FL seminar 2008-4-28

Optimal Search Control for Moving Target

FL08-03-2

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Background

Search Problem

To maximize the probability of locating the target deploying agents with the resources available.

- protection against submarine attacks
- detecting lost objects

Approach
take account of agent system explicitly
limit control energy consumption

Objective

Optimal Search Problem
maximize the probability of locating the target

+

Optimal Control Problem
limit control energy consumption

⇩

formulate ***Optimal Search Control Problem***

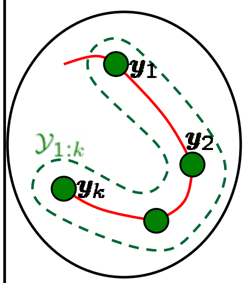
provides its approximate solution

The effectiveness of the proposed method is demonstrated through a numerical simulation

Outline

- Problem Setting
- for Motionless Target
- for Moving Target
- Conclusion, Future Works

Problem Setting



agent system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$x(t) = \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix} \quad \text{position: } y(t) \in \mathcal{R}^2$$

observation time (obs. time) :

$$t_k, k = 0, 1, 2, \cdots$$

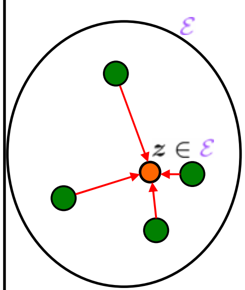
obs. point $y_k := y(t_k)$

waypoint $x_k := x(t_k)$

the set of obs. points from t_p to t_q

$$\mathcal{Y}_{p:q} := \{y_p, y_{p+1}, \cdots, y_q\}$$

Problem Setting



search area: $\mathcal{E} \subset \mathcal{R}^2, \quad z \in \mathcal{E}$

the importance of search:

$\phi(z) > 0$ (large → important)

sensing accuracy $\in [0, 1]$ (good 0, bad 1)

$$p_y(||z - y_i||) = 1 - e^{-\lambda ||z - y_i||^2}$$

obs. level $\in [0, 1]$ (good 0, bad 1)

$$p(z, \mathcal{Y}_{p:q}) := \prod_{y_i \in \mathcal{Y}_{p:q}} p_y(||z - y_i||)$$

search level (small → good)

$$S(\mathcal{Y}_{p:q}) := \int_{\mathcal{E}} \phi(z) p(z, \mathcal{Y}_{p:q}) dz$$

$S(\mathcal{Y}_{p:q}) = 0$ ⇨ target detection with probability one

Outline

- Problem Setting
- for Motionless Target
  - formulation
  - approximate solution
  - simulation 1
  - Receding Horizon Control
  - simulation 2
- for Moving Target
- Conclusion, Future Works

Theorem 1

for Motionless Target

⇨ compute search level $S(\mathcal{Y}_{1:k})$ at time t_k

Theorem 1

$S(\mathcal{Y}_{1:k})$ is monotonically decreasing with k .

$\lim_{k\to\infty} S(\mathcal{Y}_{1:k}) = 0$

→ target detection with probability one

proof:

$$S(\mathcal{Y}_{1:k+1}) - S(\mathcal{Y}_{1:k}) = \int_{\mathcal{E}} \underbrace{\phi(z)}_{>0}\underbrace{p(z,\mathcal{Y}_{1:k})}_{\geq 0}(\underbrace{p(z,\{y_{k+1}\}) - 1}_{<0})dz \leq 0$$

$S(\mathcal{Y}_{1:k+1}) - S(\mathcal{Y}_{1:k}) = 0$ iff $p(z,\mathcal{Y}_{1:k}) = 0 \ \forall z \in \mathcal{E}$

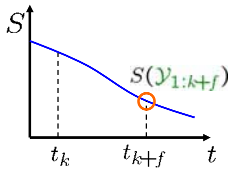
□

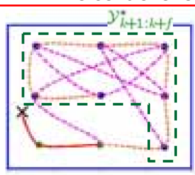
Search and Control

optimal search
minimize search level S

⇔ tradeoff

optimal control
minimize control energy





optimal search control

get $\mathcal{Y}^*_{k+1:k+f}$ to minimize search level $S(\mathcal{Y}_{1:k+f})$

rearrange $\mathcal{Y}^*_{k+1:k+f}$ taking account of control energy

get **optimal input** to minimize control energy

Optimal Search Control Problem

Problem 1 (Finite-time Optimal Search Control Problem)

$$\min_{u(t),t\in[t_k,t_{k+f}]} \sum_{i=0}^{f-1} \int_{t_{k+i}}^{t_{k+i+1}} u^T(t)Ru(t)dt$$

s.t. $y_i = y_i^*, \quad \bigcup_{i=k+1}^{k+f} \{y_i^*\} = \mathcal{Y}^*_{k+1:k+f} := \underset{\mathcal{Y}_{k+1:k+f}}{\text{argmin}}\, S(\mathcal{Y}_{1:k+f})$

It's difficult to get **global optimal solution**!

