A Brief Introduction to Distributed Cooperative Control of Multi-Agent System

FL07 -32-1
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Outline

- Introduction
- Brief Introduction to Graph
- Literature Survey
  - -Introduction to my basement
  - -Example of asymptotically convergence of consensus problem
  - -Other surveys
- Conclusion
- Future Works

Introduction

Distributed Control :

the controller elements are not central in location but are distributed throughout the system with each component sub-system controlled by one or more controllers

→ **small** amount for **calculation** for each egent and **small** communication **cost**

Cooperative Control :

a distributed control strategy that achieves specified tasks in multi-agent system

Fig. 1 School of fishes(※)

Motivation:

・Analysis of emergent and self-organized swarming behaviors in biological groups with distributed agent-to-agent interaction

・Interest in a group behavior of animals, formulation control of multi-vehicle systems and so on

Application :

Mobile sensor networks, Robot networks, and many other Multi-agent systems

※ http://www.allposters.co.jp/-sp/-Posters_i1006775_.htm

Brief Introduction to Graph

- Vertex(node) : Agent
- Edge : Information Flow
- Graph : A set of connections(Edges) of between Objects(Vertice)
  - -Directed Graph(Fig. 2) : the information flows from agent *j* to *i*
  - -Undirected Graph(Fig. 3) : the information flows to both directions

Directed Graph

・strongly connected(Fig. 4) :

there is a directly path connecting any two distinct nodes

・weakly connected(Fig. 2) :

there is a path connecting any two distinct nodes ignoring the direction

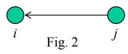


Fig. 2

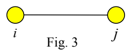


Fig. 3

Undirected Graph

・connected :

there is a path between any two distinct nodes

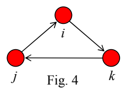


Fig. 4

Definition of Graph Laplacian

Information graph : $G := \{V, E, W\}$

The set of nodes(agents) : $V := \{v_1, \dots, v_n\}$

The set of edges(information) : $E \subseteq V \times V$

The a map assigning a weight to each edge(non-uniform reliabilty) : $W : E \to \Re^+$

(s.t. if $e_{ij} = (v_i, v_j) \in E, W(e_{ij}) = w_{ij}, w_{ij} > 0$)

Graph Laplacian : $L \in \Re^{n \times n}$

$$L_{ij} = \begin{cases} -w_{ij} & \text{if } i \neq j \text{ and } (v_i, v_j) \in E \\ 0 & \text{if } i \neq j \text{ and } (v_i, v_j) \notin E \\ \sum_{k \in N_i} w_{ik} & \text{if } i = j \end{cases}$$

Graph Laplacian has many significant properties !

Literature Survey (Vicsek et al.)

T. Vicsek et al.[1]

Propose a compelling discrete-time model of n autonomous agents

-phase transition in **nonequilibrium** systems

-the **same** speed and different headings

-each agent's set of nearest neighbors **change** with time as the system evolves

-the average of an agent's own heading plus the headings of its "neighbors"

Control Law

$$x_i(t+1) = x_i(t) + v_i(t)\Delta t \qquad t \in \{0, 1, \dots\}$$
$$\theta(t+1) = <\theta(t)>_r + \Delta\theta$$

$x_i(t)$: position $\theta(t)$: angle $v_i(t)$: velocity $<\theta(t)>_r$: average direction

$\Delta\theta$: noise

Literature Survey (Vicsek et al.)

Simulation[1]

initial state | small densities and noise | higher densities and noise | higher densities and small noise

This paper shows only simulation results !

There is **no** theoretical explanation → **Jadbabaie et al.**

Literature Survey (Jadbabaie et al.)

A. Jadbabaie et al.[2]

Provide a **theoretical explanation** for Vicsek's model

- ignore noise and the **same** speed
- undirected and simple graph
- **Graph Laplacian** use some matrix theory(stochastic, primitive, ergotic, etc.)
- "linked together" → all agents' headings **converge**
- "linked to their leader" → **leader following problem**
- by using a **dwell time**, discrete-time → continuous-time

Control Law (Jadbabaie et al.)

