ_1\hat{e}+\hat{\tilde{\omega}}^TQ_2\hat{\tilde{\omega}}+\hat{\tilde{v}}^T\hat{\tilde{v}}\}dt$$

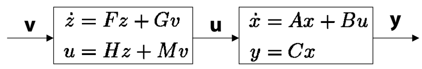
Time-domain representation

$$J=\int_0^{\infty}\{e^TQ_1e+\tilde{\omega}^TQ_2\tilde{\omega}+\tilde{v}^T\tilde{v}\}dt$$

System

Define

$$D^{-1}:\ \begin{array}{l}\dot{z}=Fz+Gv\\ u=Hz+Mv\end{array}$$



equation of state

$$\begin{pmatrix}\dot{\tilde{x}}\\ \dot{\tilde{\omega}}\\ \dot{\tilde{z}}\end{pmatrix}=\underbrace{\begin{pmatrix}A & 0 & BH\\ -C & 0 & 0\\ 0 & 0 & F\end{pmatrix}}_{A_s}\begin{pmatrix}\tilde{x}\\ \tilde{\omega}\\ \tilde{z}\end{pmatrix}+\underbrace{\begin{pmatrix}BM\\ 0\\ G\end{pmatrix}}_{B_s}\tilde{v}\qquad\begin{cases}\tilde{x}=x-x_\infty\\ \tilde{\omega}=\omega-\omega_\infty\\ \tilde{z}=z-z_\infty\end{cases}$$

$$e=(-C\quad 0\quad 0)\begin{pmatrix}\tilde{x}\\ \tilde{\omega}\\ \tilde{z}\end{pmatrix}$$

System

Riccati equation

$$A_s^TP+PA_s-PB_sR^{-1}B_s^TP+\begin{pmatrix}C^TQ_1C & 0 & 0\\ 0 & Q_2 & 0\\ 0 & 0 & 0\end{pmatrix}=0$$

Define

$$P=\begin{pmatrix}P_{11} & P_{12} & P_{13}\\ P_{21} & P_{22} & P_{23}\\ P_{31} & P_{32} & P_{33}\end{pmatrix}$$

$$Pb=\begin{pmatrix}Pb_{11} & Pb_{12}\\ Pb_{21} & Pb_{22}\end{pmatrix}=\left(\begin{array}{cc|c}P_{11} & P_{13} & P_{12}\\ P_{31} & P_{33} & P_{32}\\ \hline P_{21} & P_{23} & P_{22}\end{array}\right)$$

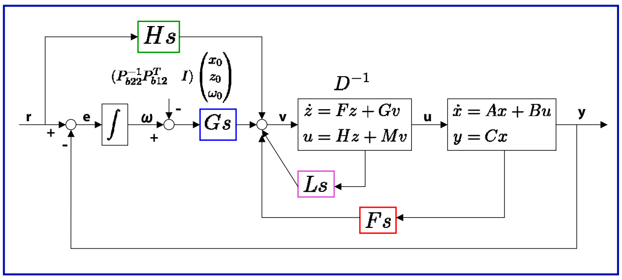
$Fs=-GP_{31}$

$Gs=-GP_{32}$

$Ls=-GP_{33}$

$$Hs=(-F_a+G_aP_{b22}^{-1}P_{b12}^T\quad -L_a+G_aP_{b22}^{-1}P_{b12}^T\quad I)\begin{pmatrix}A & BH & 0\\ 0 & F & G\\ C & 0 & 0\end{pmatrix}^{-1}\begin{pmatrix}0\\ 0\\ I\end{pmatrix}$$

System



$$v=F_sx+G_s\omega+L_sz+H_sr-(G_sP_{b22}^{-1}P_{b12}^T\quad Gs)\begin{pmatrix}x_0\\ z_0\\ \omega_0\end{pmatrix}$$

Simulation

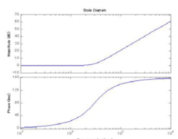
Suppose that sensors have discrete property

	resolution	cycle
Gyro	0.07(rad/s)/bit	10ms
Encoder	0.03deg/pulse	10ms

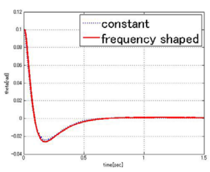
Simulation (Constant vs. Frequency-shaped)

$$x_0=\begin{pmatrix}0.1\\ 0\\ 0\\ 0\end{pmatrix}\qquad r=\begin{pmatrix}0\\ 0\end{pmatrix}$$

