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Coverage Control : Literature Survey and Anisotropic Sensors Case

FL07-22-2
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Outline

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- Literature survey
- Voronoi based coverage with anisotropic sensor
- Joint detection based isotropic sensor coverage and its extension to anisotropic case
- Future Works

Literature Survey

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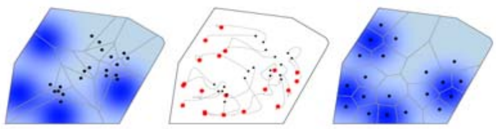
Prototype problem : locational optimization

- A. Okabe *et.al* (1997) : solving locational optimization problem through Voronoi diagram.
- Q. Du *et.al* (1999) : applications and algorithms (Lloyd's) of centroidal Voronoi tessellations.

\+

J. Cortes *et.al* (2002) :

- Voronoi based decentralized coverage control (each sensor has its own sensing area).
- convex region of interest (ROI).
- density function is known a priori to each agent.



Literature Survey

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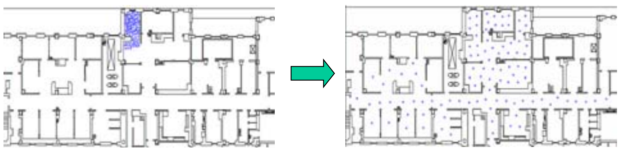
A. Howard *et.al* (2002) :

- Potential-field-based approach.
- equipped with sensor to determine the range and bearing of nearby nodes and obstacles.



S. Salapaka *et.al* (2003) :

- Adapted Deterministic Annealing (DA) approach to find globally optimal solutions.
- Modification of Lloyd algorithm (no hard partitions).

Literature Survey

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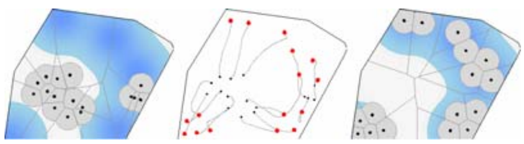
J. Cortes *et.al* (2004) :

- Deployment with limited sensing or communications radius.



W. Li *et.al* (2005) :

- Sensing performance defined by prob. model.
- Deployment algorithm : max joint detection prob.
- Incorporate communication cost (trade off between coverage and communication cost).

Literature Survey

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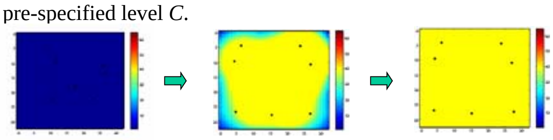
I. Hussein *et.al* (2006) :

- Dynamic coverage : agents move s.t every point is sensed with a pre-specified level *C*.



D. Rus *et.al* (2006) :

- Coverage control with unknown density function .
- The robots use sensed information to estimate the density function.

C. Gao *et.al* (2006) :

- Relationship between coverage (discrete) and averaging algorithm (acyclic diagraphs).

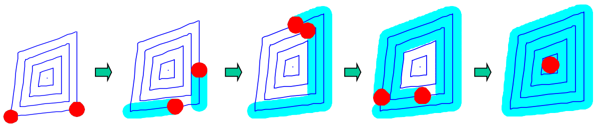
Literature Survey

I. Hussein *et.al* (2007) :

- ACC : dynamic coverage + flocking + collision avoidance
- CCA : vision-based camera sensor model (underwater vehicle)
- CDC : coverage control with heterogeneous vehicles (communication maintenance and coverage)

P. F. Hokayem *et.al* (2007) :

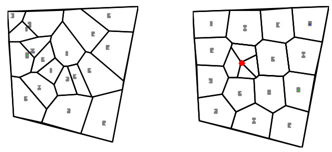
- proposed algorithm : does not depend on gradient-based, distributed
- agents do not need to exchange their positions continuously.



Literature Survey

C. H. Caicedo *et.al* (2007) :

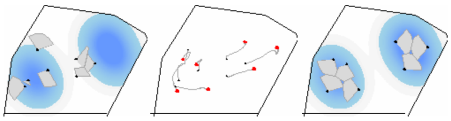
- Task switching between ensuring optimal coverage of a given area and locating events (e.g. biochemical source) that may appear.



J. Cortes *et.al* (2008) :

- limited-range comm. + anisotropic sensing probabilities.
- anisotropic : wedge-shaped region , sensing performance only depend on distance.

