Vision-Based Cooperative Control with Fixed Camera Configuration FL_07_27_2

Naoto Kobayashi

Outline

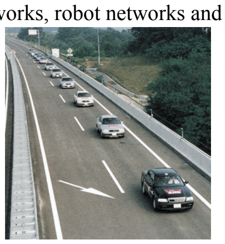
- Introduction
- Problem Formulation
- Analysis
- Simulation
- Experiment
- Conclusion / Future Works

Introduction

■Cooperative Control

- Cooperative control is a distributed control strategy that achieves specified tasks in multi-agent systems.
- It's been motivated by interests in group behavior of animals, formation control of multi-vehicle systems and so on.
- It is hoped to be applied to sensor networks, robot networks and many other multi-agents systems.

Flock of Ducks
http://www.flickr.com/photos/



Automated Highway System
http://www.its.go.jp/ITS/

Introduction

■Output Synchronization

- Output synchronization means that all of the agents' output become the same.
- This can be applied to the docking control of setellites, tracking control of multi-vehicle system and so on.

■Passivity-based Output Synchronization in SE(3) [1]

- Output synchronization in SE(3) means that all of the agents' positions and attitudes become the same.
- Consensus problem[2] and flocking problem[3] are included in reference[1].

[1]Y. Igarashi, T. Hatanaka and M. Fujita, Output Synchronization in SE(3) -Passivity-based Approach- , Proc. of the 36th SICE Symposium on Control Theory, 35/38, 2007.
[2]R. O. Saber, J. A. Fax and T. M. Murray, Consensus and Cooperation in Networked Multi-Agent Systems, Proc. of the IEEE, 95-1, 215/233, 2007.
[3]N. Moshtagh and A. Jadbabaie, Distributed Geodesic Control Laws for Flocking of Nonholonomic Agents, IEEE Trans. on Automatic Control, 52-4, 681/686, 2007.

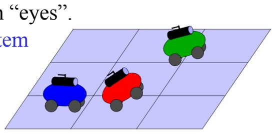
Introduction

■Passivity-based Output Synchronization in SE(3)

- Information of neighbors' relative positions and attitudes is necessary.

■Cooperative Control System with Visual Sensors

- mounted camera (local camera)
  - → Each agent can have its own "eyes".
    - → autonomous agents system

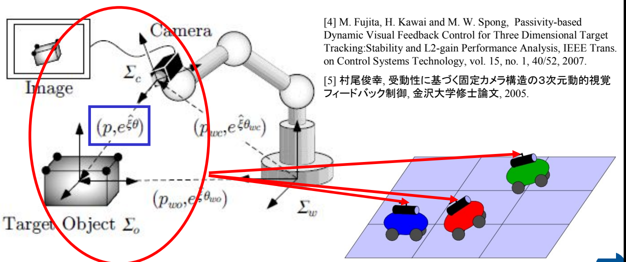


How can we calculate relative positions and attitudes between each agent and its neighbors by using cameras?

Introduction

■Visual Feedback Observer [4, 5]

- Visual feedback observer can estimate relative positions and orientations between camera and target objects.
- Our goal is realize the autonomous agents system by applying visual feedback observer to mounted cameras.



[4] M. Fujita, H. Kawai and M. W. Spong, Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking:Stability and L2-gain Performance Analysis, IEEE Trans. on Control Systems Technology, vol. 15, no. 1, 40/52, 2007.
[5] 村尾俊幸, 受動性に基づく固定カメラ構造の3次元動的視覚フィードバック制御, 金沢大学修士論文, 2005.

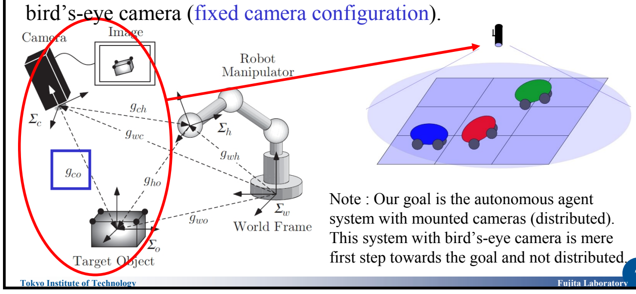
Introduction

■Visual Feedback Observer [4, 5]

•Application of visual feedback observer to cooperative control system with mounted camera (eye-in-hand configuration) is a little difficult.

↓ to make the problem easier

•I'll apply visual feedback observer to cooperative control system with bird's-eye camera (fixed camera configuration).



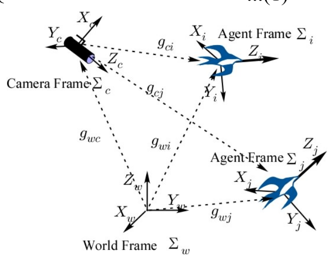
Note : Our goal is the autonomous agent system with mounted cameras (distributed). This system with bird's-eye camera is mere first step towards the goal and not distributed.

