Brief Review and Expansion of Pose Synchronization in SE(2) to Nonholonomic Agent Systems

FL08 -6-1
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Outline

Introduction

Cooperative Control

A distributed control strategy that achieves specified tasks in multi-agent system

Fig. 1 School of fishes(※)

Motivation

- Analysis of emergent and self-organized swarming behaviors in biological groups with distributed agent-to-agent interaction
- Interest in a group behavior of animals, formulation control of multi-vehicle systems and so on

Application

Mobile sensor networks, Robot networks, and many other Multi-agent systems

※ http://www.allposters.co.jp/-sp/-Posters_i1006775_.htm

Brief Review of Graph Theory

- Graph : A set of connections (Edges) of between Objects (Vertice)

  Vertex (node) : Agent Edge : Information Flow

  - Directed Graph (Fig. 2) : the information flows from agent j to i
  - Undirected Graph (Fig. 3) : the information flows to both directions

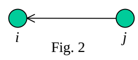


Fig. 2

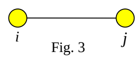


Fig. 3

- Directed Graph
  - strongly connected (Fig. 4) :

    there is a directly path connecting any two distinct nodes
  - weakly connected (Fig. 2) :

    there is a path connecting any two distinct nodes ignoring the direction

- Undirected Graph
  - connected :

    there is a path between any two distinct nodes

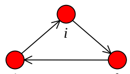


Fig. 4

Outline

Brief Review of Graph Theory

$G=(V,E,W)$: weighted digraph

$V=\{v_1,\ldots,v_n\}$: set of nodes (agents)

$E\subseteq V\times V$: set of edges (an edge of G : $e_{ij}=(v_i,v_j)$)

$W:E\to\Re^+$: map assigning a positive weight to each edge

$N_i=\{v_j\in V:(v_i,v_j)\in E\}$: set of **neighbors** of node v_i

- Adjacency Matrix : $A=[a_{ij}]$ $\begin{cases}\cdot e_{ij}\in E\Leftrightarrow a_{ij}>w_{ij}\\ \cdot a_{ii}=0\end{cases}$

- Degree Matrix : $D=[d_{ij}]$ $\begin{cases}\cdot d_{ii}=\sum_{i\neq j}w_{ij}\\ \cdot d_{ij}=0 \quad (i\neq j)\end{cases}$

Ex.)

$$A=\begin{bmatrix}0&0&w_{13}\\ w_{21}&0&w_{23}\\ 0&0&0\end{bmatrix}\qquad D=\begin{bmatrix}w_{13}&0&0\\ 0&w_{21}+w_{23}&0\\ 0&0&0\end{bmatrix}$$

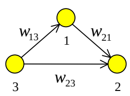


Fig. 5

Brief Review of Graph Theory

- Graph Laplacian : $L \in \Re^{n \times n}$

$$L_{ij} = \begin{cases} -w_{ij} & \text{if } i \neq j \text{ and } (v_i, v_j) \in E \\ 0 & \text{if } i \neq j \text{ and } (v_i, v_j) \notin E \\ \sum_{k \in N_i} w_{ik} & \text{if } i = j \end{cases}$$

$$\rightarrow \quad L = D - A$$

Graph Laplacian has many significant properties
E.g. ・$L\mathbf{1} = \mathbf{0}$
・positive semidifinite matrix
・the stability properties of system is completely determined by the location of the Laplacian eigenvalues of the network

Brief Review of Consensus Problem

Consensus Problem

To reach an **agreement** regarding a certain quantity of interest that depends on the state of all

Ex.) [3]

Control Law

$$\dot{x}_i(t) = u_i(t)$$
$$u_i = \sum_{v_j \in N_i} a_{ij}(x_j - x_i)$$

x_i : agent i's state
u_i : agent i's input

⇒ **Laplacian matrix L**

$$\dot{x}(t) = -Lx(t)$$

$x = (x_1, ..., x_n)^T$: network with value $x \in \mathbf{R}^n$

Consensus : $x \rightarrow \alpha\mathbf{1}$

Brief Review of Consensus Problem

Simulation

Initial States : $x_0 = [9 \quad 6 \quad 1 \quad -3 \quad -5 \quad -8]$
No Weights : $\forall w_{ij} = 1$

