Collision Avoidance for Multi-Robot System : A Set-Theoretic Approach

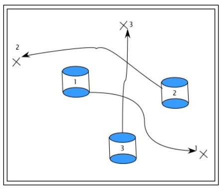
FL-07-27-1
Nopthawat Kitudomrat
21/01/2008

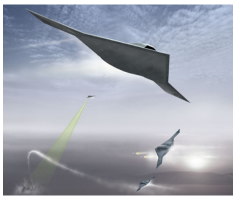
Outline

- Introduction
- Problem setting
- Previous results
- Reference Selection
- Detection range
- Summary

Introduction

- Multi-robot system
- Limited knowledge of environment
- Formation control
- Collision avoidance
- Application
  - UAV (Unmanned Aerial Vehicle)
  - Air traffic management
  - Satellite Orbit
  - Mobile Robot



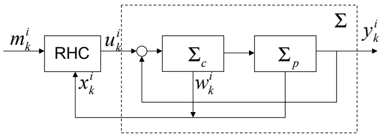


From : Journal of the air force association

Recent works

- W.B. Dunbar, et al. (2004) [Formation Stabilization]
  - Multi-robot system
  - No collision avoidance
  - Each robot are not deviated too far from the previous open loop trajectories.
  - Receding horizon update is sufficiently fast.
- S.V. Rakovic, et al. (2005) [Obstacle Avoidance]
  - Only one robot
  - Obstacle Avoidance (Not moving robot)
  - Use robust model predictive control
- T. Keviczky, et al. (2006) [Collision Avoidance]
  - Multi-robot system
  - Collision avoidance
  - Add cost of collision avoidance to the cost function
  - Applied the mixed integer linear programming (MILP)

Problem Setting



- Consider a linear discrete-time time-invariant system (for each robot)

$$\Sigma: \begin{array}{l} x_{k+1}^i = Ax_k^i + Bu_k^i \\ y_k^i = Cx_k^i = [I \quad 0]x_k^i = p_k^i \end{array}$$

where $x_k^i = [p_k^i \quad v_k^i \quad w_k^i]^T \in R^{n_p+n_v+n_w}$

$i \in \{1,\dots,N_v\}$

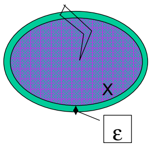
$k \in Z^+ := \{0,1,2,\dots\}$

p_k^i : Position
v_k^i : Velocity
w_k^i : Controller State
m_k^i : Target Position
u_k^i : System Input

System Assumption

- The tracking system Σ is asymptotically stable.
- Desired tracking performance of system Σ is already achieved.
- (A,B) is controllable and (C,A) is observable.
- The final output y_∞^i reaches a tracking value u_∞^i , so that $C(I-A)^{-1}B = I$
- The state x_k^i is measurable.

System Constraints



- We consider 2 types of constraint
  - Non-interactive Constraint
    - Limitation of movement (Position constraint)

$$p_{\min} \le p_k^i \le p_{\max}$$

- Maximum velocity (Velocity constraint)

$$-v_{\max} \le v_k^i \le v_{\max}$$

State Constraint (X)

- Maximum input signal (Input constraint)
  - The input that keeps the state satisfied the state constraint.

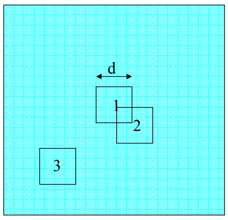
$$u_k^i \in U := \left\{ u \,\middle|\, (I-A)^{-1}Bu \in (1-\varepsilon)X, 0 < \varepsilon < 1 \right\}$$

System Constraints (2)

- Interactive Constraint
  - Collision constraint (Depend on a distance between robots)

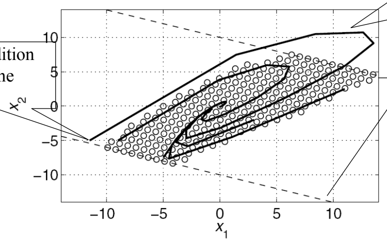
$$\left\| p_k^i - p_k^j \right\|_\infty \le d \in R^+, i \ne j$$

where d is a collision range (size of robot).