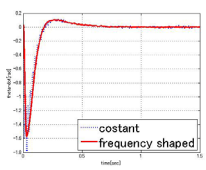
$$D=\frac{s^2+400s+90000}{90000}$$



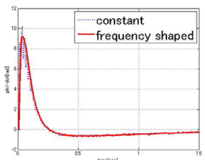
cutoff frequency : 48[Hz]



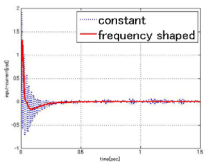
θ



$\dot{\theta}$



$\dot{\phi}_l$

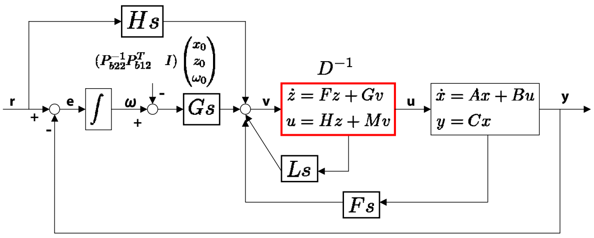


u_l

Simulation

D^{-1} is calculated in micro-computer

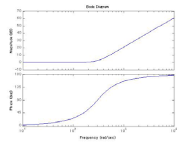
D-1 is discrete system



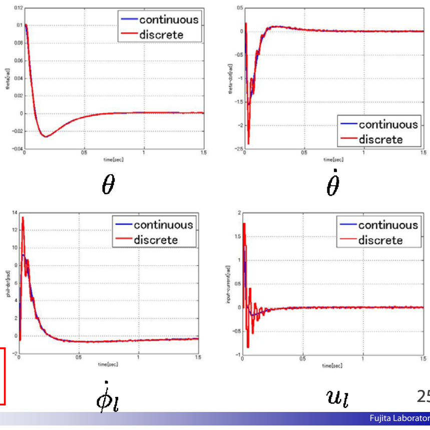
Simulation (D^{-1}: discrete T=0.01)

$$x_0 = \begin{pmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad r = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$D = \frac{s^2 + 400s + 90000}{90000}$$



cutoff frequency : 48[Hz]

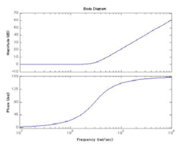


– : D^{-1} is continuous
– : D^{-1} is discrete (T=0.01[ms])

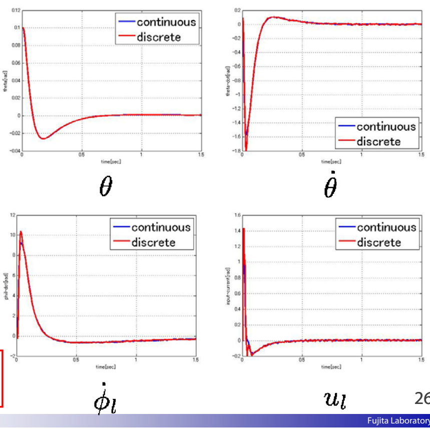
Simulation (D^{-1}: discrete T=0.005)

$$x_0 = \begin{pmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad r = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$D = \frac{s^2 + 400s + 90000}{90000}$$



cutoff frequency : 48[Hz]

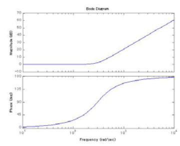


– : D^{-1} is continuous
– : D^{-1} is discrete (T=0.005[ms])

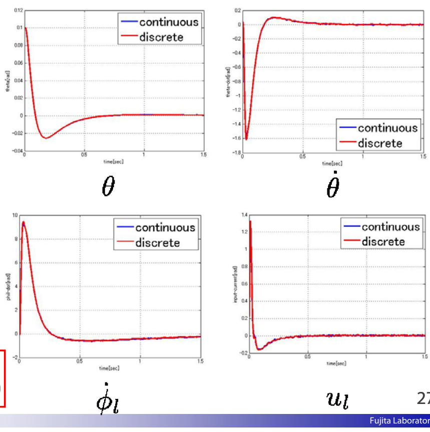
Simulation (D^{-1}: discrete T=0.001)

$$x_0 = \begin{pmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad r = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$D = \frac{s^2 + 400s + 90000}{90000}$$



cutoff frequency : 48[Hz]



– : D^{-1} is continuous
– : D^{-1} is discrete (T=0.001[ms])

Outline

✓ Default Controller

✓ Applied Controller

✓ Frequency-shaped Controller

• Conclusion

Conclusion

Conclusion

- Viewed problem of Default Controller
- Applied another controller

Future Works

- Speed up the control cycle
- Apply another controller