Average

$$\theta_i(t+1) = \langle \theta_i(t) \rangle_r \qquad t \in \{0,1,\ldots\}$$

$$\langle \theta_i(t) \rangle_r = \frac{1}{1+n_i(t)}\Big(\theta_i(t) + \sum_{j \in \mathcal{N}_i(t)} \theta_j(t)\Big)$$

$\theta_i(t)$: agent i's heading

$\mathcal{N}_i(t)$: agent i's neighbors

$n_i(t)$: the number of neighbors

Control Law

$$\theta_i(t+1) = \theta_i(t) + u_i(t) \qquad t \in \{0,1,\ldots\}$$

$$\dot{\theta}_i = u_i$$

$$\mathrm{DT}: u_i(t) = -\frac{1}{1+n_i(t)+b_i(t)}\Big((n_i(t)+b_i(t))\theta_i(t) - \sum_{j \in \mathcal{N}_i(t)} \theta_j(t) - b_i(t)\theta_0\Big)$$

$$\mathrm{CT}: u_i(t) = -\frac{1}{1+n_i(t_{ik})+b_i(t_{ik})}\Big((n_i(t_{ik})+b_i(t_{ik}))\theta_i(t) - \sum_{j \in \mathcal{N}_i(t_{ik})} \theta_j(t) - b_i(t_{ik})\theta_0\Big),$$

$$t \in [t_{ik}, t_{ik}+\tau)$$

θ_0 : the leader's fixed heading $\quad b_i(t)$ 1 whenever the leader is a neighbor of agent *i* and 0 otherwise

τ : dwell time

Literature Survey (Jadbabaie et al.)

Linked together → all agents' headings **converge**

Linked together across a time interval $[t, \tau]$:

The collection of graph $\{\mathbf{G}_t, \mathbf{G}_{t+1}, \ldots, \mathbf{G}_\tau\}$ encountered along the interval is **jointly connected**

Jointly connected :

The union of $\{\mathbf{G}_t, \mathbf{G}_{t+1}, \ldots, \mathbf{G}_\tau\}$ is a connected graph

Connected graph :

The graph has a "path" between each distinct pair of its vertices *i* and *j*

The results have been extended to the case where there are **inter-agent forces**

Motivation to More Literature Survey

Consensus? LaSalle's thorem? Lyapunov・・・?

To study not only "flocking" but more other fields

↓

survey Consensus problems, Coverage control, Coordination problems, and so on

In what follows,

I intoroduce mainly some consensus problems

Literature Survey

Consensus problem :

To reach an **agreement** regarding a certain quantity of interest that depends on the state of all

Literature Survey (Olfati et al.)

R. Olfati-Saber et al.[3]

Address the consensus problem

- A variety of assumptions on the network topology
  - **fixed** or **switching**
  - **directed** or **undirected**
  - presence or lack of communication **time-delay**
- Continuous time and Discrete time
- Balanced → Average-consensus problem
- Disagreement vector → feature of the **Algebraic connectivity**
- Algebraic connectivity → **Tradeoff** between performance and robustness

Literature Survey (Olfati et al.)

$G = (V, E, W)$: weighted digraph

$V = \{v_1, \dots, v_n\}$: set of nodes

$E \subseteq V \times V$: set of edges (an edge of $G : e_{ij} = (v_i, v_j)$)

$W : E \to \Re^+$: map assigning a positive weight to each edge

$N_i = \{v_j \in V : (v_i, v_j) \in E\}$: set of **neighbors** of node v_i

$A = [a_{ij}]$: weighted adjacency matrix with nonnegative adjacency elements a_{ij}

- $e_{ij} \in E \Leftrightarrow a_{ij} > 0$
- $a_{ii} = 0$

- example of weighted adjacency matrix(Fig. 5)

$$A = \begin{bmatrix} 0 & 0 & w_{13} \\ w_{21} & 0 & w_{23} \\ 0 & 0 & 0 \end{bmatrix}$$

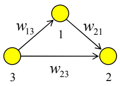


Fig. 5

Literature Survey (Olfati et al.)