1)

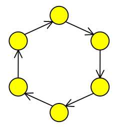
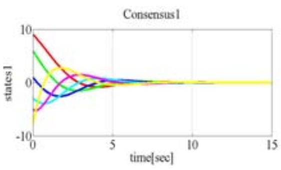


Fig. 6 Graph Structure1
(strongly, balanced)

$$L_1 = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



2)

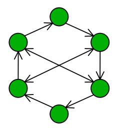
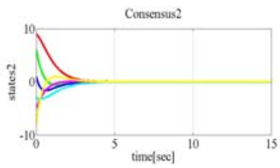


Fig. 7 Graph Structure2
(more connected)

$$L_2 = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 & -1 & 0 \\ 0 & 0 & 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 & 2 & -1 \\ -1 & 0 & -1 & 0 & 0 & 2 \end{bmatrix}$$



From Consensus to Flocking Problem

States → attitudes, positions, and both of these

⇩

Consensus Problems → **Flocking**, Formation Problems

→ Flocking Problem (Moshtagh, Igarashi's Model)

Outline

- Introduction
- Brief Review
  - Graph Theory
  - Consensus Problem
- **Flocking Problem**
  - Moshtagh's Model
    - » Control Law, Simulation, Experiment
  - Igarashi's Model
    - » Control Law, Simulation, Problem of Implementation
- Nonholonomic Constraint
  - Astolfi's Model
- Combining Model
  - Simulation
  - Experiment
- Conclusion and Future Works

Moshtagh's Model

Moshtagh's Model[5] ⇒ Attitude Coordination

Kinematic Model

$$\begin{cases} \dot{x}_i = \cos\theta_i \\ \dot{y}_i = \sin\theta_i \\ \dot{\theta}_i = \omega_i \end{cases} \qquad i = 1, \ldots, N$$

(x_i, y_i) : position
θ_i : orientation
ω_i : input angular velocity

All agents move with constant unit speed

Consensus State

$\forall \theta_i = \theta_{ss} \qquad i = 1, \ldots, N \qquad \theta_{ss}$: final value

⇒ called "**Flocking**" in the paper

Moshtagh's Model

Control Input

$$\omega_i = -\sum_{j\in N_i} \sin(\theta_i - \theta_j) \qquad i, j = 1,\ldots,N$$

N_i : i 's neighbors

$|\theta_i - \theta_j| \neq \pi$

Connected ⟹ $\forall \theta_i = \theta_{ss} \qquad i = 1,\ldots,N$

(for proof, use a simple quadratic Lyapunov function : $V = \frac{1}{2}\boldsymbol{\theta}^T\boldsymbol{\theta}$)

This model expands to 3 dimensions, leader following problem, and switching topology

This controller is the nonlinear version of the control law

$$\omega_i = -\sum_{j\in N_i} \theta_i - \theta_j$$

proposed in [2], [3] as the continuous analogue of the **Vicseck's model**[1]

Moshtagh's Model

Simulation

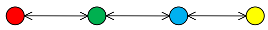
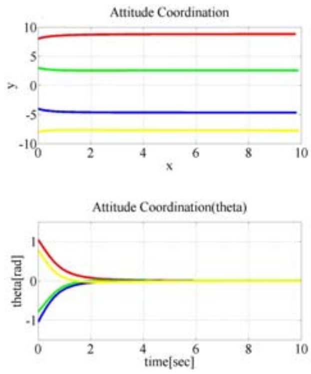


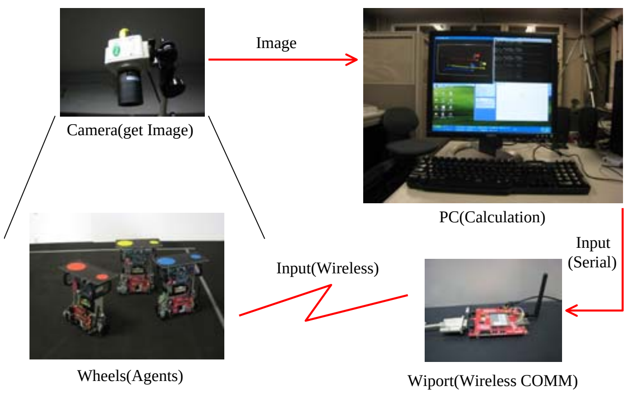
Fig. 8 Graph Structure
(Undirected, Connected)

