towards Application of
Vision-Based Cooperative Control
(progress report)
FL08_01_2

Naoto Kobayashi

Outline

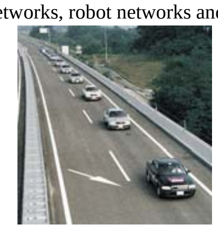
- ■Introduction
- ■Bird's Eye Camera Configuration
  - •Review (FL07_27_2)
    - ⬩Problem Formulation
    - ⬩Analysis
    - ⬩Simulation
  - •Experiments
- ■Mounted Camera Configuration
  - •Problem Formulation
  - •Analysis
  - •Simulation
- ■Conclusion / Future Works

Introduction

■Cooperative Control

• Cooperative control is a distributed control strategy that achieves specified tasks in multi-agent systems.
• It's been motivated by interests in group behavior of animals, formation control of multi-vehicle systems and so on.
• It is hoped to be applied to sensor networks, robot networks and many other multi-agents systems.

School of Fish
http://www.yunphoto.net/



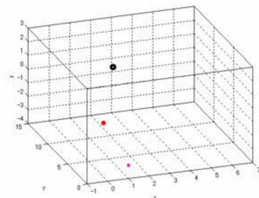
Automated Highway System
http://www.its.go.jp/ITS/

Introduction

■Cooperative Control

• Consensus Problem
: to reach an agreement regarding a certain quantity of interest that depends on the state of all agents.
• Flocking Problem
: to make all of agents' speeds be the same.
• Coverage Problem
• Formation Control Problem
:

In cooperative control problems, we assume that agents can get information of only thier neighbors.
↓
distributed control



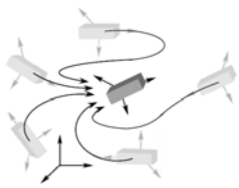
flocking (leader following)

Introduction

■ Pose Synchronization in SE(3) [1]

• Pose synchronization in SE(3) means that all of the agents' positions and attitudes become the same.

• Consensus problem[2] and a kind of flocking problem[3] are included.



Introduction

■ Pose Synchronization in SE(3)

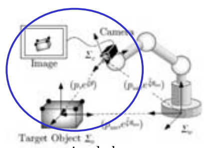
•Information of neighbors' relative positions and attitudes is necessary.
↓

How can we know relative positions and attitudes between each agent ?

⬇

■Visual Observer [4]

•Visual observer that is proposed in [4] can estimate relative positions and orientations between camera and target objects.

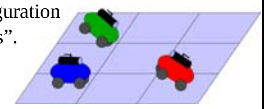


visual observer

Introduction

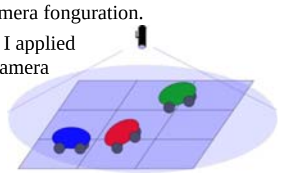
■Mounted / Bird's Eye Camera Fonfiguration

- Mounted camera (local camera) configuration
  - Each agent can have its own "eyes".
  - → autonomous agents system



mounted camera fonguration

- Bird's eye camera configuration
  - Our goal is realize the autonomous agents system by applying visual observer to mounted camera fonguration.
  - To make the problem easier, I applied visual observer to bird's eye camera fonfiguration at first.



bird's eye camera configuration

Introduction

■ Last Seminar(FL07_27_2)

- •Proof of the position synchronization (bird's eye camera configuration)
- •Simulation analysis (bird's eye camera configuration)

■ This Seminar (FL08_01_1)

- •Experiments (bird's eye camera configuration)
- •~~Proof of the position synchronization (mounted camera configuration, under some assumptions)~~
- •Simulations towards experiments(mounted camera configuration)

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- •Review (FL07_27_2)
  - ⬩Problem Formulation
  - ⬩Analysis
  - ⬩Simulation
- •Experiments

■Mounted Camera Configuration
- •Problem Formulation
- •Analysis
- •Simulation

■Conclusion / Future Works

Problem Formulation (review)

•Agents' Kinematics

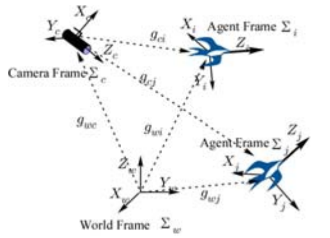
$$\begin{cases} \dot{p}_{wi} = e^{\hat{\xi}_{wi}} v^b_{wi} \\ \dot{e}^{\hat{\xi}_{wi}} = e^{\hat{\xi}_{wi}} \hat{\omega}^b_{wi} & (i=1,2,\ldots,n) \\ y_i = (p_{wi}, e^{\hat{\xi}_{wi}}) \in SE(3) & \text{...(1)} \end{cases}$$

$p_{wi} \in R^3$: position
$e^{\hat{\xi}_{wi}} \in SO(3)$: attitude
$v^b_{wi} \in R^3$: body linear velocity
$\omega^b_{wi} \in R^3$: body angular velocity
$\zeta_{wi} = \theta_{wi}\xi_{wi}$
$\xi_{wi} \in R^3$: rotation direction
$\theta_{wi} \in R$: rotation angle

$$\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix}^{\wedge} := \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

$$g_{wi} = \begin{bmatrix} e^{\hat{\xi}_{wi}} & p_{wi} \\ 0 & 1 \end{bmatrix}$$

: homogeneous representation of $g_{wi} = (p_{wi}, e^{\hat{\xi}_{wi}}) \in SE(3)$