$L = [l_{ij}]$: Graph Laplacian
$$l_{ij} = \begin{cases} -a_{ij}, & j \neq i \\ \sum_{k=1, k\neq i}^{n} a_{ik}, & j = i \end{cases}$$

Graph Laplacian has many significant properties!

e.g.
- $L\mathbf{1} = \mathbf{0}$
- nonnegative matrix
- the stability properties of system is completely determined by the location of the Laplacian eigenvalues of the network

- example of Laplacian matrix(Fig. 6)

$$L = \begin{bmatrix} w_{13} & 0 & -w_{13} \\ -w_{21} & w_{21} + w_{23} & -w_{23} \\ 0 & 0 & 0 \end{bmatrix}$$

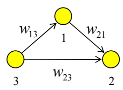


Fig. 6

Literature Survey (Olfati et al.)

Control Law

CT : $\dot{x}_i(t) = u_i(t)$

DT : $x_i(k+1) = x_i(k) + \varepsilon u_i(k)$

$$u_i = \sum_{v_j \in N_i} a_{ij}(x_j - x_i)$$

x_i : agent i's state

$\varepsilon > 0$: step-size

↓ use **Laplacian matrix L**

CT : $\dot{x}(t) = -Lx(t)$ (1)

DT : $x(k+1) = P_\varepsilon x(k)$ (2)

$x = (x_1, \dots, x_n)^T$: network with value $x \in \mathbf{R}^n$

$P_\varepsilon = I - \varepsilon L$: **Perron** matrix

Consensus : $x \rightarrow \alpha\mathbf{1}$

Literature Survey (Olfati et al.)

For **undirected** graph, graph Laplacian satisfies the following **sum-of-squares** property :

$$x^T L x = \frac{1}{2} \sum_{(i,j)\in E} a_{ij}(x_j - x_i)^2$$

Defining a quadratic disagreement function :

$$\phi(x) := x^T L x$$

By using this function,
algorithm (1) → the gradient-descent algorithm as :

$$\dot{x} = -\frac{1}{2}\nabla\phi(x)$$

This algorithm globally asymptotically converges to the agreement space provided that two conditions hold :

(i) L is a **positive semidefinite** matrix
(ii) the **only** equilibrium of (1) is $\alpha\mathbf{1}$ for some α

Literature Survey (Olfati et al.)

For **undirected** graph, **connected** graph hold both of these conditions
→ the algorithm in (1) asymptotically converge

Furthermore, the algorithm solves an **Average consensus problem** :

$$\alpha = \frac{\sum_i x_i(0)}{n} \quad (3)$$

For **directed** graph, if the graph is **strongly connected**, then the algorithm asymptotically converge to the below consensusstate with a left eigenvector $\gamma = (\gamma_1, \dots \gamma_n)$ satisfying $\gamma^T L = \mathbf{0}$.

$$\alpha = \frac{\gamma^T x(0)}{\gamma^T \mathbf{1}}$$

Verification :

$\dot{x} = -Lx \rightarrow y = \gamma^T x$ is an invariant quantity due to $\dot{y} = -\gamma^T Lx = 0, \forall x$

$\rightarrow \lim_{t\to\infty} y(t) = y(0)$ *i.e.* $\gamma^T(\alpha\mathbf{1}) = \gamma^T x(0)$

$$\rightarrow \alpha = \frac{\gamma^T x(0)}{\sum_i \gamma_i}$$

Literature Survey (Olfati et al.)

Furthermore, if graph is balanced, the consensus state converge to (3).

Balanced :

a digraph where the total weight of edges entering a node and leaving the same node are equal for all nodes. (Fig. 7)

→ any undirected graph is balanced

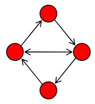


Fig. 7
(neglect weight)

For **connected undirected** graph or **strongly connected balanced digraph**, the consensus state depends on only the **initial state**.

Literature Survey (Olfati et al.)