Outline

Problem Formulation

•Agents' Kinematics

$$\begin{cases} \dot{p}_{wi} = e^{\hat{\xi}_{wi}} v^b_{wi} \\ \dot{e}^{\hat{\xi}_{wi}} = e^{\hat{\xi}_{wi}} \hat{\omega}^b_{wi} \quad (i = 1,2,\dots,n) \\ y_i = (p_{wi}, e^{\hat{\xi}_{wi}}) \in SE(3) \end{cases} \quad \text{...(1)}$$



$p_{wi} \in R^3$: position

$e^{\hat{\xi}_{wi}} \in SO(3)$: attitude

$v^b_{wi} \in R^3$: body linear velocity

$\omega^b_{wi} \in R^3$: body angular velocity

$\zeta_{wi} = \theta_{wi}\xi_{wi}$

$\xi_{wi} \in R^3$: rotation direction

$\theta_{wi} \in R$: rotation angle

$$\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix}^\wedge := \begin{bmatrix}0 & -\omega_3 & \omega_2\\ \omega_3 & 0 & -\omega_1\\ -\omega_2 & \omega_1 & 0\end{bmatrix}$$

$$g_{wi} = \begin{bmatrix} e^{\hat{\xi}_{wi}} & p_{wi} \\ 0 & 1 \end{bmatrix}$$

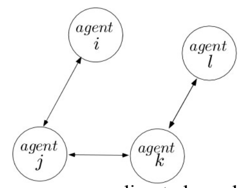
: homogeneous representation of $g_{wi} = (p_{wi}, e^{\hat{\xi}_{wi}}) \in SE(3)$

wi, ci : i th agent frame seen from world frame, camera frame

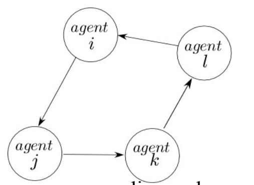
Problem Formulation

•Information Graph

- Fixed : A topology of a graph does not change.
- Balanced (undirected or cyclic) : In-degree and out-degree are same. ...(As1)



undirected graph



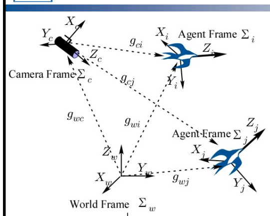
cyclic graph

•Neighbor : $N_i := \{ j \mid j \in \mathcal{V} : (i,j) \in \mathcal{E} \}$
Agents connected with agent i

•Graph Laplacian :

$$L := [L_{ij}] = \begin{cases} \sum_{j \in N_i} 1 & i = j \\ -1 & j \in N_i \\ 0 & j \notin N_i \end{cases}$$

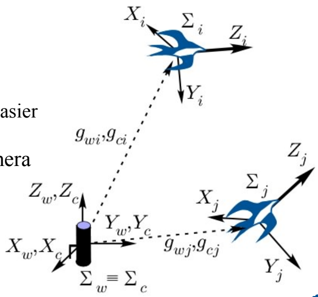
Problem Formulation



↓ to make the problem easier

•I assume that world frame and camera frame are the same. That is

$$\Sigma_w \equiv \Sigma_c \quad \text{...(As2)}$$

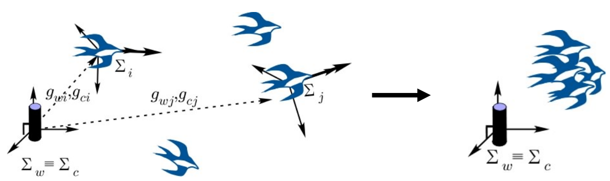


Probelm Formulation

•Control Objective

- Output Synchronization in SE(3)
: Make all of the agent outputs (position /attitude) be the same.

$$\lim_{t \to \infty} (y_i^{-1} y_j) = I_4 \quad \forall i, j$$



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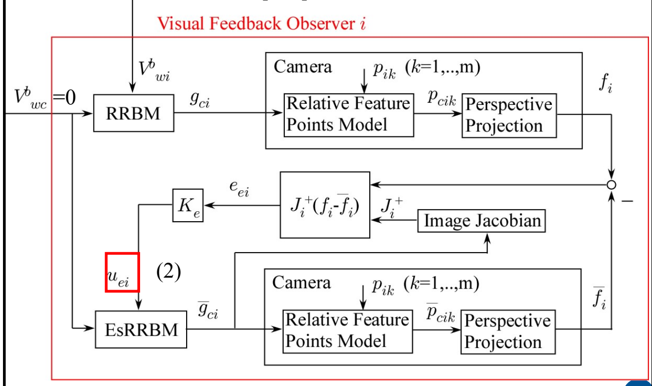
Analysis

■Analysis Outline

- Review
  - Visual Feedback Observer [4, 5]
  - Output Synchronization [1]
- Output Synchronization with Visual Feedback Observer
  - Proposal of inputs
  - Convergence Analysis

Analysis

■Visual Feedback Observer [4, 5]



Analysis

■Visual Feedback Observer [4, 5]

- Input to EsRRBM

$$u_{ei} = K_e e_{ei} \qquad \ldots(2)$$

$$K_e := \begin{bmatrix} K_{ev} & 0 \\ 0 & K_{e\omega} \end{bmatrix} \in R^{6\times 6}$$

$$K_{ev} := \mathrm{diag}\{k_{ev}, k_{ev}, k_{ev}\} \in R^{3\times 3} \qquad k_{ev} > 0$$

$$K_{e\omega} := \mathrm{diag}\{k_{e\omega}, k_{e\omega}, k_{e\omega}\} \in R^{3\times 3} \qquad k_{e\omega} > 0$$

$$p_{eei} := e^{-\hat{\bar{\xi}}_{ci}}(p_{ci} - \bar{p}_{ci})$$

$$e^{\hat{\xi}_{eei}} := e^{-\hat{\bar{\xi}}_{ci}} e^{\hat{\xi}_{ci}} \qquad \text{: estimation error}$$

$$e_{ei} := \begin{bmatrix} p_{eei} \\ \mathrm{sk}(e^{\hat{\xi}_{eei}})^{\vee} \end{bmatrix} = J_i^{+}(f_i - \bar{f}_i)$$