⇩

relaxation optimal search problem

get **local optimal solution**

$$\mathcal{Y}^*_{k+1:k+f} \in \mathcal{K}_{k+1:k+f} := \left\{\mathcal{Y}_{k+1:k+f} \,\middle|\, \frac{\partial S(\mathcal{Y}_{1:k+f})}{\partial y_i} = 0 \ \forall y_i \in \mathcal{Y}_{k+1:k+f}\right\}$$

Solution Algorithm

$\mathcal{Y}^*_{k+1:k+f} \in \mathcal{K}_{k+1:k+f}$

→ gradient method $y_i^{j+1} = y_i^j + \alpha_j \dfrac{\partial S(\mathcal{Y}^j_{1:k+f})}{\partial y_i^j}$ j : index, $i \in \{k+1,\cdots,k+f\}$

⇩ $\mathcal{Y}^*_{k+1:k+f}$

rearrange $\mathcal{Y}^*_{k+1:k+f}$ to minimize control energy

→ enumeration method $O(f!)$

→ approximate solution : Ant Colony Optimization $O(f^2)$

⇩ $y^*_{k+1}, y^*_{k+2}, \cdots, y^*_{k+f}$

$$\min_{u(t),t\in[t_k,t_{k+f}]} \sum_{i=0}^{f-1} \int_{t_{k+i}}^{t_{k+i+1}} u^T(t)Ru(t)dt$$

$$= \min_X \begin{bmatrix} x_k \\ X \end{bmatrix}^T N \begin{bmatrix} x_k \\ X \end{bmatrix}, \ X = \begin{bmatrix} x_{k+1} \\ \vdots \\ x_{k+f} \end{bmatrix} \ (\because u(t) = \psi(x_i, x_{i+1}),\ t \in [t_i, t_{i+1}])$$

$$= \begin{bmatrix} x_k \\ Y^* \end{bmatrix}^T H \begin{bmatrix} x_k \\ Y^* \end{bmatrix}, \ Y^* = \begin{bmatrix} y^*_{k+1} \\ \vdots \\ y^*_{k+f} \end{bmatrix}, \ \dot{Y}^* = G \begin{bmatrix} x_k \\ Y^* \end{bmatrix}$$

⇩ $u(t), \ t \in [t_k, t_{k+f}]$

Simulation 1

agent system

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u(t)$$

$x_0 = \begin{bmatrix} 12 & 12 & 0 & 0 \end{bmatrix}^T$

$\mathcal{E} = [0,40] \times [0,40]$

$\phi(z) = 1$

$t_i = ih, \quad h = 5$

$R = diag(1,1)$

$\lambda = 0.02$

$f = 10$

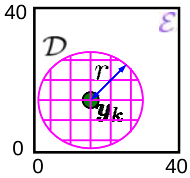
random walk based method

$\mathcal{D} := \left\{ z \in \mathcal{E}_d \,\middle|\, ||z - y_k|| < r \right\}$

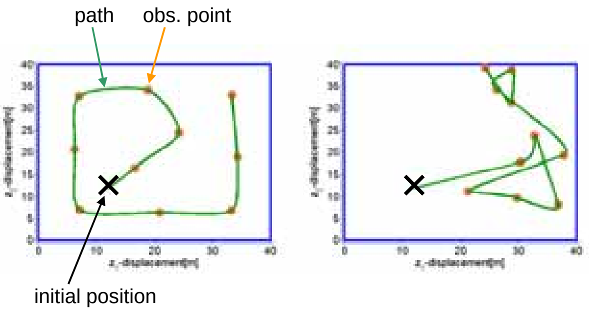
$\mathcal{E}_d$: discretize $\mathcal{E}$

select $y_{i+1} \in \mathcal{D}$ randomly

$r = 20$

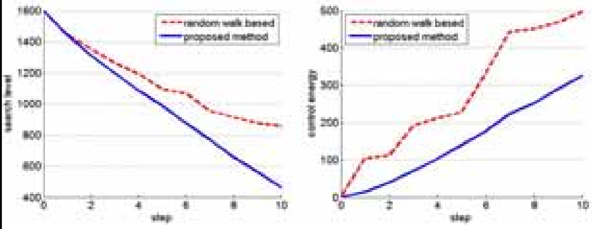


Simulation 1



proposed method — random walk based method

Simulation 1



search level $S(\mathcal{Y}_{1:k})$ at t_k — the sum of control energy

Receding Horizon Control

at time t_k

horizon : $t \in [t_k, t_{k+f}]$

Optimal Search Control Problem

$\boldsymbol{u}^*(t), t \in [t_k, t_{k+f}]$

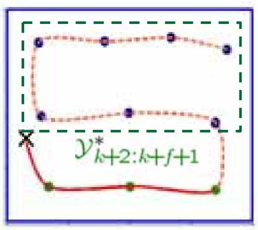
$\boldsymbol{u}^*(t), t \in [t_k, t_{k+1})$

