towards Application of Vision-Based Cooperative Control (progress report) FL08_01_2

Naoto Kobayashi

Outline

- Introduction
- Bird’s Eye Camera Configuration
  - Review (FL07_27_2)
  - Problem Formulation
  - Analysis
  - Simulation
  - Experiments
- Mounted Camera Configuration
  - Problem Formulation
  - Analysis
  - Simulation
- Conclusion / Future Works

Introduction

Cooperative Control
- Cooperative control is a distributed control strategy that achieves specified tasks in multi-agent systems.
- It’s been motivated by interests in group behavior of animals, formation control of multi-vehicle systems and so on.
- It is hoped to be applied to sensor networks, robot networks and many other multi-agents systems.

In cooperative control problems, we assume that agents can get information of only their neighbors.

Pose Synchronization in SE(3) [1]
- Pose synchronization in SE(3) means that all of the agents’ positions and attitudes become the same.

How can we know relative positions and attitudes between each agent?

- Visual observer that is proposed in [4] can estimate relative positions and orientations between camera and target objects.
**Introduction**

- Our goal is to realize the autonomous agents system by applying a visual observer to mounted camera configuration.

**Mounted / Bird’s Eye Camera Configuration**

- Each agent can have its own “eyes”.
- Our goal is to realize the autonomous agents system.

**Last Seminar (FL07_27_2)**

- Proof of the position synchronization (bird’s eye camera configuration)
- Simulation analysis (bird’s eye camera configuration)

**This Seminar (FL08_01_1)**

- Experiments (bird’s eye camera configuration)
- Proof of the position synchronization (mounted camera configuration, under some assumptions)
- Simulations towards experiments (mounted camera configuration)

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**Problem Formulation (review)**

- Agents’ Kinematics
  \[
  p_{i} \in \mathbb{R}^{3} \quad \text{position} \\
  \theta_{i} \in \mathbb{R}^{3} \quad \text{rotation angle} \\
  \omega_{i} \in \mathbb{SO}(3) \quad \text{rotation direction} \\
  v_{i}^{b} \in \mathbb{R}^{3} \quad \text{body linear velocity} \\
  \omega_{i}^{b} \in \mathbb{R}^{3} \quad \text{body angular velocity} \\
  \]

- Information Graph
  - Fixed: A topology of a graph does not change.
  - Balanced (undirected or cyclic): In-degree and out-degree are same. ...(As1)

- Neighbor: \( N_{i} = \{ j | j \in V, (i, j) \in E \} \)

- Graph Laplacian:
  \[
  L = \begin{bmatrix}
  \sum_{j \in N_{i}} & \cdots & \sum_{j \in N_{i}} \\
  \vdots & \ddots & \vdots \\
  \sum_{j \in N_{i}} & \cdots & \sum_{j \in N_{i}} \\
 \end{bmatrix}
  \]

- Control Objective
  - Make all of the agent poses (position, attitude) be the same.
  \[
  \lim_{t \to \infty} \| y_{i} - y_{j} \| = I_{4} \quad \forall i, j
  \]

- I assume that world frame and camera frame are the same. That is \( \sum_{i} = \sum_{i} \) ...(As2)
Analysis (review)

Structure of Agent with Visual Observer (Bird’s eye camera)

Visual Observer

• Estimation error $e_v$ is asymptotically stable with input (2) when $V_{obs} = 0$.

$$V_{obs} = \frac{1}{2} \| p_{av} \|^2 + \phi(e^{e_{av}})$$

$$\dot{V}_{obs} = p_{av}^T \dot{p}_{av} + \phi(e^{e_{av}})$$

$$= (u_v) - e_v$$

$$= -e_v^T K e_v < 0$$

$$\lim_{t \to \infty} (y_i, y_j) = I_k \quad \forall i, j$$

Proposal of Inputs

• Input to EsRRBM

$$u_i = K e_i \quad \ldots(2)$$

$$K_i := \begin{bmatrix} K_{rv} & 0 \\ 0 & K_{rv} \end{bmatrix} \in \mathbb{R}^{k \times k}$$