Maximal Output Admissible Set (MOA)

- The maximal output admissible set is the largest constraint admissible positively invariant set or, in other word, the set of all initial conditions such that the trajectories never exceed the specified constraints [Hirata,2005].



The trajectory violates the constraints.

Initial condition is outside the MOA.

Constraints

Maximal Output Admissible Set (2)

- The maximal output admissible set can be defined as

$$S_x = \left\{ x_0 \in \Re^n \,\middle|\, y(t;x_0) \in Y, \forall t \in Z^+ \right\}$$

where Y is an output constraint.

- In our work, we concentrate on keeping the trajectory of a state x_k^i inside the constraint set with the constant reference input u_k^i.

$$S_x = \left\{ (x_0, u) \,\middle|\, x(t;x_0,u) \in X, x_0 \in X, u \in U, \forall t \in Z^+ \right\}$$

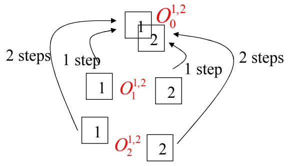
- Remark : Only the initial condition and reference input are required for guaranteeing the constraint satisfaction.
- The details of MOA calculation are studied in Gilbert's work[Gilbert,1991].

Collision Region

- To find the constraint set that can guarantee collision avoidance, we first consider the predictive collision set.
- Definition (Predictive Collision Set)
  - Predictive Collision Set contains the initial state and input such that the collision occurs at time k. It can be described as follow.

$$O_0^{i,j} = \left\{ (x^i,u^i,x^j,u^j) \,\middle|\, (x^i,u^i) \in S_x, (x^j,u^j) \in S_x, \left\| p^i - p^j \right\|_\infty \le d \right\}$$

$$O_k^{i,j} = \left\{ (x^i,u^i,x^j,u^j) \,\middle|\, \begin{array}{l} (x^i,u^i) \in S_x, (x^j,u^j) \in S_x, \\ (Ax^i+Bu^i,u^i,Ax^j+Bu^j,u^j) \in O_{k-1}^{i,j} \end{array} \right\}$$



Collision Region (2)

- Definition (Collision region)
  - Collision region is a set that contains all of the pair of initial state and reference input such that the collision is guaranteed to be occur.

$$O_\infty^{i,j} = \bigcup_{p=0}^{\infty} O_p^{i,j}$$

  - From the previous result, if there exists time k such that $O_{k+1}^{i,j} \subseteq O_k^{i,j}$, then

$$O_\infty^{i,j} = \bigcup_{p=0}^{k} O_p^{i,j}$$

  - The inclusion $O_{k+1}^{i,j} \subseteq O_k^{i,j}$ can easily be evaluated by checking the redundancy.

Safe Region

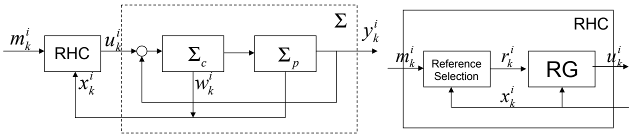
- Definition (Safe region)
  - The safe region is the duality set of the collision region.

$$S^{i,j} = \left\{ (x^i, u^i, x^j, u^j) \middle| \begin{array}{l} (x^i, u^i) \in S_x, (x^j, u^j) \in S_x \\ , (x^i, u^i, x^j, u^j) \notin O_\infty^{i,j} \end{array} \right\}$$

  - The safe region can be computed by the following set equation.

$$S^{i,j} = S_x \times S_x \setminus O_\infty^{i,j}$$

Reference (Input) Selection



- In the previous work, we chose the direction of the reference input r^i_k based on only the position of neighbor. Here, we add more factor to consider the reference.
- Definition (Brake, Avoid)
  - ‘Brake’ is a mode that the robot tries to stop at current position.

$$r_k^i = p_k^i$$

  - ‘Avoid’ is a mode that the robot moves to the avoidance zone.