Initial States

$$\begin{cases} x_1(0) = (0, 8, \frac{\pi}{3}) \\ x_2(0) = (0, 3, -\frac{\pi}{4}) \\ x_3(0) = (0, -3, -\frac{\pi}{3}) \\ x_4(0) = (0, -8, \frac{\pi}{4}) \end{cases}$$



Experimental Circumstance



Moshtagh's Model

Experiment

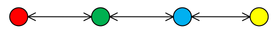
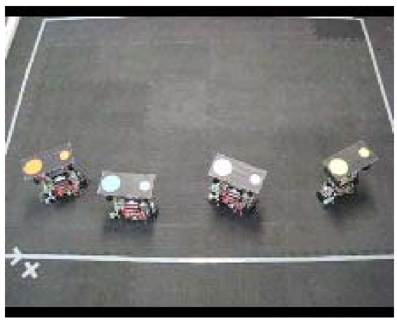


Fig. 9 Graph Structure
(Undirected, Connected)



Moshtagh's Model

Moshtagh's model considers only attitudes!

⇩

Igarashi's Model (consider positions too)

- Consider attitudes and positions (called pose)
- Consider weights
- Consider in 3 dimensions
- Possess passivity

Igarashi's Model in SE(3)

Igarashi's Model[6] ⟹ Pose Synchronization in SE(3)

Kinematic Model

$$\begin{cases} \dot{p}_i = e^{\hat{\xi}_i} v_i \\ \dot{e}^{\hat{\xi}_i} = e^{\hat{\xi}_i}\hat{\omega}_i \qquad \zeta_i = \theta_i \xi_i \\ y_i = [p_i \quad e^{\hat{\xi}_i}] \qquad i = 1,\ldots,N \end{cases}$$

$p_i \in \Re^3$: position
$e^{\hat{\xi}_i} \in SO(3)$: orientation
$v_i \in \Re^3$: body velocity
$\omega_i \in \Re^3$: angular velocity
$\theta_i \in \Re$: rotation angle
$\xi_i \in \Re^3$: rotation axis
y_i : output

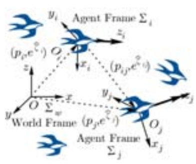


Fig. 10 Rigid Body Motion in SE(3)

$\wedge$: wedge
the skew-symmetric operator from to the space of 3 × 3 skew-symmetric matrices

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}^{\wedge} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \longrightarrow \hat{\omega}a = \omega \times a$$

$\vee$: vee
the inverse operator to '$\wedge$'

- skew-symmetric components

$$\mathrm{sk}(e^{\hat{\xi}_i}) := \frac{1}{2}(e^{\hat{\xi}_i} - e^{-\hat{\xi}_i})$$

Igarashi's Model in SE(3)

Assumptions

- The rigid-bodies' orientation matrices, $e^{\hat{\xi}_i} \forall i$ are positive definite $\rightarrow |\theta_i| < \frac{\pi}{2}$
- Information Graph is fixed, strongly connected and each weights are positive

Control Input

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = K_i \sum_{j \in N_i} \omega_{ij} \begin{bmatrix} e^{-\hat{\xi}_i} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} p_j - p_i \\ \mathrm{sk}(e^{-\hat{\xi}_i} e^{\hat{\xi}_j})^{\vee} \end{bmatrix} \quad \forall i$$

$K_i = \begin{bmatrix} k_{pi} I & 0 \\ 0 & k_{ei} I \end{bmatrix}$: i's gain matrix $\quad N_i$: i's neighbors $\quad \omega_{ij}$: weight of edge

Pose Synchronization

$$\lim_{t \to \infty} |y_i - y_j| = 0 \qquad \forall i, j$$

Igarashi's Model in SE(3)