Analysis

■Visual Feedback Observer [4, 5]

- Estimation error e_{ei} is asymptotically stable with input (2) when $V^b_{wi} = 0$.

$$V_{obsi} = \frac{1}{2}\| p_{eei} \|^2 + \phi(e^{\hat{\xi}_{eei}})$$

$$\begin{aligned} \dot{V}_{obsi} &= p_{eei}^T \dot{p}_{eei} + \dot{\phi}(e^{\hat{\xi}_{eei}}) \\ &= \ldots \\ &= (u_{ei})^T(-e_{ei}) \\ &= -e_{ei}^T K_e e_{ei} < 0 \end{aligned}$$

➡ $e_{ei} (= \begin{bmatrix} p_{eei} \\ \mathrm{sk}(e^{\hat{\xi}_{eei}})^{\vee} \end{bmatrix}) \to 0$

Analysis

■Output Synchronization [1]

- Velocity Input

$$\begin{bmatrix} v^b_{wi} \\ \omega^b_{wi} \end{bmatrix} = \sum_{j \in N_i} \begin{bmatrix} e^{-\hat{\xi}_{wi}} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} p_{wj} - p_{wi} \\ \mathrm{sk}(e^{-\hat{\xi}_{wi}} e^{\hat{\xi}_{wj}}) \end{bmatrix} \qquad (i = 1, 2, \ldots, n)$$

- Velocity input (3) achieves output synchronization.

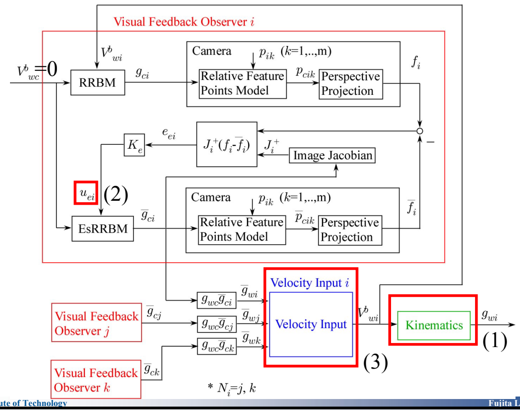
$$V_{syn} := \sum_{i=1}^{n} \{ \frac{1}{2} p_{wi}^T p_{wi} + \phi(e^{\hat{\xi}_{wi}}) \}$$

$$\begin{aligned} \dot{V}_{syn} &= \sum_{i=1}^{n} \{ p_{wi}^T \dot{p}_{wi} + \dot{\phi}(e^{\hat{\xi}_{wi}}) \} \\ &= \ldots \\ &\le 0 \end{aligned}$$

➡ $\lim_{t \to \infty} (y_i^{-1} y_j) = I_4 \quad \forall i, j$

Analysis

■Structure of Agent i with Observer (Bird's eye camera)



Analysis

■Proposal of Inputs

•Input to EsRRBM

$$u_{ei} = K_e e_{ei} \qquad ...(2)$$

$$K_e := \begin{bmatrix} K_{ev} & 0 \\ 0 & K_{e\omega} \end{bmatrix} \in R^{6\times 6}$$

$$K_{ev} := \mathrm{diag}\{k_{ev}, k_{ev}, k_{ev}\} \in R^{3\times 3} \qquad k_{ev} > 0$$

$$K_{e\omega} := \mathrm{diag}\{k_{e\omega}, k_{e\omega}, k_{e\omega}\} \in R^{3\times 3} \qquad k_{e\omega} > 0$$

•Velocity Input

$$\begin{bmatrix} v^b_{wi} \\ \omega^b_{wi} \end{bmatrix} = \sum_{j\in N_i} \begin{bmatrix} e^{-\hat{\xi}_{wi}} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{p}_{wj} - \bar{p}_{wi} \\ \mathrm{sk}(e^{-\hat{\xi}_{wi}} e^{\hat{\xi}_{wj}}) \end{bmatrix} \quad (i = 1,2,\ldots,n) \qquad ...(3)$$

Analysis

■Analysis of Output Synchronization

•I analysed synchronization of agents' position and attitude separately.

• I have proved that position synchronization is achieved already, but have not proved that attitude synchronization is achieved yet.

■Analysis of Position Synchronization

Input (2), (3) achieve position synchronization when the condition $(K_{ev}\otimes I_n)-(L\otimes I_3)\geq 0$ is satisfied.

Proof :

Define the energy function as follows.

$$V := \sum_{i=1}^{n}\{\frac{1}{2}\|p_{eei}\|^2 + \frac{1}{2}\|p_{wi}\|^2\}$$

Observer Output Synchronization

Differentiating this energy function yields

$$\dot{V} = \sum_{i=1}^{n}\{p^T_{eei}\dot{p}_{eei} + p^T_{wi}\dot{p}_{wi}\} = \ldots$$