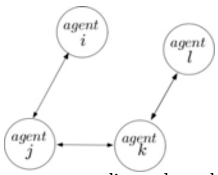


wi, ci : i th agent frame seen from world frame, camera frame

Problem Formulation (review)

•Information Graph

- Fixed : A topology of a graph does not change.
- Balanced (undirected or cyclic) : In-degree and out-degree are same. ...(As1)

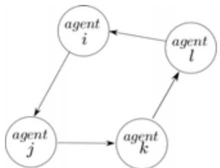


undirected graph

•Neighbor : $N_i := \{ j \mid j \in \mathcal{V} : (i,j) \in \mathcal{E} \}$
Agents connected with agent i

•Graph Laplacian :

$$L := [L_{ij}] = \begin{cases} \sum_{j \in N_i} 1 & i = j \\ -1 & j \in N_i \\ 0 & j \notin N_i \end{cases}$$



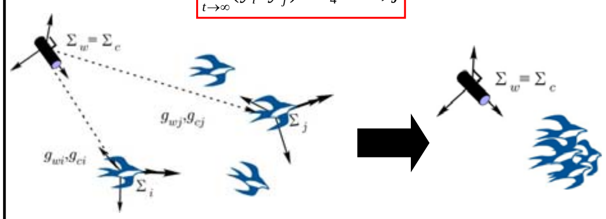
cyclic graph

Problem Formulation (review)

•Control Objective

- Pose Synchronization in SE(3)
  : Make all of the agent poses (position /attitude) be the same.

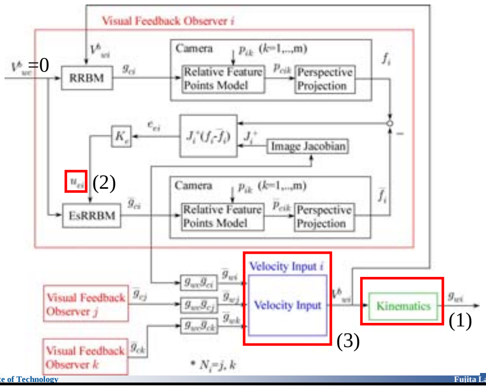
$$\lim_{t\to\infty}(y_i^{-1} y_j) = I_4 \quad \forall i, j$$



*I assume that world frame and camera frame are the same.
That is $\sum_w \equiv \sum_c$ ...(As2)

Analysis (review)

■Structure of Agent i with Visual Observer (Bird's eye camera)



Analysis (review)

■Visual Observer [4]

•Input to EsRRBM

$$u_{ei} = K_e e_{ei} \qquad ...(2)$$

$$K_e := \begin{bmatrix} K_{ev} & 0 \\ 0 & K_{e\omega} \end{bmatrix} \in R^{6\times 6}$$

$$K_{ev} := \mathrm{diag}\{k_{ev}, k_{ev}, k_{ev}\} \in R^{3\times 3} \qquad k_{ev} > 0$$

$$K_{e\omega} := \mathrm{diag}\{k_{e\omega}, k_{e\omega}, k_{e\omega}\} \in R^{3\times 3} \qquad k_{e\omega} > 0$$

$$p_{eei} := e^{-\hat{\xi}_{ci}}(p_{ci} - \bar{p}_{ci})$$

$$e^{\hat{\xi}_{eei}} := e^{-\hat{\bar{\xi}}_{ci}} e^{\hat{\xi}_{ci}} \qquad \text{: estimation error}$$

$$e_{ei} := \begin{bmatrix} p_{eei} \\ \mathrm{sk}(e^{\hat{\xi}_{eei}})^{\vee} \end{bmatrix} = J_i^{+}(f_i - \bar{f}_i)$$

Analysis (review)

■Visual Observer [4]

•Estimation error e_{ei} is asymptotically stable with input (2) when $V^b_{wi} = 0$.

$$V_{obsi} = \frac{1}{2}\| p_{eei} \|^2 + \phi(e^{\hat{\xi}_{eei}})$$

$$\dot{V}_{obsi} = p^T_{eei}\dot{p}_{eei} + \dot{\phi}(e^{\hat{\xi}_{eei}})$$

$$= \dots$$

$$= (u_{ei})^T(-e_{ei})$$

$$= -e^T_{ei} K_e e_{ei} < 0$$

➡ $e_{ei}\left(= \begin{bmatrix} p_{eei} \\ \mathrm{sk}(e^{\hat{\xi}_{eei}})^{\vee} \end{bmatrix}\right) \to 0$

Analysis (review)

■Pose Synchronization [1]