Expansion of Inputs

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} e^{-\hat{\xi}_i} & 0 \\ 0 & e^{-\hat{\xi}_i} \end{bmatrix} \begin{bmatrix} v_c \\ e^{-\hat{\xi}_c} \omega_c \end{bmatrix} + K_i \sum_{j \in N_i} \omega_{ij} \begin{bmatrix} e^{-\hat{\xi}_i} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} p_j - p_i \\ \mathrm{sk}(e^{-\hat{\xi}_i} e^{\hat{\xi}_j})^{\vee} \end{bmatrix} \quad \forall i$$

v_c, ω_c : desired velocity(angular velocity) after the pose synchronization

$$\lim_{t \to \infty} v_i = e^{-\hat{\xi}_i} v_c \qquad \lim_{t \to \infty} \omega_i = e^{-\hat{\xi}_i} e^{-\hat{\xi}_c} \omega_c \quad \forall i, j$$

Achieve pose synchronization and desired motion!!

Take SE(2) version in order to consider vehicle systems

Igarashi's Model in SE(2)

Kinematic Model

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}$$

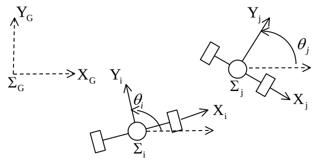


Fig. 11 Rigid Body Motion in SE(2)

Control Input

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} + \begin{bmatrix} k_{pi} & 0 & 0 \\ 0 & k_{pi} & 0 \\ 0 & 0 & k_{ei} \end{bmatrix} \sum_{j \in N_i} \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_j - x_i \\ y_j - y_i \\ \sin(\theta_j - \theta_i) \end{bmatrix}$$

$$\begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta}_c \end{bmatrix} \qquad \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Igarashi's Model in SE(2)

Simulation1 (without v_c, ω_c)

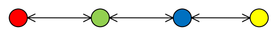
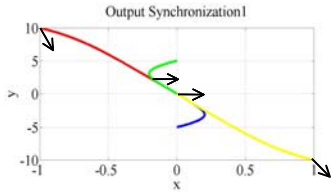
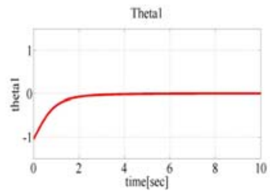
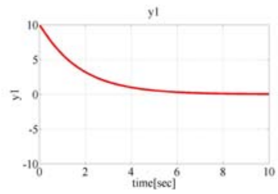


Fig. 12 Graph Structure
(Undirected, Connected)







Igarashi's Model in SE(2)

Simulation2 (with v_c, ω_c)

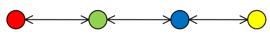
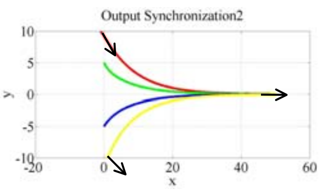
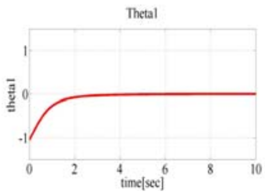
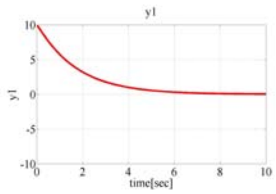


Fig. 13 Graph Structure
(Undirected, Connected)

$v_d = (5,0) \qquad \omega_d = 0$







Igarashi's Model in SE(2)

Implementation Problem

- No links between positions and attitudes!
- Agents move kinematically!

→ Difficulty of applying to vehicles, airplane, and so on

⇨ Expand to **Nonholonomic Agent Systems**

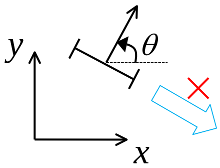
Next, consider the nonholonomic constraint

Outline

Nonholonomic Constraint

Vehicle System

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{\theta} = \omega \end{cases}$$



Cannot move just beside

Fig. 14 Vehicle Model

Constraint of Vehicle System

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0 \qquad \left(\frac{\dot{y}}{\dot{x}} = \tan\theta\right)$$

Constraints like this are called "**Nonholonomic Constraint**"

Nonholonomic Constraint

Nonholonomic Constraint

$$h(q)\dot{q} = 0$$

(Holonomic constraint : $h(q) = 0$)