$$K_{rv} := \text{diag}\{k_{rv}, k_{rv}, k_{rv}\} \in \mathbb{R}^{k \times k}$$

$$k_{rv} > 0$$

$$K_{rv} := \text{diag}\{k_{rv}, k_{rv}, k_{rv}\} \in \mathbb{R}^{k \times k}$$

$$k_{rv} > 0$$

$$p_{av} := e^{e_{av}} (p_{av} - p_{av})$$

$$e^{e_{av}} := e^{e_{av}} e^{e_{av}}$$

: estimation error

$$e_v := \begin{bmatrix} p_{av}^T \\ \text{sk}(e^{e_{av}} \cdot) \end{bmatrix} = J_i (f_i - \bar{f})$$
Analysis of Position Synchronization

Input (2), (3) achieve position synchronization under the assumptions As1, As2 if the condition \((K_{\alpha} \otimes I_2) - (L \otimes I_2) \geq 0\) is satisfied.

**Proof:**

Define the energy function as follows.

\[
V = \sum_{i=1}^{n} \frac{1}{2} \| p_{ei} \|_2^2 + \frac{1}{2} \| p_{wi} \|_2^2
\]

By differentiating this energy function, we can prove that position synchronization is achieved. Q.E.D

Analysis of Attitude Synchronization

• I have not been able to prove attitude synchronization yet.

Simulation Results:

- Simulation Results : \(K_{\alpha} = 5 \times I_6\)

Simulation Results:

- Simulation Results : \(K_{\alpha} = 3 \times I_6\)

Simulation Results:

- Sufficient Condition
  
  Assume \(K_{\alpha} = k \times I_3\), \(k > 0\)
  
  Then
  
  \[
  \begin{cases} 
  (K_{\alpha} \otimes I_2) - (L \otimes I_2) \geq 0 & k \geq 3.55 \\
  (K_{\alpha} \otimes I_2) - (L \otimes I_2) < 0 & k < 3.55 
  \end{cases}
  \]

Experiment

- Experimental system schematic
  
  - HALCON, SIMULINK, DS1104
  - Vehicle (Mini-2), RF Transmitter, Camera
  - Experiment is performed on 2-Dimension plane

- Problem formulation
  
  - graph : balanced graph
  
  - initial states

Experiment

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- Problem formulation
  
  - graph : balanced graph
  
  - initial states
### Problem Formulation

#### Information Graph

- **Fixed**: A topology of a graph that consists of a single cycle.
- **Cyclic**: A graph that consists of a single cycle. (very strong restriction) ...(As3)

- **Control Objective**
  - Pose Synchronization in SE(3): Make all of the agent poses (position / attitude) be the same.

\[
\lim_{t \to \infty} (x_i^j, y_i^j) = \tilde{I}_i \quad \forall i, j
\]

- **Agents’ Kinematics**

\[
\begin{align*}
& p_{w, i} \in R^3: \text{position} \\
& e_{w, i} \in SO(3): \text{attitude} \\
& v_{w, i} \in R^3: \text{body linear velocity} \\
& \omega_{w, i} \in R^3: \text{body angular velocity} \\
& \zeta_i = (p_{w, i}, e_{w, i}) \in SE(3) \quad \text{(1)}
\end{align*}
\]

- **Structure of Agent with Visual Observer (Mounted camera)**

### Analysis

- **Structure of Agent with Visual Observer (Mounted camera)**

\[
\begin{align*}
& v_{\text{Camera}} \quad p_{\text{Camera}}(u, v) \\
& \text{Relative Position} \quad P_i^j \quad \text{Points Model} \\
& \text{Perspective Projection} \\
& \text{Image Jacobian} \\
& \text{Velocity Input} \\
& v_{\text{Camera}} \quad N \quad \tilde{J}_i \\
& \text{Kinematics}
\end{align*}
\]
Input to EsRRBM

\[ u_{ij} = K_{ij} e_{ij} \]  
\[ e_{ij} = (\gamma_{ij}) (sk_{i}, \hat{e}_{i}) \]