•Velocity Input

$$\begin{bmatrix} v^b_{wi} \\ \omega^b_{wi} \end{bmatrix} = \sum_{j \in N_i} \begin{bmatrix} e^{-\hat{\xi}_{wi}} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} p_{wj} - p_{wi} \\ \mathrm{sk}(e^{-\hat{\xi}_{wi}} e^{\hat{\xi}_{wj}}) \end{bmatrix} \quad (i = 1, 2, \dots, n)$$

•Velocity input (3) achieves output synchronization.

$$V_{syn} := \sum_{i=1}^{n} \{\frac{1}{2} p^T_{wi} p_{wi} + \phi(e^{\hat{\xi}_{wi}})\}$$

$$\dot{V}_{syn} = \sum_{i=1}^{n} \{p^T_{wi}\dot{p}_{wi} + \dot{\phi}(e^{\hat{\xi}_{wi}})\}$$

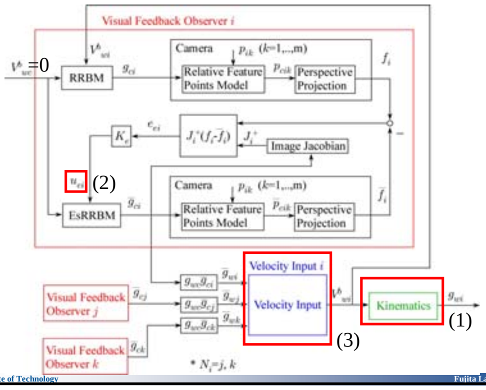
$$= \dots$$

$$\le 0$$

➡ $\lim_{t\to\infty}(y_i^{-1} y_j) = I_4 \quad \forall i, j$

Analysis (review)

■Structure of Agent i with Visual Observer (Bird's eye camera)



Analysis (review)

■Proposal of Inputs

•Input to EsRRBM

$$u_{ei} = K_e e_{ei} \qquad ...(2)$$

$$K_e := \begin{bmatrix} K_{ev} & 0 \\ 0 & K_{e\omega} \end{bmatrix} \in R^{6\times 6}$$

$$K_{ev} := \mathrm{diag}\{k_{ev}, k_{ev}, k_{ev}\} \in R^{3\times 3} \qquad k_{ev} > 0$$

$$K_{e\omega} := \mathrm{diag}\{k_{e\omega}, k_{e\omega}, k_{e\omega}\} \in R^{3\times 3} \qquad k_{e\omega} > 0$$

•Velocity Input

$$\begin{bmatrix} v^b_{wi} \\ \omega^b_{wi} \end{bmatrix} = \sum_{j \in N_i} \begin{bmatrix} e^{-\hat{\bar{\xi}}_{wi}} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{p}_{wj} - \bar{p}_{wi} \\ \mathrm{sk}(e^{-\hat{\bar{\xi}}_{wi}} e^{\hat{\bar{\xi}}_{wj}}) \end{bmatrix} \quad (i = 1, 2, \dots, n) \qquad ...(3)$$

Analysis (review)

■Analysis of Position Synchronization

Input (2), (3) achieve position synchronization under the assumptions As1, As2 if the condition $(K_{ev} \otimes I_n) - (L \otimes I_3) \geq 0$ is satisfied.

Proof :

Define the energy function as follows.

$$V := \sum_{i=1}^{n} \{ \underbrace{\frac{1}{2} \| p_{eei} \|^2}_{\text{Observer}} + \underbrace{\frac{1}{2} \| p_{wi} \|^2}_{\text{Position Synchronization}} \}$$

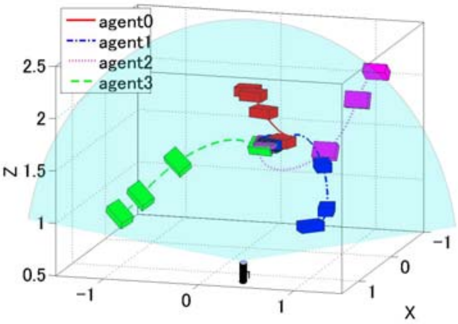
By differentiating this energy function, we can prove that position synchronization is achieved. Q.E.D

■Analysis of Attitude Synchronization

•I have not be able to prove attitude synchronization yet.

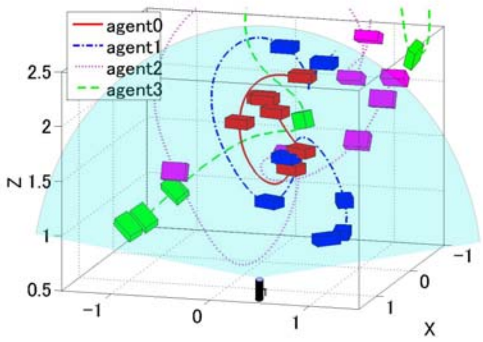
Simulation (review)

■Simulation Results : $K_e = 5 \times I_6$



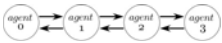
Simulation (review)

■Simulation Results : $K_e = 3 \times I_6$



Simulation (review)

■Simulation Results



• Sufficient Condition

Assume

$K_{ev} = k \times I_3 \qquad k > 0$

$* \; L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

Then

$$\begin{cases} (K_{ev} \otimes I_n) - (L \otimes I_3) \geq 0 & k \geq 3.55 \\ (K_{ev} \otimes I_n) - (L \otimes I_3) < 0 & k < 3.55 \end{cases}$$