•Vehicle System

$$\begin{bmatrix}\sin(\theta) & \cos(\theta) & 0\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = 0$$

•This model cannot be stabilized by a continuous, memoryless state feedback
•This model can be stabilized by following methods
➢Time-varying Control
➢Discontinuous(Switching) Control
➢Chained Form [7], [10]

To control positions and attitudes ⇨ **Astolfi's Model**

Astolfi's Model

Astolfi's Model[9] ⇨ AchieveDesired Position and Attitude

Kinematic Model

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{\theta} = \omega \end{cases} \xrightarrow{\text{polar coordinate}} \begin{cases} \dot{\rho} = -v\cos\alpha \\ \dot{\alpha} = \dfrac{\sin\alpha}{\rho}v - \omega \\ \dot{\phi} = -\omega \end{cases} \quad (1)$$

$R(x, y, \theta)$: current states
$G(0,0,0)$: origin
v : input velocity
ω : input angular velocity

$$\begin{cases} \rho = \sqrt{x^2 + y^2} \\ \alpha = -\theta + \tan^{-1}\left(\dfrac{-y}{-x}\right) \\ \phi = \dfrac{\pi}{2} - \theta \end{cases} \mod\left(\frac{\pi}{2}\right) \quad (2)$$

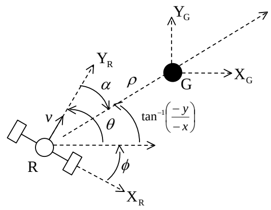


Fig. 15 Robot Kinematics

Astolfi's Model

Achievement

$(\rho, \alpha, \phi) = (0, 0, 0)$: Equilibrium point

•Globally exponentially stabilize
•The floor path does not contain cusps
→The agent can go back due to the initial state
•Use a discontinuous time invariant state feedback control law

Astolfi's Model

Control Input

$$\begin{cases} v = k_\rho \rho \\ \omega = k_\alpha \alpha + k_\phi \phi \end{cases} \qquad -\frac{\pi}{2} < \alpha \le \frac{\pi}{2}$$

k_ρ, k_α, k_ϕ : gain

$-\pi < \alpha \le -\frac{\pi}{2}, \ \frac{\pi}{2} < \alpha \le \pi \Rightarrow v = -v$ (reverse)

$$(1) \Rightarrow \begin{cases} \dot{\rho} = -k_p\cos\alpha \\ \dot{\alpha} = -k_\alpha\alpha - k_\phi\phi + k_\rho\sin\alpha \\ \dot{\phi} = -k_\alpha\alpha - k_\phi\phi \end{cases} \quad (3)$$

This system has the unique equilibrium point $(\rho, \alpha, \phi) = (0, 0, 0)$

There is the indeterminacy of the control law at $\rho = 0 (\because (1))$, but such an indeterminacy can be resolved setting $v|_{(0,0,\theta)} = 0, \omega|_{(0,0,\theta)} = (k_\alpha + k_\phi)\left(\frac{\pi}{2} - \theta\right)$

Stability Issue

Consider only the local stability (omit the global stability)

Linear Approximation of system (2)

$$(3) \quad \Rightarrow \quad \begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -k_\rho & 0 & 0 \\ 0 & -(k_\alpha - k_\rho) & -k_\phi \\ 0 & -k_\alpha & -k_\phi \end{bmatrix} \begin{bmatrix} \rho \\ \alpha \\ \phi \end{bmatrix} \quad + \quad \text{h.o.t.}$$

Hence, the system is locally exponentially stable iff the eigenvalues of the matrix have all negative real part. Such eigenvalues are the roots of the polynomial

$$p_A(\lambda) = (\lambda + k_\rho)(\lambda^2 + \lambda(k_\alpha + k_\phi - k_\rho) - k_\rho k_\phi).$$

Thus, the necessary and sufficient condition of the local stability is following.