at time t_{k+1}

horizon : $t \in [t_{k+1}, t_{k+1+f}]$

same above …

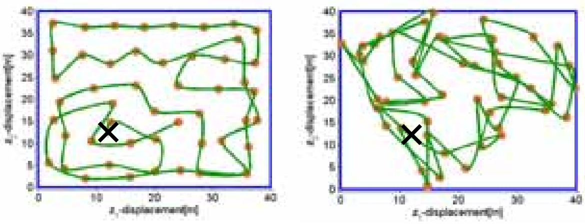
⋮



Simulation 2

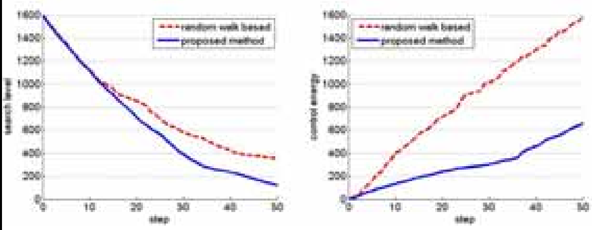
$f = 20 \quad \lambda = 0.05 \quad r = 15$

50 steps



proposed method — random walk based method

Simulation 2



search level $S(\mathcal{Y}_{1:k})$ at t_k — the sum of control energy

Outline

- Problem Setting
- for Motionless Target
- for Moving Target
  - formulation
  - solution algorithm
  - simulation 3
- Conclusion, Future Works

Search and Control

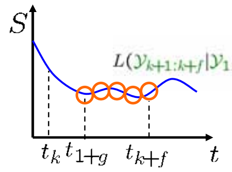
for Moving Target

⇨ compute search level $S(\mathcal{Y}_{k-g:k})$ at time t_k

attention : suppose $f \geq g+2$

optimal search minimize search level S	⇔ tradeoff	***optimal control*** minimize control energy

$$L(\mathcal{Y}_{k+1:k+f}|\mathcal{Y}_{1:k}) = \sum_{i=\max\{1+g,k+1\}}^{k+f} S(\mathcal{Y}_{i-g:i})$$



Optimal Search Control Problem

Problem 2 (Finite-time Optimal Search Control Problem)

$$\min_{u(t),t\in[t_k,t_{k+f}]} \sum_{i=0}^{f-1} \int_{t_{k+i}}^{t_{k+i+1}} \boldsymbol{u}^T(t) R \boldsymbol{u}(t) dt$$

s.t. $\boldsymbol{y}_i = \boldsymbol{y}_i^*$, $\bigcup_{i=k+1}^{k+f} \{\boldsymbol{y}_i^*\} = \mathcal{Y}^*_{k+1:k+f} :=$ argmin$_{\mathcal{Y}_{k+1:k+f}} L(\mathcal{Y}_{k+1:k+f}|\mathcal{Y}_{1:k})$

⇩

It's difficult to get **global optimal solution** as well as Problem 1.

⇩

relaxation optimal search problem

get **local optimal solution**

$$\mathcal{Y}^*_{k+1:k+f} \in \bar{\mathcal{K}}_{k+1:k+f} := \left\{ \mathcal{Y}_{k+1:k+f} \middle| \frac{\partial L(\mathcal{Y}_{k+1:k+f}|\mathcal{Y}_{1:k})}{\partial \boldsymbol{y}_i} = 0 \ \forall \boldsymbol{y}_i \in \mathcal{Y}_{k+1:k+f} \right\}$$

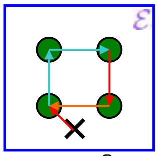
Solution Algorithm

at time t_0

1. $\mathcal{Y}^*_{1:1+g} \in \mathcal{K}_{1:1+g}$
   → gradient method $y_i^{j+1} = y_i^j + \alpha_j \frac{\partial S(\mathcal{Y}^j_{1:1+g})}{\partial y_i^j}$ j : index, $i \in \{1, \cdots, 1+g\}$
2. rearrange $\mathcal{Y}^*_{1:1+g}$, and $\boldsymbol{y}_{2+g} = \boldsymbol{y}_1$ to minimize control energy
   ⇨ get a round path to minimize control energy