Analysis

■Analysis of Position Synchronization

$$\dot{V} = \sum_{i=1}^{n}\{p^T_{eei}\dot{p}_{eei} + p^T_{wi}\dot{p}_{wi}\}$$

$$= \sum_{i=1}^{n}\{p^T_{eei}e^{\hat{\xi}_{eei}}v^b_{eei} + p^T_{wi}\dot{p}_{wi}\}$$

$$= \sum_{i=1}^{n}\{p^T_{eei}e^{\hat{\xi}_{eei}}(-\begin{bmatrix} e^{-\hat{\xi}_{eei}} & -e^{-\hat{\xi}_{eei}}\hat{p}_{eei} \end{bmatrix}u_{ei} + v^b_{wi}) + p^T_{wi}\dot{p}_{wi}\}$$

$$= \sum_{i=1}^{n}\{-p^T_{eei}K_{ev}p_{eei} + p^T_{eei}e^{\hat{\xi}_{eei}}v^b_{wi} + p^T_{wi}\dot{p}_{wi}\}$$

$$= \sum_{i=1}^{n}\{-p^T_{eei}K_{ev}p_{eei} + 2p^T_{wi}\dot{p}_{wi} - \bar{p}^T_{wi}\dot{p}_{wi}\}$$

$$= \ldots$$

Analysis

■Analysis of Position Synchronization

$$\dot{V} = \sum_{i=1}^{n}\{-p^T_{eei}K_{ev}p_{eei} + 2p^T_{wi}\dot{p}_{wi} - \bar{p}^T_{wi}\dot{p}_{wi}\}$$

Define p_{wo} and p_{ee} as stuck vector of p_{wi} and p_{eei} as following.

$$p_{wo} := \begin{bmatrix} p_{w1} \\ \vdots \\ p_{wn} \end{bmatrix} \qquad p_{ee} := \begin{bmatrix} p_{ee1} \\ \vdots \\ p_{een} \end{bmatrix}$$

Then using p_{wo} and p_{ee}, $\dot{V}$ is rewrited as

$$\dot{V} = -p^T_{ee}(K_{ev}\otimes I_n)p_{ee} + 2p^T_{wo}\dot{p}_{wo} - \bar{p}^T_{wo}\dot{p}_{wo} \qquad ...(4)$$

where $\otimes$ denotes the Kronecker matrix product.

* Kronecker matrix product $A\otimes B = \begin{bmatrix} A_{11}B & \cdots & A_{1n}B \\ \vdots & \ddots & \\ A_{m1}B & & A_{mn}B \end{bmatrix}$

Analysis

■Analysis of Position Synchronization

Using graph Laplacian L , $\dot{p}_{wo}$ is expressed as

$$\dot{p}_{wo} = \begin{bmatrix} e^{\hat{\xi}_{w1}}v^b_{w1} \\ \vdots \\ e^{\hat{\xi}_{wn}}v^b_{wn} \end{bmatrix} = \begin{bmatrix} \sum_{j\in N_1}(\bar{p}_j - \bar{p}_1) \\ \vdots \\ \sum_{j\in N_n}(\bar{p}_j - \bar{p}_n) \end{bmatrix} = L\bar{p}_{wo} \qquad ...(5)$$

Substituting (5) into (4) yields

$$\dot{V} = -p^T_{ee}(K_{ev}\otimes I_n)p_{ee} - 2p^T_{wo}(L\otimes I_3)\bar{p}_{wo} + \bar{p}^T_{wo}(L\otimes I_3)\bar{p}_{wo}$$

$$= -(p^T_{wo} - \bar{p}^T_{wo})\{(K_{ev}\otimes I_n) - (L\otimes I_3)\}(p_{wo} - \bar{p}_{wo})$$

$$- p^T_{wo}(L\otimes I_3)p_{wo}$$

Analysis

■Analysis of Position Synchronization

$$\dot{V} = -(p_{wo}^T - \bar{p}_{wo}^T)\{(K_{ev} \otimes I_n) - (L \otimes I_3)\}(p_{wo} - \bar{p}_{wo}) - p_{wo}^T (L \otimes I_3) p_{wo}$$

If the condition $(K_{ev} \otimes I_n) - (L \otimes I_3) \geq 0$ is satisfied, the first term satisfies

$$-(p_{wo}^T - \bar{p}_{wo}^T)\{(K_{ev} \otimes I_n) - (L \otimes I_3)\}(p_{wo} - \bar{p}_{wo}) \leq 0$$

Then by the positive-semidefinite property of graph Laplacian L
(∵ balanced graph)

$$\dot{V} \leq 0$$

Define set E as

$$E := \{p_{eei} \in R^3, p_{wi} \in R^3 \ {}^{\forall} i \in \{1, \ldots, n\} \mid \dot{V} = 0\}$$

Analysis

■Analysis of Position Synchronization

Because of the following property of graph Laplacian

$$L \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = 0$$

set E can be rewrited as follows.

$$E = \{p_{eei} \in R^3, p_{wi} \in R^3 \ {}^{\forall} i \in \{1, \ldots, n\} \mid p_{eei} = 0 \ {}^{\forall} i \in \{1, \ldots, n\} \text{ and } p_{wi} = p_{wj} \ {}^{\forall} i, j \in \{1, \ldots, n\}\}$$

Then by LaSalle's Invariance Principle, it is proved that estimation error is assymptotically stable and position synchronization is achieved when $(K_{ev} \otimes I_n) - (L \otimes I_3) \geq 0$ is satisfied. ■

Analysis

■Analysis of Position Synchronization

•Sufficient Condition

$$(K_{ev} \otimes I_n) - (L \otimes I_3) \geq 0$$

- Bigger λ_2 , faster convergence.
- Bigger λ_{max}, shorter time-delay limitation.

* limitation of time-delay [6] : $\tau^* = \dfrac{\pi}{2\lambda_{max}}$

(* in case of undirected graph)

[6] R. Olfati-Saber and R. Murray, Consensus Problems in Networks of Agents with Switching Topology and Time-Delays, IEEE Trans. on Automztic Control, vol. 49, no. 9, 1520/1533, 2004.
[7] Y. Igarashi, Consensus問題と行列理論について, FL06_12_2, 2006.