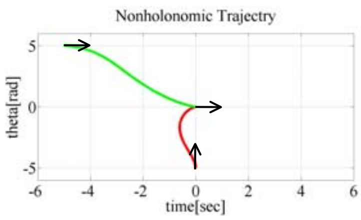
$$\begin{cases} k_\rho > 0 \\ k_\phi < 0 \\ k_\alpha + k_\phi - k_\rho > 0 \end{cases}$$

Simulation1

Gain

$$\begin{cases} k_\rho = 1 \\ k_\alpha = 8 \\ k_\phi = -1.5 \end{cases}$$

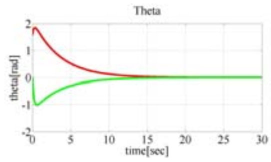
$$(k_\alpha + k_\phi - k_\rho > 0)$$



Initial States

Red : $(x_{r0}, y_{r0}, \theta_{r0}) = (-5, -5, \frac{\pi}{2})$

Green : $(x_{g0}, y_{g0}, \theta_{g0}) = (-5, 5, 0)$



Coordinate Transformation

Transfer the equilibrium point(origin) to the desired one

$(0, 0, 0) \quad \rightarrow \quad (x_d, y_d, \theta_d)$

$R(x_R, y_R, \theta_R)$: current states

$D(x_d, y_d, \theta_d)$: desired states

Coordinate Transformation

$$(2) \quad \Rightarrow \quad \begin{cases} \rho' = \sqrt{(x_d - x_R)^2 + (y_d - y_R)^2} \\ \alpha' = -\theta_R + \tan^{-1}\left(\dfrac{y_d - y_R}{x_d - x_R}\right) \\ \phi' = \phi_R - \phi_d \end{cases}$$

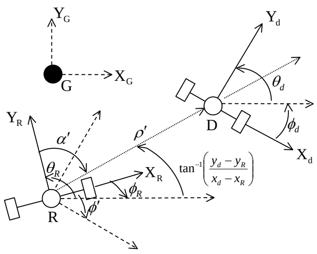


Fig. 16 Robot Kinematics

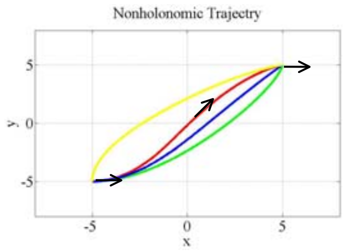
Simulation2

Initial Position

$(x_0, y_0) = (-5, -5)$

Target Position

$(x_d, y_d) = (5, 5)$



Initial and Target Angle

Red : $(\theta_0, \theta_d) = (0, 0)$ Blue : $(\theta_0, \theta_d) = (0, \frac{\pi}{4})$

Green : $(\theta_0, \theta_d) = (0, \frac{\pi}{2})$ Yellow : $(\theta_0, \theta_d) = (\frac{\pi}{2}, 0)$

Experiment1 $\theta_d = 0$

Experiment2 $\theta_d = \frac{\pi}{2}$

Astolfi's Model

Using this control law, achieve cooperative task!

⇩ combine Igarashi and Astolfi's model

Achieve Pose Synchronization in vehicle systems

Outline

- Introduction
- Brief Review
  - Graph Theory
  - Consensus Problem
- Flocking Problem
  - Moshtagh Model
    - » Control Law, Simulation, Experiment
  - Igarashi Model
    - » Control Law, Simulation, Problem of Implementation
- Nonholonomic Constraint
  - Astolfi Model
- **Combining Model**
  - Simulation
  - Experiment
- Conclusion and Future Works

Problem Setting

System

$$\begin{cases}\dot{x}_i = v_i\cos\theta_i\\ \dot{y}_i = v_i\sin\theta_i\\ \dot{\theta}_i=\omega_i\end{cases}$$

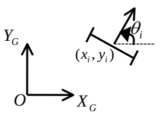


Fig. 17 Vehicle Model

v_i : input velocity

ω_i : input angular velocity

Objective

To achieve the pose synchronization via the vehicle system possessing nonholonomic constraint

$$\lim_{t\to\infty}|x_i - x_j| = 0$$

$$\lim_{t\to\infty}|y_i - y_j| = 0 \qquad \forall i, j$$

$$\lim_{t\to\infty}|\theta_i - \theta_j| = 0$$

Problem Setting

Problem Setting

Determine input velocity (v, ω) to achieve pose synchronization in vehicle systems possessing nonholonomic constraint