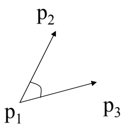
Avoidance Zone

- Definition (Avoidance Zone)
  - Avoidance Zone A_z^i is the set of the position such that the angle, between the vector from current robot to considered position and to obstacle, is more than π/2.

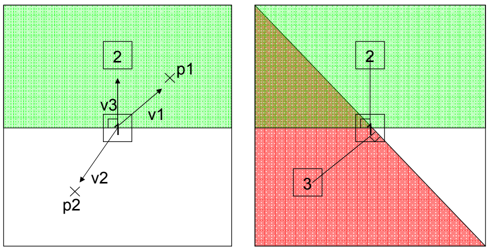
$$D(p_1, p_2, p_3) = \cos^{-1}\left(\frac{(p_1 - p_2)(p_1 - p_3)}{|p_1 - p_2||p_1 - p_3|} \right)$$

$$A_z^{i,j} = \left\{ p \in R^{n_p} \middle| D(p^i, p, p^j) \geq \frac{\pi}{2} \right\}$$

$$A_z^i = \bigcap_{j \in N(i)} A_z^{i,j}$$



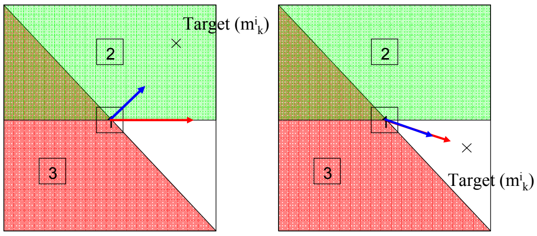
Avoidance Zone (2) - Example



The white area is the avoidance zone A_z^i.

Reference Selection (2)

- The reference input for ‘Avoid’ mode is a position that minimizes the angle between the vector from current position to the target and to the considered position.



Reference Selection (3)

- The reference input for ‘Avoid’ mode can be described as follow.

$$r_k^i = \underset{p \in A_z^i}{\arg\min}\ \deg(p_k^i, p, m_k^i)$$

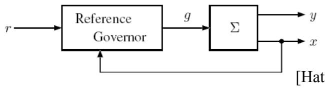
$$\deg(p_1, p_2, p_3) = \arctan 2\begin{pmatrix} |(p_1 - p_2) \times (p_1 - p_3)|, \\ (p_1 - p_2) \cdot (p_1 - p_3) \end{pmatrix}$$

- Note : The arctan2 function range is

$$-\pi < \arctan 2(\cdot) \leq \pi$$

Reference Governor

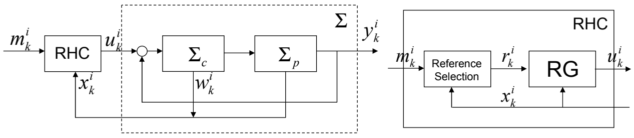
- Reference Governor (RG) is a nonlinear device based on predictive control. The aim of this RG device is to modify the reference in such a way that the constraints are enforced [Bemporad, 1998].



[Hatanaka, 2005]

- From the figure, the reference input r is modified and the modified control signal g is inputted to the primal compensated system Σ.

Reference Governor (2)



- In our work, the input u^i_k is computed based on the reference input r^i_k as the following equation :

$$u_k^i = Kr_k^i + (1-K)p_k^i, K \in [0,1]$$

- r^i_k is a reference input which will be described later.
- To make the input u^i_k close to the reference input r^i_k, the gain K is maximized.
- To satisfy the system constraints, the model predictive control is applied.