- K_{ev} have to be sufficiently large to estimate agents' position promptly.

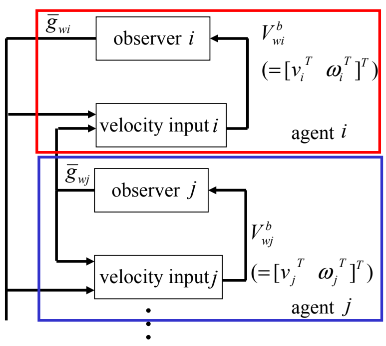
■Analysis of Attitude Synchronization

•I have not proved attitude synchronization yet.

Outline

Simulation

■Problem Formulation for Simulation



•Input to EsRRBM

$$u_{ei} = K_e e_{ei} \quad \ldots(2)$$

•Velocity Input

$$V_{wi}^b \left(= \begin{bmatrix} v_{wi}^b \\ \omega_{wi}^b \end{bmatrix}\right) = \sum_{j \in N_i} \begin{bmatrix} e^{-\hat{\bar{\xi}}_{wi}} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{p}_{wj} - \bar{p}_{wi} \\ \mathrm{sk}(e^{-\hat{\bar{\xi}}_{wi}} e^{\hat{\bar{\xi}}_{wj}}) \end{bmatrix} \quad \ldots(3)$$

Simulation

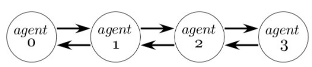
■Problem Formulation for Simulation

•Number of Agents : 4 agents

• Initial Condition :

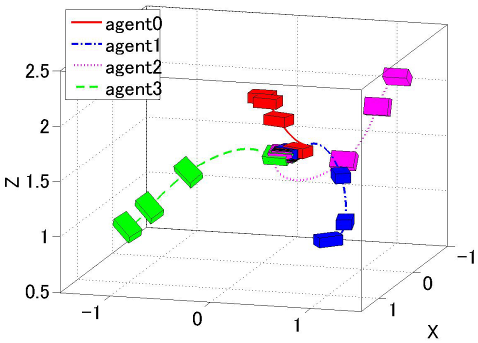
$$p_{w0}(0) = [0 \ \ 0 \ \ 2]^T \quad \zeta_{w0}(0) = [0 \ \ 0 \ \ 0]^T$$
$$p_{w1}(0) = [1 \ \ 1 \ \ 1]^T \quad \zeta_{w1}(0) = \begin{bmatrix} 0 & 0 & \frac{\sqrt{2}}{2} \end{bmatrix}^T$$
$$p_{w2}(0) = [-1 \ \ 1 \ \ 2]^T \quad \zeta_{w2}(0) = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & 0 \end{bmatrix}^T$$
$$p_{w3}(0) = [1 \ \ -1 \ \ 1]^T \quad \zeta_{w3}(0) = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}^T$$
$$\bar{p}_{w0}(0) = [0 \ \ 0 \ \ 2]^T \quad \bar{\zeta}_{w0}(0) = [0 \ \ 0 \ \ 0]^T$$
$$\bar{p}_{w1}(0) = [0 \ \ 0 \ \ 2]^T \quad \bar{\zeta}_{w0}(0) = [0 \ \ 0 \ \ 0]^T$$
$$\bar{p}_{w2}(0) = [0 \ \ 0 \ \ 2]^T \quad \bar{\zeta}_{w2}(0) = [0 \ \ 0 \ \ 0]^T$$
$$\bar{p}_{w3}(0) = [0 \ \ 0 \ \ 2]^T \quad \bar{\zeta}_{w3}(0) = [0 \ \ 0 \ \ 0]^T$$

•Graph :



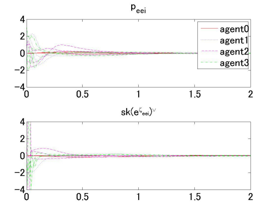
Simulation

■Simulation Results : $K_e = 5\times I_6$

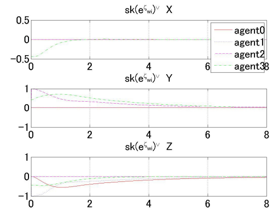


Simulation

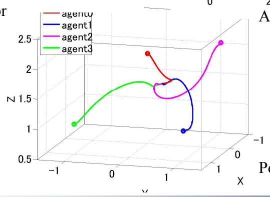
■Simulation Results : $K_e = 5\times I_6$



Estimation Error



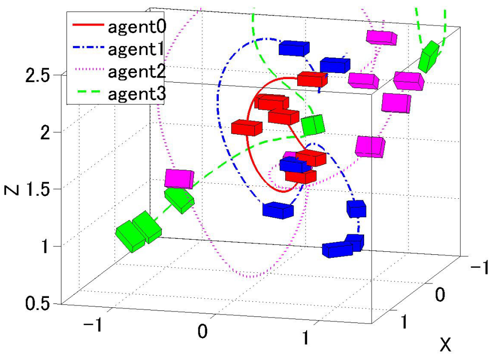
Attitude Error Vector



Position

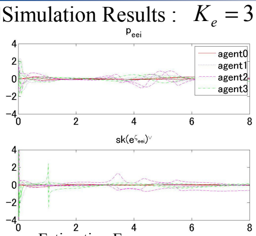
Simulation

■Simulation Results : $K_e = 3\times I_6$

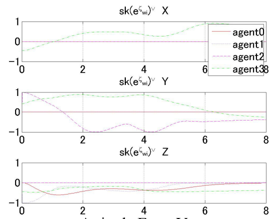


Simulation

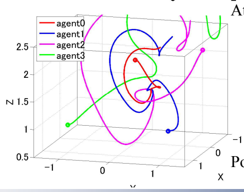
■Simulation Results : $K_e = 3\times I_6$