Maneuver

Inputs $(\dot{x}, \dot{y}, \dot{\theta})$ achieved by Igarashi's control law

⇩

Target pose (x_d, y_d, θ_d) of Astolfi's control law

$$\begin{bmatrix}x_d\\ y_d\\ \theta_d\end{bmatrix} = K\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix}$$

(x_d, y_d, θ_d) : target pose

$(\dot{x}, \dot{y}, \dot{\theta})$: velocities achieved by Igarashi's control law

K : gain matrix of each state

$$K = \begin{bmatrix}k_x & 0 & 0\\ 0 & k_y & 0\\ 0 & 0 & k_\theta\end{bmatrix}$$

Block Diagram

Block Diagram

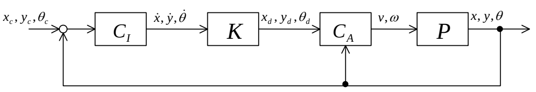


Fig. 18 Block Diagram

Plant

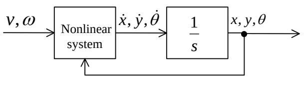


Fig. 19 Block Diagram(Plant)

C_A : Astolfi's controller

C_I : Igarashi's controller in SE(2)

K : gain

P : plant

Control Law

Control Input

i, j : indices of agents

$$\left(\begin{bmatrix}v_c\\ \omega_c\end{bmatrix} = \begin{bmatrix}\dot{x}_c\\ \dot{y}_c\\ \omega_c\end{bmatrix}\right)$$

$$\begin{bmatrix}v_i\\ \omega_i\end{bmatrix} = \begin{bmatrix}k_\rho\sqrt{(x_i - x_{di})^2 + (y_i - y_{di})^2}\\ k_\alpha\left(\tan^{-1}\left(\dfrac{y_{di} - y_i}{x_{di} - x_i}\right) - \theta_i\right) + k_\phi(\theta_{di} - \theta_i)\end{bmatrix}$$

$$= \begin{bmatrix}k_\rho\sqrt{\left(x_i - k_x\left(\dot{x}_c + k_{pi}\sum_{j\in N_i}(x_j - x_i)\right)\right)^2 + \left(y_i - k_y\left(\dot{y}_c + k_{pi}\sum_{j\in N_i}(y_j - y_i)\right)\right)^2}\\ k_\alpha\left(\tan^{-1}\left(\dfrac{\dot{y}_c + k_{pi}\sum_{j\in N_i}(y_j - y_i) - y_i}{\dot{x}_c + k_{pi}\sum_{j\in N_i}(x_j - x_i) - x_i}\right) - \theta_i\right) + k_\phi\left(\omega_c + \sum_{j\in N_i}\sin(\theta_j - \theta_i) - \theta_i\right)\end{bmatrix}$$

Simulation

Initial States

Red : $(x_{r0}, y_{r0}, \theta_{r0}) = (0, 10, -\frac{\pi}{4})$

Green : $(x_{g0}, y_{g0}, \theta_{g0}) = (0, 5, \frac{\pi}{3})$

Blue : $(x_{b0}, y_{b0}, \theta_{b0}) = (0, -5, \frac{\pi}{4})$

Yellow: $(x_{y0}, y_{y0}, \theta_{y0}) = (0, -10, -\frac{\pi}{3})$

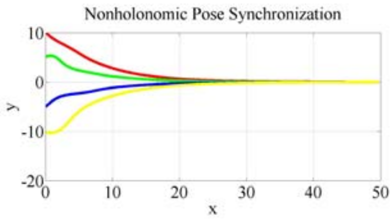
Target Velocities

$$\begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

Gains

$$\begin{bmatrix} k_\rho \\ k_\alpha \\ k_\phi \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -1.5 \end{bmatrix}$$