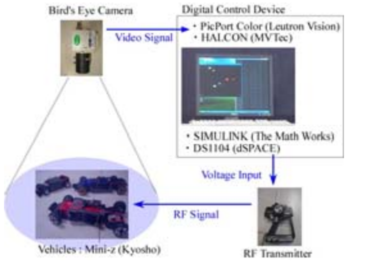
•Simulation Results

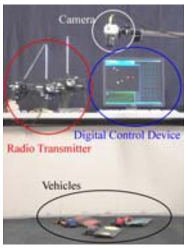
- Output synchronization is achied when $K_e = 5 \times I_6 > 3.55 \times I_6$
- Output synchronization is not achieved when $K_e = 3 \times I_6 < 3.55 \times I_6$

Experiment

■Experiment

• Experimental system schematic
  - HALCON, SIMULINK, DS1104
  - Vehicle (Mini-z), RF Transmitter, Camera
  - Experiment is performed on 2-Dimension plane

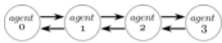




Experiment

■Experiment

• Problem formulation
  - graph : balanced graph



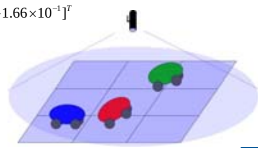
  - initial states

$p0(0) = [-0.92 \; -0.59 \; 2.05]^T, \xi 0(0) = [0 \; 0 \; -1.48 \times 10^{-1}]^T$

$p1(0) = [-0.90 - 0.18 \; 2.05]^T, \xi 1(0) = [0 \; 0 \; 2.36 \times 10^{-1}]^T$

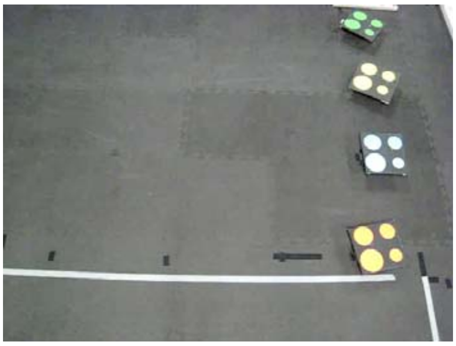
$p2(0) = [-0.93 \; 0.28 \; 2.05]^T, \xi 2(0) = [0 \; 0 \; 3.31 \times 10^{-1}]^T$

$p3(0) = [-0.87 \; 0.69 \; 2.05]^T, \xi 3(0) = [0 \; 0 \; -1.66 \times 10^{-1}]^T$



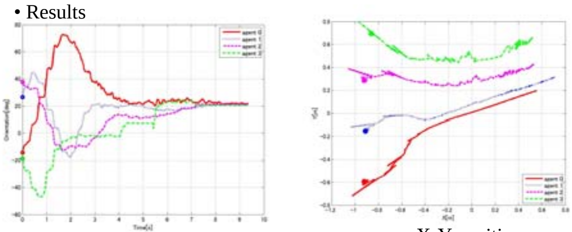
Experiment

■Experiment
• Results



Experiment

■Experiment
• Results



orientation

X-Y position

Pose(position and attitude) synchronization is achieved.

*Note : for collision avoidance, I assume that position synchronization is achieved when distances between each vehicle become less than 20cm.

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⬩Simulation
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•Analysis
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■Conclusion / Future Works

Problem Formulation

•Agents' Kinematics

$$\begin{cases} \dot{p}_{wi} = e^{\hat{\xi}_{wi}} v^b_{wi} \\ \dot{e}^{\hat{\xi}_{wi}} = e^{\hat{\xi}_{wi}} \hat{\omega}^b_{wi} \\ y_i = (p_{wi}, e^{\hat{\xi}_{wi}}) \in SE(3) \end{cases} \quad (i = 1, 2, \ldots, n) \quad \text{...(1)}$$

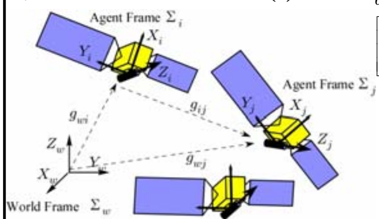
$p_{wi} \in R^3$: position
$e^{\hat{\xi}_{wi}} \in SO(3)$: attitude
$v^b_{wi} \in R^3$: body linear velocity
$\omega^b_{wi} \in R^3$: body angular velocity
$\zeta_{wi} = \theta_{wi}\xi_{wi}$
$\xi_{wi} \in R^3$: rotation direction
$\theta_{wi} \in R$: rotation angle

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}^\wedge := \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

$$g_{wi} = \begin{bmatrix} e^{\hat{\xi}_{wi}} & p_{wi} \\ 0 & 1 \end{bmatrix}$$

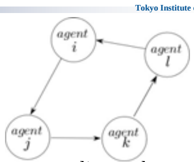
: homogeneous representation of $g_{wi} = (p_{wi}, e^{\hat{\xi}_{wi}}) \in SE(3)$

wi : i th agent frame seen from world frame



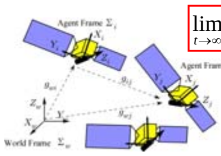
Problem Formulation

•Information Graph
- Fixed : A topology of a graph does not change.
- Cyclic : a graph that consists of a single cycle. (very strong restriction) ...(As3)

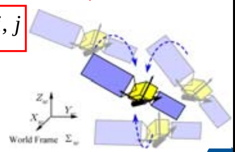


cyclic graph

•Control Objective
- Pose Synchronization in SE(3)
: Make all of the agent poses (position /attitude) be the same.