- Velocity Input

\[ \frac{\partial \phi}{\partial t} = \sum_{j \neq i} \left( \frac{\theta}{\gamma_{ij}} \right) \]  
\[ (i = 1, 2, \ldots, n) \]  

Proposal of Inputs

- Assumption
  - Visual observer can estimate relative positions and attitudes of neighbor agents wherever they are.

\[ V = \sum_{i \neq j} \sum_{k=1}^{n} \left( \frac{1}{2} \| p_{i,k} - p_{j,k} \| ^2 \right) \]

Analysis of Position Synchronization

- Assumption
  - Input (4), (5) achieve position synchronization under the condition ...

\[ \frac{1}{2} \| p_{i,k} - p_{j,k} \| ^2 \]

\[ \Xi = \hat{e}_{i} - \hat{e}_{j} \]

Analysis of Attitude Synchronization

- I have not been able to prove attitude synchronization yet.

Simulation towards Experiments

- Assumption
  - In some cases, pose (quasi-)synchronization is achieved.

\[ \phi(0) = [0 \ 0 \ 0]^T \]

\[ \phi(0) = [4 \ 4 \ 4]^T \]

\[ \phi(0) = [-3 \ 3 \ 3]^T \]

\[ \phi(0) = [0 \ 0 \ 0]^T \]

- Agent 0 cannot detect any other agents so it does not move.

- If there are more than 2 agents that cannot detect any other agents, synchronization is not achieved.

I want to prevent this situation in experiments.
Simulation towards Experiments

To prevent such a situation ...
- limit the initial configuration
  (line-configuration, center-directed configuration, ...)
- By mounting the another camera to each agent to see backward, they come to be able to detect all other agents.

- I’ll do this experiment ASAP. After that, I’ll try to modify the theory of mounted camera configuration.

Leader following with line-configuration

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Conclusion / Future Works

- Bird’s view camera configuration
  - Review of the previous seminar (FL07_27_2)
  - Experiments
- Mounted camera configuration
  - analysis of position synchronization
  - some simulations towards experiments

- I’ll do this experiment ASAP. After that, I’ll try to modify the theory of mounted camera configuration.

Future Works

- I’ll modify experimental systems (camera, image board and etc).
- I’ll do experiments of mounted camera configuration ASAP.
- I’ll make theories of mounted camera configuration.

References


Appendix

FL07_16_2

*Note: Notation is different in some part.
Visual Feedback Observer -RRBM-

Relative Rigid Body Motion

\[
\sum \omega_c \cdot \text{world frame} \\
\sum \omega_t \cdot \text{camera frame} \\
\sum \omega_r \cdot \text{object frame}
\]

\[
\mathbf{g}_m = R_{m,w} \mathbf{g}_w
\]

configuration of a frame \( \sum \) relative to a frame \( \sum \)

\[
\mathbf{\tilde{g}} = \mathbf{g}_m + \mathbf{g}_r
\]

Relative Rigid Body Motion

\[
\mathbf{g}_m = R_{m,w} \mathbf{g}_w + \mathbf{g}_r 
\]

\[
\mathbf{\tilde{g}} = \mathbf{g}_m + \mathbf{g}_r 
\]

Visual Feedback Observer -Camera Model-

Camera Model

\[
\mathbf{p}_i \in \mathbb{R}^3 \quad \text{position vector of the } i\text{-th feature point relative to } \sum \noindent \text{feature point relative to } \sum
\]

\[
\mathbf{p}_i = \left[ x_i, y_i, z_i \right]^T
\]

\[
\mathbf{f}_i = \mathbf{f}_i^0 + \mathbf{f}_i^1
\]

\[
\mathbf{f}_i = \left[ f_x, f_y, f_z \right] \quad \text{perspective projection of the } i\text{-th feature point onto the image plane}
\]

\[
\mathbf{p}_i = \left[ x_i, y_i, z_i \right] \quad \text{image plane}
\]

\[
\mathbf{f}_i = \lambda_i \mathbf{f}_i^1
\]

\[
\mathbf{f}_i = \lambda_i \mathbf{f}_i^1
\]

\[
\mathbf{f}_i = \lambda_i \mathbf{f}_i^1
\]