Estimation Error



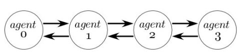
Attitude Error Vector



Position

Simulation

■Simulation Results



•Sufficient Condition

Assume

$K_{ev} = k\times I_3 \qquad k>0$

* $L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

Then

$$\begin{cases} (K_{ev}\otimes I_n)-(L\otimes I_3)\geq 0 & k\geq 3.55 \\ (K_{ev}\otimes I_n)-(L\otimes I_3)<0 & k<3.55 \end{cases}$$

•Simulation Results

- Output synchronization is achied when $K_e = 5\times I_6 > 3.55\times I_6$
- Output synchronization is not achieved when $K_e = 3\times I_6 < 3.55\times I_6$

Outline

•Introduction
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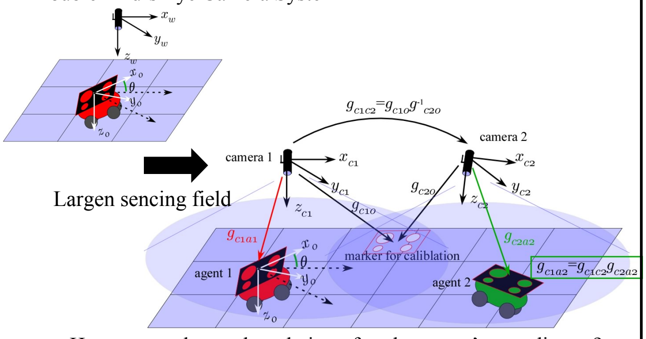
•Analysis
•Simulation
•Experiment
•Conclusion / Future Works

Experiment

■Modification of the Esperimental Environment

•Double Bird's Eye Camera System



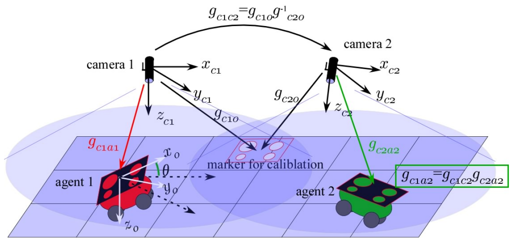
How can we know the relation of each camera's coordinates?

Experiment

■Modification of the Esperimental Environment

•Double Bird's Eye Camera System

- Using visual feedback observer, two cameras estimate the configuration of common marker (g_{c1_mark}, g_{c2_mark}) . Then we can calculate the relative configuration between camera1 and camera2 (g_{c1_c2}).

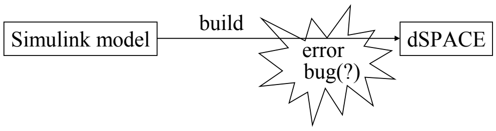


Experiment

■Modification of the Esperimental Environment

•Double Bird's Eye Camera System

- Problem : S-function block dosen't work when Simulink model is built and implemented into dSPACE.



Outline

Conclusion

■Conclusion

•As a first step toward the autonomous agents system (with mounted camera), I study on output synchronization with bird's eye camera system.

•Position synchronization has been proved.

•I validated this results by numerical simulations.

•Due to some problems, I have not performed experiments yet.

Future Works

■Future Works

•as soon as possible

- remove bugs and make experiments on output synchronization with bird's eye camera

•near future

- proof of attitude sunchronization with bird's eye camera

•future

- proof of output synchronization with mounted camera
- experiment on output synchronization with mounted camera

References

[1]Y. Igarashi, T. Hatanaka and M. Fujita, Output Synchronization in SE(3) -Passivity-based Approach- ,Proc. of the 36th SICE Symposium on Control Theory, 35/38, 2007.

[2]R. O. Saber, J. A. Fax and T. M. Murray, Consensus and Cooperation in Networked Multi-Agent Systems, Proc. of the IEEE, 95-1, 215/233, 2007.

[3]N. Moshtagh and A. Jadbabaie, Distributed Geodesic Control Laws for Flocking of Nonholonomic Agents, IEEE Trans. on Automatic Control, 52-4, 681/686, 2007.

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[5] 村尾俊幸, 受動性に基づく固定カメラ構造の3次元動的視覚フィードバック制御, 金沢大学修士論文, 2005.

[6] R. Olfati-Saber and R. Murray, Consensus Problems in Networks of Agents with Switching Topology and Time-Delays, IEEE Trans. on Automztic Control, vol. 49, no. 9, 1520/1533, 2004.

[7] Y. Igarashi, Consensus問題と行列理論について, FL06_12_2, 2006.

Appendix

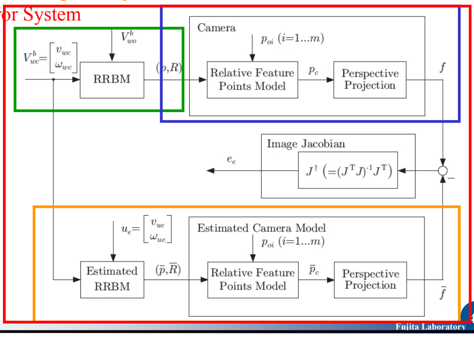
FL07_16_2

*Note : Notation is different in some part.