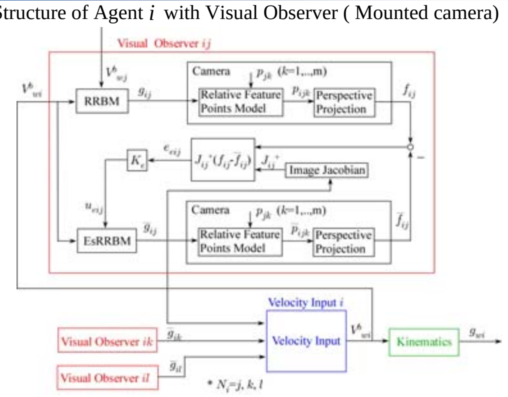


$$\lim_{t\to\infty}(y_i^{-1} y_j) = I_4 \quad \forall i, j$$



Analysis

■Structure of Agent i with Visual Observer (Mounted camera)



Analysis

■Proposal of Inputs

•Input to EsRRBM

$u_{eij} = K_e e_{eij}$ ...(4)

$K_e := \begin{bmatrix} K_{ev} & 0 \\ 0 & K_{e\omega} \end{bmatrix} \in R^{6\times 6}$

$K_{ev} := \mathrm{diag}\{k_{ev}, k_{ev}, k_{ev}\} \in R^{3\times 3} \quad k_{ev} > 0$

$K_{e\omega} := \mathrm{diag}\{k_{e\omega}, k_{e\omega}, k_{e\omega}\} \in R^{3\times 3} \quad k_{e\omega} > 0$

$p_{eeij} := e^{-\hat{\bar{\xi}}_{ij}} (p_{ij} - \bar{p}_{ij})$

$e^{\hat{\xi}_{eeij}} := e^{-\hat{\bar{\xi}}_{ij}} e^{\hat{\xi}_{ij}}$: estimation error

$e_{eij} := \begin{bmatrix} p_{eeij} \\ \mathrm{sk}(e^{\hat{\xi}_{eeij}})^{\vee} \end{bmatrix} = J_{ij}^{+} (f_{ij} - \bar{f}_{ij})$

•Velocity Input

$\begin{bmatrix} v_{wi}^b \\ \omega_{wi}^b \end{bmatrix} = \sum_{j\in N_i} \begin{bmatrix} \bar{p}_{ij} \\ \mathrm{sk}(e^{\hat{\bar{\xi}}_{ij}}) \end{bmatrix} \quad (i = 1, 2, \ldots, n)$ ...(5)

Analysis

■Analysis of Position Synchronization

•Assumption

- Visual observer can estimate relative positions and attitudes of neighbor agents wherever they are. ...(As4)

→ impossible assumption

•Proposition

Input (4), (5) achieve position synchronization under the assumption As3, As4 if the condition $(K_{ev} \otimes I_n) - (L \otimes I_3) \geq 0$ is satisfied.

Proof : Define the energy function as follows.

$$V := \sum_{i=1}^{n} \{ \sum_{j\in N_i} \{ \frac{1}{2} \| p_{eeij} \|^2 \} + \frac{1}{2} \| p_{wi} \|^2 \}$$

Observer — Position Synchronization

By differentiating this energy function, we can prove that position synchronization is achieved. Q.E.D

Analysis

■Analysis of Position Synchronization

As4 is not proper assumption : a camera cannot see targets which are too near nor behind it .

⬇

•New Assumption

- Agent i can sense its neighbors N_i those satisfy $z_{ij}^i > d_{short}$.

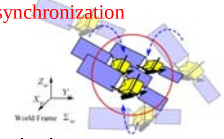
z_{ij}^i : distance in the direction of the optical axis

↓

approch the neighbors whom agent i can see

↓

I have to investigate if the position (quasi-)synchronization is achieved under this assumption.



■Analysis of Attitude Synchronization

•I have not been able to prove attitude synchronization yet.

Simulation towards Experiments

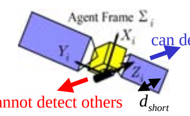
* Note : Following simulation is not a simulation for the foregoing theory but for the preparation for experiments in terms of that it considers only one camera on each agent (s.t. they can see only forward).

However if we consider two camera on each agent, they can see both forward and backward. This is almost the same situation with foregoing theory.

•Assumption

- switching topology

$(i, j) \in \mathcal{E}$ if $z_{ij}^i > d_{short}$



- each agent knows initial relative configurations with other agents approximately.