Visual Feedback Observer -EES

Estimation Error System

\[
\mathbf{g}_m = \left( \mathbf{p}_i, \mathbf{e}_i^c \right) \\
= R_{m,w} \mathbf{g}_w \quad \text{estimation error}
\]

\[
\mathbf{p}_i = \mathbf{f}_i^0 + \left( \mathbf{f}_i - \mathbf{p}_i \right)
\]

\[
\mathbf{e}_i^c = \mathbf{f}_i - \mathbf{e}_i^c
\]

\[
\mathbf{e}_i = \left[ e_{i,x}, e_{i,y}, e_{i,z} \right] \quad \text{estimation error vector} \\
= \frac{1}{2}(\mathbf{e}_i^c - \mathbf{e}_i^c)^T \\
f - \tilde{f} = J(\mathbf{g}_m) \mathbf{e}_i
\]

* take 3 or more feature points so that \( J(\mathbf{g}_m) \) be column full rank

Visual Feedback Observer -EsRRBM-

Estimated Relative Rigid Body Motion

\[
\mathbf{\tilde{v}}^R = -\mathbf{Ad}_{\omega} \mathbf{v}^R + \mathbf{u}_e \quad \text{EsRRBM model}
\]

\[
\mathbf{p}_i = \left[ x_i, y_i, z_i \right]
\]

\[
\mathbf{f}_i = \lambda_i \mathbf{f}_i^1
\]

\[
\mathbf{f}_i = \lambda_i \mathbf{f}_i^1
\]

Visual Feedback Observer -EES

Estimation Error System

\[
\mathbf{g}_m = \left( \mathbf{p}_i, \mathbf{e}_i^c \right)
\]

\[
\mathbf{p}_i = \mathbf{f}_i^0 + \left( \mathbf{f}_i - \mathbf{p}_i \right)
\]

\[
\mathbf{e}_i^c = \mathbf{f}_i - \mathbf{e}_i^c
\]

\[
\mathbf{e}_i = \left[ e_{i,x}, e_{i,y}, e_{i,z} \right] \quad \text{estimation error vector} \\
= \frac{1}{2}(\mathbf{e}_i^c - \mathbf{e}_i^c)^T \\
f - \tilde{f} = J(\mathbf{g}_m) \mathbf{e}_i
\]

* take 3 or more feature points so that \( J(\mathbf{g}_m) \) be column full rank
Visual Feedback Observer -EES

**Estimation Error System**

\[ V'_e = \begin{bmatrix} e' \cdot p_e \\ (\varepsilon' \cdot e' \cdot p_e) \end{bmatrix} \]

If \( V'_e = 0 \), then estimation error system satisfies

\[ \dot{V}'_e = -Ad_p + F_p + F_e \quad \text{estimation error motion model} \]

**Passivity of the Estimation Error System**

If \( V'_e = 0 \), then estimation error system satisfies

\[ \int_0^T u'_e (-e) dt \geq -\beta \]

\[ \beta > 0 \]

**Proof:** Consider the following positive definite function

\[ \dot{\phi} = \frac{1}{2} \| p_e \|^2 + \phi (\varepsilon' \cdot e') \]

differentiating \( \dot{\phi} \) with respect to time yields

\[ \dot{\phi} = p'_e \cdot p_e + e'_p (\varepsilon' \cdot e') \dot{\phi} \]

Choose \( u'_e = Ke_e \), then

- Estimation error is asymptotically stable if \( V'_e = 0 \).
- Estimation error is L2-gain stable if \( V'_e = 0 \), \( K = \frac{1}{2} \frac{1}{2} I \geq 0 \).