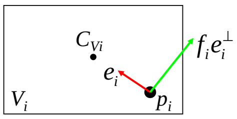
Literature Survey



M. Schwager *et.al* (2008) :

- Control strategy inspired by hunting tactics of ladybugs.
- Coverage + exploration : avoiding locally optimal configurations.

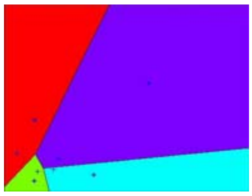
$$u_i = k\left(e_i + f_i e_i^{\perp}\right) \qquad e_i = C_{V_i} - p_i$$



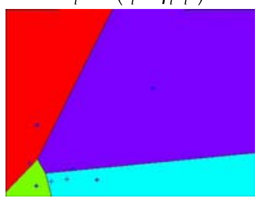
Literature Survey

Simulation :

$u_i = k(e_i)$

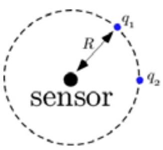


$u_i = k\left(e_i + f_i e_i^{\perp}\right)$



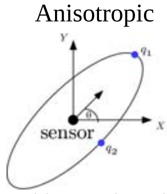
Voronoi Based Anisotropic Sensor

Isotropic



position

Anisotropic



position + orientation

Assumption : orientation of all agents is fixed and equal

Assumption : orientation of all agents is fixed

Voronoi Based Anisotropic Sensor

Objective function :

$$H(P,\Theta,W) = \sum_{i=1}^{n} \int_{W_i} f\left(\|q - p_i\|_{L_i}\right)\phi(q)dq$$

$$H(P) = \sum_{i=1}^{n} \int_{V_i} \|q - p_i\|_{L_i}^2 \phi(q)dq \quad \Rightarrow \text{minimize}$$

Definition :

A Voronoi tessellation is called an anisotropic centroidal Voronoi configuration if

$$p_i = C_{V_i}, \forall i;$$

where

$$C_{V_i} = \left(\int_{V_i} L_i \phi(q)dq\right)^{-1}\left(\int_{V_i} L_i q\phi(q)dq\right)$$

$$L_i = F_i^T F_i \quad \text{where } F_i = \left[\begin{pmatrix} \frac{c}{a} & 0 \\ 0 & \frac{c}{b} \end{pmatrix}\begin{pmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{pmatrix}\right]$$

Voronoi Based Anisotropic Sensor

Proposition 1 :

The objective function is minimized by the anisotropic centroidal Voronoi configuration

Proof :

$$\frac{\partial H}{\partial p_i} = 0$$

For a fixed Voronoi partition,

$$\frac{\partial H}{\partial p_i} = \int_{V_i} \frac{\partial}{\partial p_i} \left\| q - p_i \right\|_{L_i}^2 \phi(q)dq$$

$$= \int_{V_i} -2L_i(q - p_i)\phi(q)dq$$

$$= 2\int_{V_i} L_i \phi(q)dq p_i - 2\int_{V_i} L_i q \phi(q)dq$$

setting $p_i = \left(\int_{V_i} L_i \phi(q)dq\right)^{-1} \left(\int_{V_i} L_i q \phi(q)dq\right)$ will result to $\frac{\partial H}{\partial p_i} = 0$

QED

Voronoi Based Anisotropic Sensor

For dynamical systems : $\dot{p}_i = u_i$

Set : $u_i = -k\left(p_i - C_{V_i}\right)$

Proposition 2 :

By applying the above control law, the agents will converge asymptotically to the set of critical points i.e the set of anisotropic centroid Voronoi configurations. If this set is finite, the agents converge to one of them.

Proof :

Consider H as Lyapunov function.

$$\frac{d}{dt}H = \sum_{i=1}^{n} \frac{\partial H}{\partial p_i} \dot{p}_i$$

Voronoi Based Anisotropic Sensor

By applying Reynolds Transport Theorem,

$$\frac{\partial H}{\partial p_i} = \underbrace{\frac{\partial}{\partial p_i}\int_{V_i} f\left(\left\| q - p_i \right\|_{L_i}\right)\phi(q)dq}_{I} + \underbrace{\frac{\partial}{\partial p_i}\sum_{j=1, j\neq i}\int_{V_j} f\left(\left\| q - p_j \right\|_{L_j}\right)\phi(q)dq}_{II}$$