Reference Governor (3)

$$\max\left(K_k^i + \sum_{j\in N(i)} K_k^j \right) \quad \Longrightarrow \quad \text{Maximize gain k}$$

$$\text{s.t.} \quad i \in \{1,\ldots,N_v\}, j \in N(i)$$

$$x_{k+1}^i = Ax_k^i + Bu_k^i$$

$$u_k^i = Kr_k^i + (1-K)p_k^i$$

$$\left.\begin{aligned} &(x_k^i, u_k^i) \in S_x \\ &(x_k^j, u_k^{j,i}) \in S_x \end{aligned}\right\} \quad \Longrightarrow \quad \text{Non-interactive Constraint}$$

$$\left.\begin{aligned} &(Ax_k^i + Bu_k^i, p_k^i) \in S_x \\ &(Ax_k^j + Bu_k^{j,i}, p_k^j) \in S_x \end{aligned}\right\} \quad \Longrightarrow \quad \text{Next state : Brake}$$

$$(x_k^i, u_k^i, x_k^j, u_k^{j,i}) \in S^{i,j} \quad \Longrightarrow \quad \text{Collision Avoidance}$$

$$\left.\begin{aligned} &(x_k^i, u_k^i, x_k^j, p_k^j) \in S^{i,j} \\ &(x_k^i, p_k^i, x_k^j, u_k^{j,i}) \in S^{i,j} \end{aligned}\right\} \quad \Longrightarrow \quad \text{Observed constraints}$$

Detection Range

- The robot can detect others if the distance between them is less than a detection range d_s.
- A detection range d_s can be computed by the following equation.

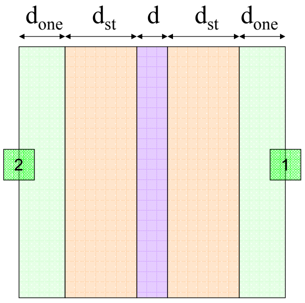
$$d_s \geq 2(d_{one} + d_{st}) + d$$

where

- d_{one} is a maximum distance that the robot can move in one time step.
- d_{st} is a maximum distance that the robot uses to brake until it stops..
- d is a collision range.

Detection Range (2)

$$d_s = 2(d_{one} + d_{st}) + d$$



Proof for Collision Avoidance

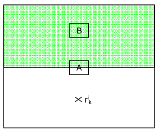
- From the definition of the detection range (d_s), it's obvious for "Brake" mode that the collision will not occur because the robot can stop before reaching neighbors.
- We will prove that the "Avoid" mode also guarantee the collision avoidance.
- First, we assume that the components of matrix B of system Σ are > 0 (This is a mild assumption because we consider the tracking system and matrix B should increase the effect of input u)
- Suppose that we have 2 robots where the first one tries to avoid and another tries to brake for all time t.
- The proof is based on one dimension and the feasibility is assumed.

Proof (2)

- The initial condition for each robot is the same.

$$x_k^1 = x_k^2 \rightarrow p_k^1 = p_k^2, v_k^1 = v_k^2$$

- From the reference selection, let $r_k^1 - p_k^1 < 0$

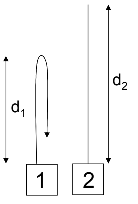


$$x_{k+1}^1 = Ax_k^1 + B(Kr_k^1 + (1-K)p_k^1), K \in (0,1]$$
$$x_{k+1}^2 = Ax_k^2 + Bp_k^2$$
$$x_{k+1}^1 - x_{k+1}^2 = BK(r_k^1 - p_k^1)$$
$$\therefore p_{k+1}^1 - p_{k+1}^2 = [1 \quad 0]BK(r_k^1 - p_k^1) < 0 \rightarrow p_{k+1}^1 < p_{k+1}^2$$
$$\therefore v_{k+1}^1 - v_{k+1}^2 = [0 \quad 1]BK(r_k^1 - p_k^1) < 0 \rightarrow v_{k+1}^1 < v_{k+1}^2$$

Hence, in 1 step, the avoid mode can decrease velocity and use less distance than the Brake mode.

$$\therefore d_1 < d_2 \because d_2 \le d_{st} < \frac{d_s}{2} \therefore d_1 < \frac{d_s}{2}$$

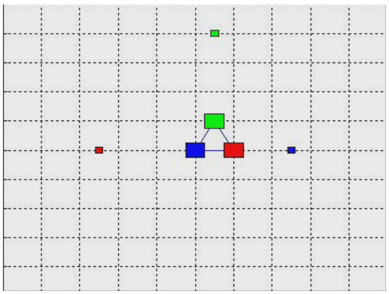
The collision is proved.



