Discrete-time Lloyd Descent for Coverage Control

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Outline

- Algorithm
- Experiment
- Future work

Algorithm

○Locational optimization

Let Q be convex polytope in $\Re^N$

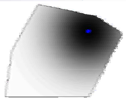

Fig. 1. Contour plot on a polygonal environment of the Gaussian density function $\phi = \exp(-x^2-y^2)$.

ϕ:density function

$P = \{p_1, p_2, \dots, p_n\}$:location of n sensors

$W = \{W_1, W_2, \dots, W_n\}$:the partition of Q

$f(\|q-p_i\|)$:distance function

consider the following objective function

$$H(P,W) = \sum_{i=1}^{n} \int_{W_i} f(\|q-p_i\|)\phi(q)dq \quad (1)$$

The task

Minimizing the objective function

Algorithm

○Voronoi Partitions

Voronoi Partitions generated by the points $\{p_1, p_2, \dots, p_n\}$

$$V(P) = \{q \in Q \mid \|q-p_i\| \le \|q-p_j\|, \forall j \ne i\} \quad (2)$$

Then, define

$$H_V(P) = H(P, V(P)) \quad (3)$$

Define the mass, centroid and polar moment of inertia about a region $V' \subset \Re^N$

$$M_{V'} = \int_{V'} \rho(q)dq \qquad C_{V'} = \frac{1}{M_{V'}} \int_{V'} q\rho(q)dq$$

$$J_{V'} = \int_{V'} \|q-p\|^2 \rho(q)dq$$

Algorithm

Let $T: Q^n \to Q^n$ be a mapping satisfying properties (a) and (b)

(a)for all $i \in \{1, \dots, n\}$, $\|T_i(P) - C_{V_i(P)}\| \le \|p_i - C_{V_i(P)}\|$

where T_i denotes the ith component of T

(b)If P is not centroidal, then there exists a j such that

$\|T_j(P) - C_{V_j(P)}\| < \|p_j - C_{V_j(P)}\|$

Then the sequence $\{T^m(P_0) \mid m \in \mathrm{N}\}$,if this set is finite, converge to a centroidal Voronoi configuration

Algorithm

Proof: about $H(P,W) = \sum_{i=1}^{n} \int_{W_i} f(\|q-p_i\|)\phi(q)dq$

using the parallel axis theorm

$$H(P,W) = \sum_{i=1}^{n} J_{W_i, C_{W_i}} + \sum_{i=1}^{n} M_{W_i} \|p_i - C_{W_i}\|^2 \quad (4)$$

Independent of P — dependent on P

therefore

$$H(P', W) \le H(P, W) \quad (5)$$

as long as $\|p_i' - C_{W_i}\| \le \|p_i - C_{W_i}\|$ for all $i \in \{1, 2, \dots, n\}$

Algorithm

Moreover, since Voronoi partition is the optimal one

$$H(P,V(P)) \le H(P,W) \quad (6)$$

Because of property (a) of T

$$\underline{H(T(P),V(P))} \le H(P,V(P)) = H_V(P) \quad (7)$$

And because of (6)

$$H_V(T(P)) = H(T(P),V(T(P))) \le \underline{H(T(P),V(P))} \quad (8)$$

Hence

$$H_V(T(P)) \le H_V(P) \quad (9)$$

H_V is a descent function for the argorithm T

Experiment

Dimension : 1D The number of agents: 3

$$p(k+1) = Ap(k) + B \quad \begin{bmatrix} p_1' \\ p_2' \\ p_3' \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/2 \end{bmatrix}$$

1:Update Voronoi partition and all $moving[i]=1$

2:agent i move toward $p_i(k+1)$

3:if agent i arrive at $p_i(k+1)$, $moving[i]=0$

4:if all $moving[i]=0$, return to (1) until convergence

Future Works

・carry out the coverage control (1-D) experiment

・consider the result of the expriment