•Initial states

$p0(0) = [0\ 0\ 3]^T, \xi 0(0) = [0\ 0\ 1]^T$

$p1(0) = [3\ 4\ 2]^T, \xi 1(0) = [\frac{1}{2}\ 0\ -\frac{1}{2}]^T$

$p2(0) = [-3\ 1\ 1]^T, \xi 2(0) = [0\ 0\ 1]^T$

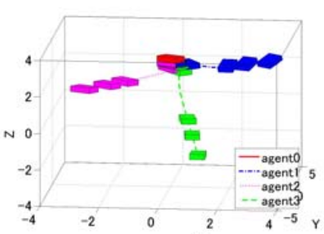
$p3(0) = [1\ -1\ -2]^T, \xi 3(0) = [0\ 0\ 0]^T$

Simulation towards Experiments

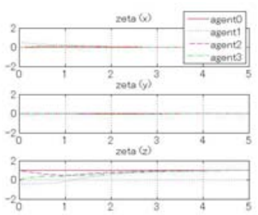
•Assumption

- switching topology

$(i, j) \in \mathcal{E}$ if $z_{ij}^i > d_{short}$



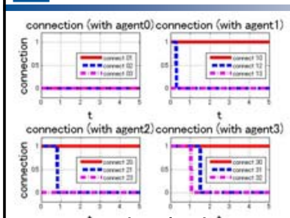
position



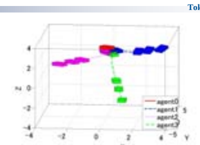
attitude

- In some case, pose (quasi-)synchronization is achieved.

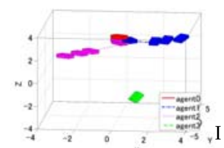
Simulation towards Experiments



switching of topology



- Agent0 cannot detect any other agents so it does not move. (a kind of leader following)



- If there are more than 2 agents that cannot detect any other agents, synchronization is not achieved.

↓

I want to prevent this situation in experiments.

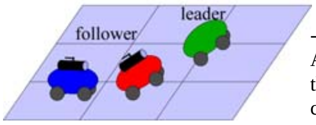
Simulation towards Experiments

To prevent such a situation ...

- limit the initial configuration
  (line-configuration, center-directed configuration, ...)
- By mounting the another camera to each agent to see backward, they come to be able to detect all other agents.

⬇ the easiest experiment is ...

Leader following with line-configuration



- I'll do this experiment ASAP. After that, I'll try to modify the theory of mounted camera configuration.

Outline

Conclusion / Future Works

■Conclusion
- Bird's view camera configuration
  - Review of the previous seminar (FL07_27_2)
  - Experiments
- Mounted camera configuration
  - ~~analysis of position synchronization~~
  - some simulations towards experiments

■Future Works
- I'll modify experimental systems (camera, image board and etc).

↓

- I'll do experiments of mounted camera configuration ASAP.

↓

- I'll make theories of mounted camera configuration.

References

[1]Y. Igarashi, T. Hatanaka and M. Fujita, Output Synchronization in SE(3) -Passivity-based Approach- ,Proc. of the 36th SICE Symposium on Control Theory, 35/38, 2007.

[2]R. O. Saber, J. A. Fax and T. M. Murray, Consensus and Cooperation in Networked Multi-Agent Systems, Proc. of the IEEE, 95-1, 215/233, 2007.

[3]N. Moshtagh and A. Jadbabaie, Distributed Geodesic Control Laws for Flocking of Nonholonomic Agents, IEEE Trans. on Automatic Control, 52-4, 681/686, 2007.

[4] M. Fujita, H. Kawai and M. W. Spong, Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking:Stability and L2-gain Performance Analysis, IEEE Trans. on Control Systems Technology, vol. 15, no. 1, 40/52, 2007.

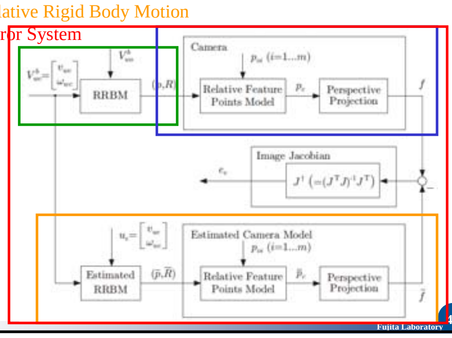
Appendix

FL07_16_2

*Note : Notation is different in some part.