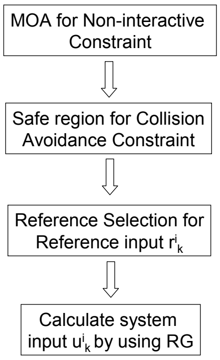
Simulation

$$x_{k+1}^i = \begin{bmatrix} 0.86 & 0 & 0.32 & 0 \\ 0 & 0.86 & 0 & 0.32 \\ -0.46 & 0 & 0.32 & 0 \\ 0 & -0.46 & 0 & 0.32 \end{bmatrix} x_k^i + \begin{bmatrix} 0.14 & 0 \\ 0 & 0.14 \\ 0.46 & 0 \\ 0 & 0.46 \end{bmatrix} u_k^i, \quad y_k^i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x_k^i$$

$$d = 0.5$$
$$d_s = 2.47$$
$$-10 \le p_k^i \le 10, -1 \le v_k^i \le 1$$
$$x_0^1 = [0 \quad 0 \quad 0 \quad 0]^T$$
$$x_0^2 = [1 \quad 0 \quad 0 \quad 0]^T$$
$$x_0^3 = [0.5 \quad 1 \quad 0 \quad 0]^T$$
$$m_0^1 = [4 \quad 0]^T$$
$$m_0^2 = [-4 \quad 0]^T$$
$$m_0^3 = [0.5 \quad 4]^T$$



Summary and Future Works



MOA for Non-interactive Constraint

Safe region for Collision Avoidance Constraint

Reference Selection for Reference input r^i_k

Calculate system input u^i_k by using RG

- Summary
  - Set Constraints
  - Reference selection
  - Minimum detection range
- Future Works
  - Proof for feasibility and stability
  - Experiment
  - Nonlinear model

Thank you

Reference

- W.B. Dunbar and R.M. Murray, “Receding Horizon Control of Multi-Vehicle Formations: A Distributed Implementation”, Proceedings of the 43rd IEEE Conference on Decision and Control, Vol.2, December 2004, pp. 1995-2002.
- S.V. Rakovic and D.Q. Mayne, “Robust model predictive control for obstacle avoidance: discrete time case,” Proceedings of the International Workshop on Assessment and Future Directions of Nonlinear Model Predictive Control NMPC05, Freudenstadt - Lauterbad, Germany, August 2005, pp. 347-356.
- T. Keviczky, B. Vanek, F. Borrelli and G.J. Balas, “Hybrid decentralized receding horizon control of vehicle formations”, Proceedings of the American Control Conference, June 2006, pp.3358-3363.
- E.G. Gilbert and K.T. Tan, “Linear Systems with State and Control Constraints: The Theory and Application of Maximal Output Admissible Sets”, IEEE Transactions on automatic control, Vol.36, No.9, September 1991, pp.1008-1020.

Reference (2)

- K. Hirata and Y. Ohta, “Exact Determinations of the Maximal Output Admissible Set for a Class of Nonlinear Systems”, 44th IEEE Conference on Decision and Control, and the European Control Conference 2005, Seville, Spain, December 2005, pp.8276-8281.
- A. Bemporad, A. Casavola and E. Mosca, ”A predictive reference governor for constrained control systems,”Computers in Industry, Vol.36, Issue 1-2, April 1998, pp. 55-64.
- T. Hatanaka and K. Takaba, “Output Feedback Reference Governor for Nonlinear Systems”, 44th IEEE Conference on Decision and Control, and the European Control Conference 2005, Seville, Spain, December 2005, pp. 7558-7563.