Visual Feedback Observer -Outline-

■Outline

- ●Relative Rigid Body Motion
- ●Camera Model
- ●Estimated Relative Rigid Body Motion
- ●Estimation Error System



Visual Feedback Observer -RRBM-

■Relative Rigid Body Motion

Σ_w:world frame

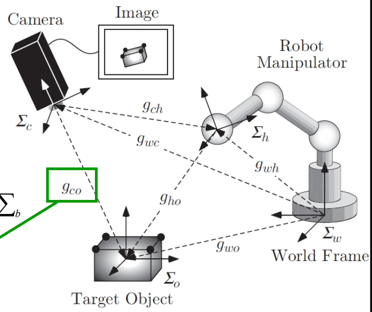
Σ_c:camera frame

Σ_o:object frame

$g_{ab} = (p_{ab}, e^{\hat{\xi}ab})$

$= \begin{bmatrix} e^{\hat{\xi}ab} & p_{ab} \\ 0 & 1 \end{bmatrix} \in SE(3)$

:configuration of a frame Σ_b relative to a frame Σ_a



$g_{co} = g_{wc}^{-1} g_{wo}$

$\dot{g}_{co} = \dot{g}_{wc}^{-1} g_{wo} + g_{wc}^{-1} \dot{g}_{wo}$

$= -g_{wc}^{-1} \dot{g}_{wc} g_{wc}^{-1} g_{wo} + g_{wc}^{-1} \dot{g}_{wo}$ ...(1)

Visual Feedback Observer -RRBM-

■Relative Rigid Body Motion

$$\hat{V}_{ab}^b = g_{ab}^{-1} \dot{g}_{ab} = \begin{bmatrix} e^{-\hat{\xi}_{ab}} \dot{e}^{\hat{\xi}_{ab}} & e^{-\hat{\xi}_{ab}} \dot{p}_{ab} \\ 0 & 0 \end{bmatrix}$$

$$V_{ab}^b = \begin{bmatrix} v_{ab}^b \\ \omega_{ab}^b \end{bmatrix} = \begin{bmatrix} e^{-\hat{\xi}_{ab}} \dot{p}_{ab} \\ (e^{-\hat{\xi}_{ab}} \dot{e}^{\hat{\xi}_{ab}})^{\vee} \end{bmatrix}$$

$\hat{V}_{ab}^b \in se(3)$, $V_{ab}^b \in R^6$:body velocity

$v_{ab}^b \in R^3$:body translation velocity

$\omega_{ab}^b \in R^3$:body angular velocity

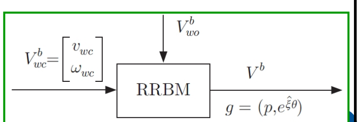
$$\dot{g}_{co} = -g_{wc}^{-1} \dot{g}_{wc} g_{wc}^{-1} g_{wo} + g_{wc}^{-1} \dot{g}_{wo} \quad ...(1)$$

$$= -\hat{V}_{wc}^b g_{co} + g_{co} \hat{V}_{wo}^b$$

$$\hat{V}_{co}^b = g_{co}^{-1} \dot{g}_{co} = -g_{co}^{-1} \hat{V}_{wc}^b g_{co} + \hat{V}_{wo}^b$$

$V_{co}^b = -Ad_{(g_{co}^{-1})} V_{wc}^b + V_{wo}^b$:RRBM model

$$\mathrm{Ad}_{(g_{ab})} = \begin{bmatrix} e^{\hat{\xi}_{ab}} & \hat{p}_{ab} e^{\hat{\xi}_{ab}} \\ 0 & e^{\hat{\xi}_{ab}} \end{bmatrix}$$



Visual Feedback Observer -Camera Model-

■Camera Model

$p_{oi} \in R^3$:position vector of the i-th feature point relative to Σ_o

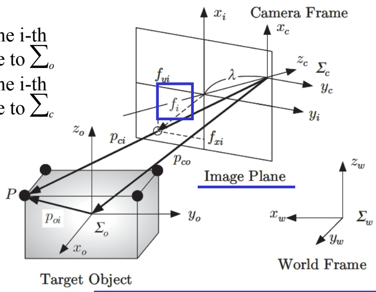
$p_{ci} \in R^3$:position vector of the i-th feature point relative to Σ_c

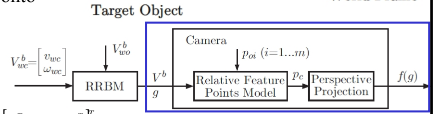
$p_{ci} := [x_{ci} \quad y_{ci} \quad z_{ci}]$
$= g_{co} p_{oi}$

$f_i := [f_{xi} \quad f_{yi}]^T$

:**perspective projection** of the i-th feature point onto the **image plane**

$$f_i = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix}$$





$p_c = [p_{c1}^T \quad \cdots \quad p_{cm}^T]^T$, $f = [f_1^T \quad \cdots \quad f_m^T]^T$

Visual Feedback Observer -EsRRBM-

■Estimated Relative Rigid Body Motion

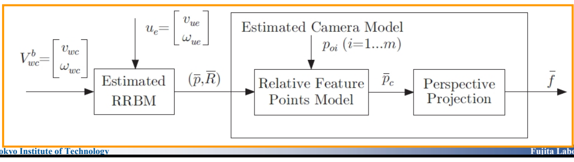
$\dot{\bar{V}}_{co}^b = -Ad_{(\bar{g}_{co}^{-1})} V_{wc}^b + u_e$:EsRRBM model

u_e :input to EsRRBM

$\bar{p}_{ci} := [\bar{x}_{ci} \quad \bar{y}_{ci} \quad \bar{z}_{ci}]$

$= \bar{g}_{co} p_{oi}$

$\bar{f}_i = \frac{\lambda}{\bar{z}_{ci}} \begin{bmatrix} \bar{x}_{ci} \\ \bar{y}_{ci} \end{bmatrix}$ $\quad \bar{p}_c = [\bar{p}_{c1}^T \ \cdots \ \bar{p}_{cm}^T]^T$, $\quad \bar{f} = [\bar{f}_1^T \ \cdots \ \bar{f}_m^T]^T$