Visual Feedback Observer -Outline-

■Outline
- ●Relative Rigid Body Motion
- ●Camera Model
- ●Estimated Relative Rigid Body Motion
- ●Estimation Error System



Visual Feedback Observer -RRBM-

■Relative Rigid Body Motion

Σ_w:world frame

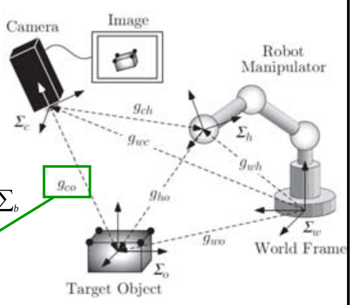
Σ_c:camera frame

Σ_o:object frame

$g_{ab} = (p_{ab}, e^{\hat{\xi}\theta_{ab}})$

$= \begin{bmatrix} e^{\hat{\xi}\theta_{ab}} & p_{ab} \\ 0 & 1 \end{bmatrix} \in SE(3)$

:configuration of a frame Σ_b relative to a frame Σ_a



$g_{co} = g_{wc}^{-1} g_{wo}$

$\dot{g}_{co} = \dot{g}_{wc}^{-1} g_{wo} + g_{wc}^{-1}\dot{g}_{wo}$

$= -g_{wc}^{-1}\dot{g}_{wc} g_{wc}^{-1} g_{wo} + g_{wc}^{-1}\dot{g}_{wo}$ ...(1)

Visual Feedback Observer -RRBM-

■Relative Rigid Body Motion

$\hat{V}_{ab}^b = g_{ab}^{-1}\dot{g}_{ab}$

$= \begin{bmatrix} e^{-\hat{\xi}_{ab}}\dot{e}^{\hat{\xi}_{ab}} & e^{-\hat{\xi}_{ab}}\dot{p}_{ab} \\ 0 & 0 \end{bmatrix}$

$V_{ab}^b = \begin{bmatrix} v_{ab}^b \\ \omega_{ab}^b \end{bmatrix}$

$= \begin{bmatrix} e^{-\hat{\xi}_{ab}}\dot{p}_{ab} \\ (e^{-\hat{\xi}_{ab}}\dot{e}^{\hat{\xi}_{ab}})^\vee \end{bmatrix}$

$\hat{V}_{ab}^b \in se(3)$

$V_{ab}^b \in R^6$:body velocity

$v_{ab}^b \in R^3$:body translation velocity

$\omega_{ab}^b \in R^3$:body angular velocity

$\dot{g}_{co} = -g_{wc}^{-1}\dot{g}_{wc} g_{wc}^{-1} g_{wo} + g_{wc}^{-1}\dot{g}_{wo}$ ...(1)

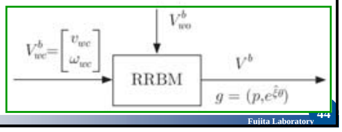
$= -\hat{V}_{wc}^b g_{co} + g_{co}\hat{V}_{wo}^b$

$\hat{V}_{co}^b = g_{co}^{-1}\dot{g}_{co}$

$= -g_{co}^{-1}\hat{V}_{wc}^b g_{co} + \hat{V}_{wo}^b$

$V_{co}^b = -Ad_{(g_{co}^{-1})} V_{wc}^b + V_{wo}^b$:RRBM model

$\mathrm{Ad}_{(g_{ab})} = \begin{bmatrix} e^{\hat{\xi}_{ab}} & \hat{p}_{ab} e^{\hat{\xi}_{ab}} \\ 0 & e^{\hat{\xi}_{ab}} \end{bmatrix}$



Visual Feedback Observer -Camera Model-

■Camera Model

$p_{oi} \in R^3$:position vector of the i-th feature point relative to Σ_o

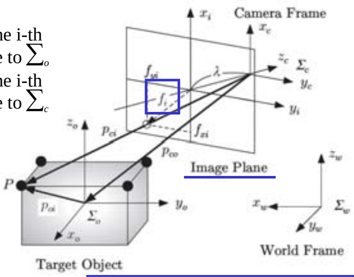
$p_{ci} \in R^3$:position vector of the i-th feature point relative to Σ_c

$p_{ci} := [x_{ci} \quad y_{ci} \quad z_{ci}]$

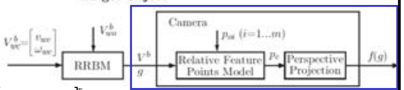
$= g_{co} p_{oi}$

$f_i := [f_{xi} \quad f_{yi}]^T$

:**perspective projection** of the i-th feature point onto the **image plane**



$f_i = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix}$



$p_c = [p_{c1}^T \ \cdots \ p_{cm}^T]^T$, $f = [f_1^T \ \cdots \ f_m^T]^T$

Visual Feedback Observer -EsRRBM-

■Estimated Relative Rigid Body Motion

$\bar{V}_{co}^b = -Ad_{(\bar{g}_{co}^{-1})} V_{wc}^b + u_e$:EsRRBM model

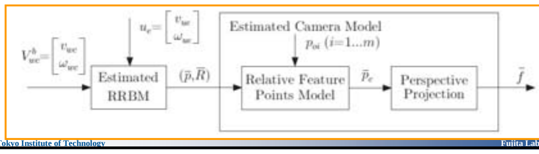
u_e :input to EsRRBM

$\bar{p}_{ci} := [\bar{x}_{ci} \quad \bar{y}_{ci} \quad \bar{z}_{ci}]$

$= \bar{g}_{co} p_{oi}$

$\bar{f}_i = \frac{\lambda}{\bar{z}_{ci}} \begin{bmatrix} \bar{x}_{ci} \\ \bar{y}_{ci} \end{bmatrix}$