at time t_k, $k = 0, 1, 2, \ldots$

3. $\boldsymbol{y}_{i+g+1} = \boldsymbol{y}_i$, $i = 1, 2, \ldots$



$g = 3$

Lemma 1

Lemma 1

$$\mathcal{Y}_{1:1+g} = \mathcal{Y}_{2:2+g} = \cdots = \mathcal{Y}_{k-g:k} = \cdots \quad --- \ (1)$$

$$S(\mathcal{Y}_{1:1+g}) = S(\mathcal{Y}_{2:2+g}) = \cdots = S(\mathcal{Y}_{k-g:k}) = \cdots \quad --- \ (2)$$

$$\mathcal{Y}_{k-g:k} \in \mathcal{K}_{k-g:k} = \left\{ \mathcal{Y}_{k-g:k} \middle| \frac{\partial S(\mathcal{Y}_{k-g:k})}{\partial \boldsymbol{y}_i} = 0 \ \forall \boldsymbol{y}_i \in \mathcal{Y}_{k-g:k} \right\} \quad --- \ (3)$$

proof:

(1) → $\boldsymbol{y}_i \cup \mathcal{Y}_{i+1:i+g} = \mathcal{Y}_{i+1:i+g} \cup \boldsymbol{y}_{i+g+1}$ $(\because \boldsymbol{y}_{i+g+1} = \boldsymbol{y}_i)$
$\mathcal{Y}_{i:i+g} = \mathcal{Y}_{i+1:i+g+1}, i = 1, 2, \ldots$

(2) → obviously by (1) ⇨ at time $t_k \geq t_{1+g}$, $S(\mathcal{Y}_{k-g:k}) = const.$

(3) → obviously by $\mathcal{Y}_{1:1+g} \in \mathcal{K}_{1:1+g}$ and (1)

□

Theorem 2

Theorem 2

$\mathcal{Y}^*_{k+1:k+f}$ gotten by using Algorithm 2 is one local optimal obs. points of Problem 2, so that, $\mathcal{Y}^*_{k+1:k+f} \in \bar{\mathcal{K}}_{k+1:k+f}$.

proof:

$\forall i \in \{k+1, \cdots, k+f\}$,

$$\frac{\partial L(\mathcal{Y}^j_{k+1:k+f}|\mathcal{Y}_{1:k})}{\partial \boldsymbol{y}_i^j} = \sum_{i'=\max\{1+g,k+1\}}^{k+f} \frac{\partial S(\mathcal{Y}_{i'-g:i'})}{\partial \boldsymbol{y}_i^j}$$

$$= 0$$

because $\frac{\partial S(\mathcal{Y}_{i'-g:i'})}{\partial \boldsymbol{y}_i^j} = 0 \ \forall i' \in \{\max\{1+g, k+1\}, \ldots, k+f\}$ by Lemma 1 (3).

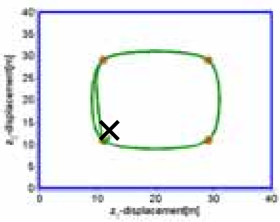
So $\mathcal{Y}^*_{k+1:k+f} \in \bar{\mathcal{K}}_{k+1:k+f}$.

□

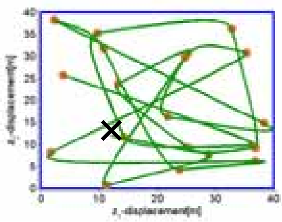
Simulation 3

$f = 5$ $g = 3$ $\lambda = 0.02$ $r = 60$

20 steps

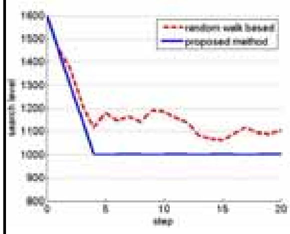


proposed method

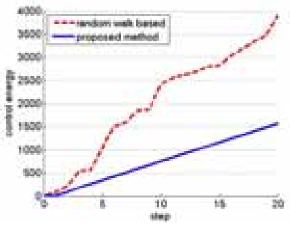


random walk based method

Simulation 3



search level $S(\mathcal{Y}_{k-g:k})$ at t_k



the sum of control energy

Proposition 1

Proposition 1

Suppose $t_{i+1} - t_i = t_{i+1+g} - t_{i+g},\ i = 0, 1, 2, \ldots$ and $T = t_{i+g} - t_i$.

$$\begin{aligned} \boldsymbol{x}(t) &= \boldsymbol{x}(t+T) \\ \boldsymbol{u}(t) &= \boldsymbol{u}(t+T) \end{aligned} \quad (\text{as } t \to \infty)$$

I have not proved this yet …

Conclusion and Future Works

Conclusion

- formulation of Optimal Search Control Problem
- its approximate solution
- The effectiveness of the proposed method was demonstrated through a numerical simulation

Future Works

- theoretical verification
- reduce computation time
- search control under non-convex state constraints
- pursuit