Others : 1



To **avoid colliding**,

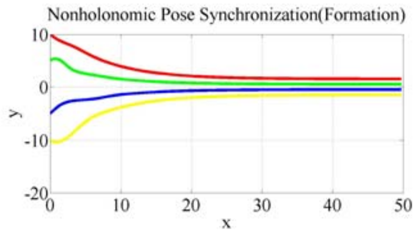
$$\lim_{t\to\infty} |x_i - x_j| = 0 \qquad \lim_{t\to\infty} |x_i - (x_j + d_{xij})| = 0$$

$$\lim_{t\to\infty} |y_i - y_j| = 0 \quad \Rightarrow \quad \lim_{t\to\infty} |y_i - (y_j + d_{yij})| = 0 \quad \forall i, j$$

$$\lim_{t\to\infty} |\theta_i - \theta_j| = 0 \qquad \lim_{t\to\infty} |\theta_i - \theta_j| = 0$$

d_{xij}, d_{yij} : desired distances

$$\begin{cases} d_{xij} = 0 \\ |d_{yij}| = 1 \quad \forall i, j \end{cases}$$



Experiment1

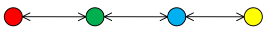


Fig. 20 Graph Structure
(Undirected, Connected, Fixd)

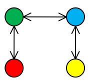
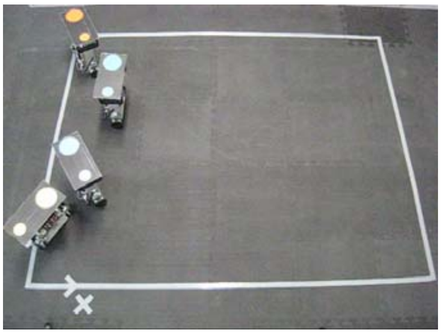


Fig. 21 Formation Structure



Experiment2

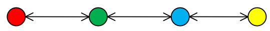


Fig. 22 Graph Structure
(Undirected, Connected, Fixed)

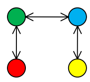
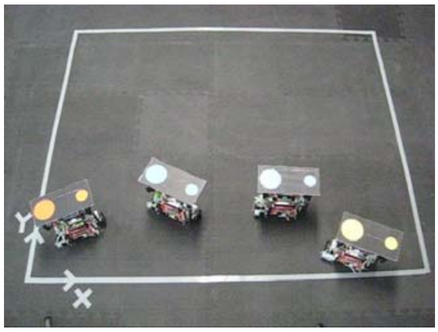


Fig. 23 Formation Structure



For attitude coordination target velocities (v_c, ω_c) are necessary!

⇩ Why?

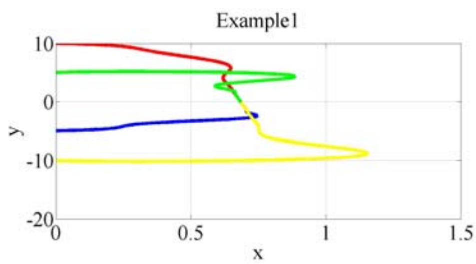
For position synchronization, orientations can be $|\theta_i - \theta_j| > \frac{\pi}{2}$

→ Show examples in next slide

I implement only simulation and experiment!
I haven't consider any mathematical Analysis yet!
I don't know the convergence condition, initial condition which guarantee the convergence, etc.
→ **Future Work**

Without Target Velocities

Ex.1)



Orientations don't synchronize!

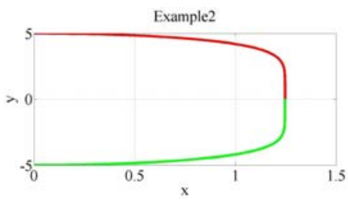
Problems

Ex.2)

Initial States

Red : $(x_{r0}, y_{r0}, \theta_{r0}) = (0, 10, -\frac{\pi}{4})$

Green : $(x_{g0}, y_{g0}, \theta_{g0}) = (0, 5, \frac{\pi}{3})$



Because ω_i contains $\sin(\theta_j - \theta_i)$, if $|\theta_i - \theta_j| > \frac{\pi}{2}$ occurs, then

$$|\theta_i - \theta_j| \quad \rightarrow \quad \pi$$

Outline

Conclusion and Future Work

Conclusion

- Review some matters briefly
- Introduce to my study
- Find problems and next challenges

Future Works

- **Mathematical Analysis**
  - Initial Condition
  - Convergence Condition
  - etc.
- More Survey
  - Learning Technical Matters
- More Experiment
  - Increasing Agents, Desired Motion, etc.

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