Visual Feedback Observer -EES

■Estimation Error System

$g_{ee} = (p_{ee}, e^{\hat{\xi}_{ee}})$

$= \bar{g}_{co}^{-1} g_{co}$:estimation error

$p_{ee} = \bar{e}^{-\hat{\xi}_{co}} (p_{co} - \bar{p}_{co})$

$e^{\hat{\xi}_{ee}} = \bar{e}^{-\hat{\xi}_{co}} e^{\hat{\xi}_{co}}$

$e_e = [p_{ee}^T \quad e_R^T(e^{\hat{\xi}_{ee}})]^T$:estimation error vector $\quad * \ e_R(e^{\hat{\xi}_{ab}}) = \frac{1}{2}(e^{\hat{\xi}_{ab}} - e^{-\hat{\xi}_{ab}})^{\vee}$

$f - \bar{f} = J(\bar{g}_{co}) e_e$:estimation error of feature point

$J_i(g_{co}) = \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} \bar{e}^{\hat{\xi}_{co}} [I \quad -\hat{p}_{oi}]$ **:image jacobian**

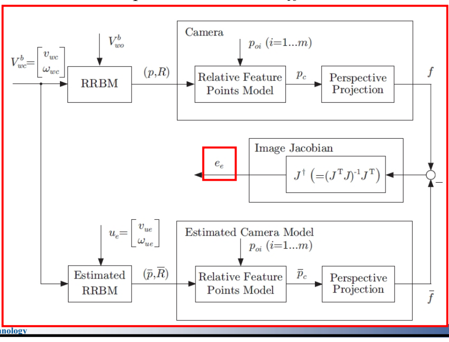
$J(g_{co}) = [J_1^T(g_{co}) \ \cdots \ J_m^T(g_{co})]^T$

Visual Feedback Observer -EES

■Estimation Error System

$e_e = J^{+}(\bar{g}_{co})(f - \bar{f})$

*take 3 or more feature points so that $J(\bar{g}_{co})$ be column full rank



Visual Feedback Observer -EES

■Estimation Error System

$V_{ee}^b = (g_{ee}^{-1} \dot{g}_{ee})^{\vee} = \begin{bmatrix} e^{-\hat{\xi}_{ee}} \dot{p}_{ee} \\ (e^{-\hat{\xi}_{ee}} \dot{e}^{\hat{\xi}_{ee}})^{\vee} \end{bmatrix}$

$V_{ee}^b = -\mathrm{Ad}_{(g_{ee}^{-1})} u_e + V_{wo}^b$:estimation error motion model

■Passivity of the Estimation Error System

If $V_{wo}^b = 0$, then estimation error system satisfies $\int_o^T u_e^T (-e_e) dt \geq -\beta_e$

$* \ \beta_e > 0$

proof : Consider the following positive definite function

✞ $= \frac{1}{2} \| p_{ee} \|^2 + \phi(e^{\bar{\xi}_{ee}})$ $\quad * \ \phi(e^{\hat{\xi}_{ab}}) = \frac{1}{2} \mathrm{tr}(I - e^{\hat{\xi}_{ab}})$

differentiating ✞ with respect to time yields

✞̇ 🖬 $p_{ee}^T \dot{p}_{ee} + e_R^T(e^{\bar{\xi}_{ee}}) \omega_{ee}^s$

$= ....$ ...(2)

Visual Feedback Observer -EES

■Passivity of the Estimation Error System

✞̇ 🖬 $p_{ee}^T \dot{p}_{ee} + e_R^T(e^{\bar{\xi}_{ee}}) \omega_{ee}^s$ ...(2)

$= ....$

$= u_e^T (-e_e)$

integrating ✞̇ from 0 to T, we obtain

$\int_0^T u_e^T (-e_e) d\tau = \int_0^T$ ✞̇ $d\tau$

= ✞☎T) − ✞☎) ≥ −✞☎) ≥ $-\beta_e$ ■

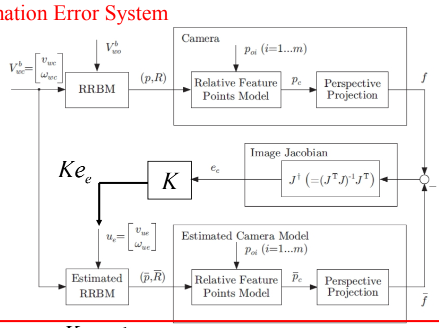
$u_e = K e_e \qquad K > 0$

$\Rightarrow$ ✞̇ < 0

$\Rightarrow e_e (= \begin{bmatrix} p_{ee} \\ e_R(e^{\bar{\xi}_{ee}}) \end{bmatrix}) \to 0$

Visual Feedback Observer -EES

■Estimation Error System



γ :L2-gain

Choose $u_e = K e_e$, then

- Estimation error is asymptotically stable if $V_{wo}^b = 0$.
- Estimation error is L2-gain stable if $V_{wo}^b \neq 0$, $K - \frac{1}{2\gamma^2} I - \frac{1}{2} I \geq 0$.