Algebraic connectivity :

The **second smallest** eigenvalue of Laplacian is a measure of **performance**/**speed** of consensus algorithms → **disagreement vector**

Disagreement vector :

For **connected** undirected graph or **SC balanced** digraph, α is invariant

→ disagreement vector : $\delta = x - \alpha \mathbf{1} \quad (\mathbf{1}^T \delta = 0)$

(1), (2) → $\dot{\delta}(t) = -L\delta(t)$, $\delta(k+1) = P\delta(k)$

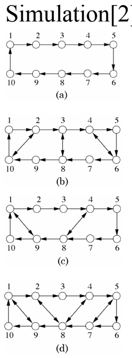
$\phi(\delta) := \delta^T \delta$: **Valid Lyapunov function**

→ $\dot{\phi} \le -2\lambda_2 \phi, \quad \|\delta\| = \phi(\delta)^{\frac{1}{2}}$ λ_2 : the second largest eigenvalue of L

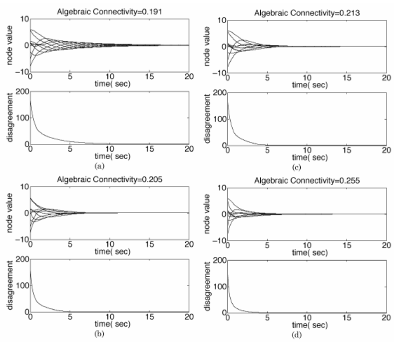
The disagreement vector exponentially vanishes with a speed of at least λ_2

Literature Survey (Olfati et al.)

Simulation[2]



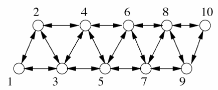
Balanced graphs

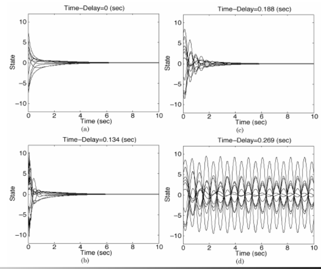


Literature Survey (Olfati et al.)

Other studies

- Switching topology
- Time-delay
- Tradeoff between robustness of a protocol to time-delay and its performance





Literature Survey (Others)

- H. G. Tanner et al.[4],[5]
  - Fixed and dynamic topology
  - Double integrator dynamics
  - Collision avoidance ← only if Graph is **complete**
  - **Artificial potential function** (Fig. 8)
  - graph representing agent interconnections remains **connected at all times**

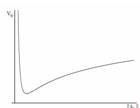
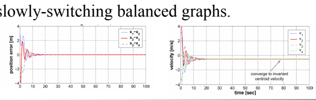


Fig. 8

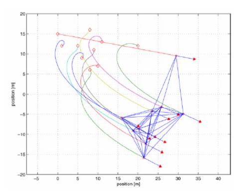
- D. Lee and W. Spong[7]
  - **Inertial effect**
  - Agents evolve on a balanced information graph
  - **passive decompose** the dynamics into a **shape** system and a **locked** system
  - By using **dwell-time** concept the proposed framework is also applicable for slowly-switching balanced graphs.



Literature Survey (Others)

- C. Reynolds[8]
  - Approach based on computer simulation
  - Three rules
    - Separation : steer to avoid crowding local flockmates
    - Alignment : steer towards the average heading of local flockmates
    - Cohesion : steer to move toward the average position of local flockmates

- N. Moshtagh, and A. Jadbabaie[9]
  - Kinematic **nonholonomic** agents
  - Leader following problem
  - **Misalignment energy**(Lyapunov fanction)
  - **Dwell-time**



Conclusion

- Introduced to my work briefly
- Got preliminary knowledge for my study
- Raised motivation for my study
  - Learned to be interested in survey
  - Knew pleasantness of survey
  - Willing to experiment

Future Works

- More survey
- Learning experimental circumstance
- Simulation
- Implementation of a flocking behavior with "e-nuvo WHEEL" by applying Igarashi's control law[10] to the system

References

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