$\bar{p}_c = [\bar{p}_{c1}^T \ \cdots \ \bar{p}_{cm}^T]^T$, $\bar{f} = [\bar{f}_1^T \ \cdots \ \bar{f}_m^T]^T$



Visual Feedback Observer -EES

■Estimation Error System

$g_{ee} = (p_{ee}, e^{\hat{\xi}_{ee}})$

$= \bar{g}_{co}^{-1} g_{co}$:estimation error

$p_{ee} = \bar{e}^{-\hat{\xi}_{co}}(p_{co} - \bar{p}_{co})$

$e^{\hat{\xi}_{ee}} = \bar{e}^{-\hat{\xi}_{co}} e^{\hat{\xi}_{co}}$

$e_e = [p_{ee}^T \quad e_R^T(e^{\hat{\xi}_{ee}})]^T$:estimation error vector

$* \ e_R(e^{\hat{\xi}_{ab}}) = \frac{1}{2}(e^{\hat{\xi}_{ab}} - e^{-\hat{\xi}_{ab}})^\vee$

$f - \bar{f} = J(\bar{g}_{co}) e_e$:estimation error of feature point

$J_i(g_{co}) = \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} \bar{e}^{\hat{\xi}_{co}} [I \quad -\hat{p}_{oi}]$:**image jacobian**

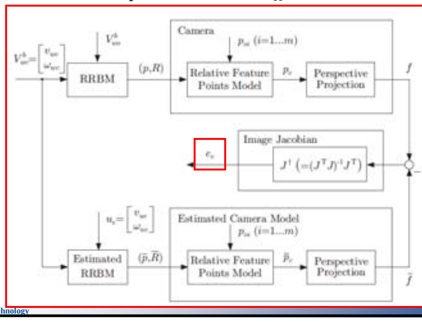
$J(\bar{g}_{co}) = [J_1^T(g_{co}) \ \cdots \ J_m^T(g_{co})]^T$

Visual Feedback Observer -EES

■Estimation Error System

$e_e = J^+(\bar{g}_{co})(f - \bar{f})$

*take 3 or more feature points so that $J(\bar{g}_{co})$ be column full rank



Visual Feedback Observer -EES

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■Estimation Error System

$$V_{ee}^b = (g_{ee}^{-1}\dot{g}_{ee})^\vee = \begin{bmatrix} e^{-\hat{\xi}_{ee}}\dot{p}_{ee} \\ (e^{-\hat{\xi}_{ee}}\dot{e}^{-\hat{\xi}_{ee}})^\vee \end{bmatrix}$$

$V_{ee}^b = -\mathrm{Ad}_{(g_{ee}^{-1})}u_e + V_{wo}^b$:estimation error motion model

■Passivity of the Estimation Error System

If $V_{wo}^b = 0$, then estimation error system satisfies $\int_0^T u_e^T(-e_e)dt \geq -\beta_e$

$* \ \beta_e > 0$

proof : Consider the following positive definite function

$✞ = \frac{1}{2}\| p_{ee} \|^2 + \phi(e^{\bar{\xi}_{ee}})$ $\quad * \ \phi(e^{\hat{\xi}_{ab}}) = \frac{1}{2}\mathrm{tr}(I - e^{\hat{\xi}_{ab}})$

differentiating ✞ with respect to time yields

$✞ ■ \ p_{ee}^T\dot{p}_{ee} + e_R^T(e^{\bar{\xi}_{ee}})\omega_{ee}^s$

$= ....$ ...(2)

Visual Feedback Observer -EES

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■Passivity of the Estimation Error System

$✞ ■ \ p_{ee}^T\dot{p}_{ee} + e_R^T(e^{\bar{\xi}_{ee}})\omega_{ee}^s$ ...(2)

$= ....$

$= u_e^T(-e_e)$

integrating ✞ from 0 to T, we obtain

$\int_0^T u_e^T(-e_e)d\tau = \int_0^T ✞ \, d\tau$

$= ✞☎) - ✞☎) \geq -✞☎) \geq -\beta_e$ ■

$u_e = Ke_e \quad K > 0$

$\Rightarrow ✞ < 0$

$\Rightarrow e_e (= \begin{bmatrix} p_{ee} \\ e_R(e^{\bar{\xi}_{ee}}) \end{bmatrix}) \to 0$

Visual Feedback Observer -EES

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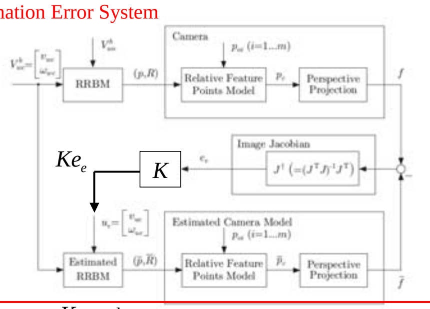
■Estimation Error System



γ :L2-gain

Choose $u_e = Ke_e$, then

- Estimation error is asymptotically stable if $V_{wo}^b = 0$.
- Estimation error is L2-gain stable if $V_{wo}^b \neq 0$, $K - \frac{1}{2\gamma^2}I - \frac{1}{2}I \geq 0$.