$$I = \int_{V_i} \frac{\partial}{\partial p_i} f\left(\left\| q - p_i \right\|_{L_i}\right)\phi(q)dq + \int_{\partial V_i} \eta_i^t \frac{\partial \partial V_i}{\partial p_i} f\left(\left\| q - p_i \right\|_{L_i}\right)\phi(q)dq$$

$$II = \sum_{j=1, j\neq i}^{n} \int_{V_j} \frac{\partial}{\partial p_i} f\left(\left\| q - p_j \right\|_{L_j}\right)\phi(q)dq + \sum_{j=1, j\neq i}^{n} \int_{\partial V_j} \eta_j^t \frac{\partial \partial V_j}{\partial p_i} f\left(\left\| q - p_j \right\|_{L_j}\right)\phi(q)dq$$

$$= \sum_{j \in N_i} \int_{\partial V_j} \eta_j^t \frac{\partial \partial V_j}{\partial p_i} f\left(\left\| q - p_j \right\|_{L_j}\right)\phi(q)dq$$

Note that : $\sum_{j \in N_i} \frac{\partial \partial V_j}{\partial p_i} = \frac{\partial \partial V_i}{\partial p_i}$

Voronoi Based Anisotropic Sensor

At the boundary $f\left(\left\| q - p_i \right\|_{L_i}\right) = f\left(\left\| q - p_j \right\|_{L_j}\right)$, then

$$\sum_{j \in N_i} \int_{\partial V_j} \eta_j^t \frac{\partial \partial V_j}{\partial p_i} f\left(\left\| q - p_j \right\|_{L_j}\right)\phi(q)dq = -\int_{\partial V_i} \eta_i^t \frac{\partial \partial V_i}{\partial p_i} f\left(\left\| q - p_i \right\|_{L_i}\right)\phi(q)dq$$

Therefore

$$\frac{\partial H}{\partial p_i} = \frac{\partial}{\partial p_i}\int_{V_i} f\left(\left\| q - p_i \right\|_{L_i}\right)\phi(q)dq + \frac{\partial}{\partial p_i}\sum_{j=1, j\neq i}\int_{V_j} f\left(\left\| q - p_j \right\|_{L_j}\right)\phi(q)dq$$

$$= \int_{V_i} \frac{\partial}{\partial p_i} f\left(\left\| q - p_i \right\|_{L_i}\right)\phi(q)dq + \int_{\partial V_i} \eta_i^t \frac{\partial \partial V_i}{\partial p_i} f\left(\left\| q - p_i \right\|_{L_i}\right)\phi(q)dq$$

$$- \int_{\partial V_i} \eta_i^t \frac{\partial \partial V_i}{\partial p_i} f\left(\left\| q - p_i \right\|_{L_i}\right)\phi(q)dq$$

Assume

$$f\left(\left\| q - p_i \right\|_{L_i}\right) = \left\| q - p_i \right\|_{L_i}^2$$

Voronoi Based Anisotropic Sensor

$$\frac{\partial H}{\partial p_i} = 2\left[M_{V_i} p_i - M_{V_i} C_{V_i}\right]$$

where $M_{V_i} = \int_{V_i} L_i \phi(q)dq$

hence

$$\frac{d}{dt}H = \sum_{i=1}^{n} \frac{\partial H}{\partial p_i} \dot{p}_i$$

$$= \sum_{i=1}^{n} -2M_{V_i}\left[p_i - C_{V_i}\right]^2 \leq 0$$

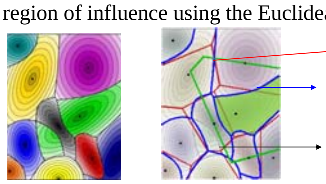
By LaSalle's Principle, the agents converge to the largest invariant set which is the set of anisotropic centroid Voronoi configurations.

QED

Voronoi Based Anisotropic Sensor

How to make the control law distributed ?

- Assuming that each agent is equipped with isotropic (omnidirectional) communication s.t. the neighbors of each agent is the same with the neighbors of the isotropic sensor case.
- By adapting the idea in Feng *et.al* (2007) : reducing the possible region of influence using the Euclidean Voronoi tessellation.



standard Voronoi

anisotropic Voronoi

localized anisotropic Voronoi

$$V_i = \left\{ p \in D \middle| i \in I_p \text{ and } d(p_i, p) \leq d(p_j, p)\ \forall j \in I_p, i \neq j \right\}$$

$$I_p = \left\{ i \in I \middle| (p_i - p_j)(p - p_j) \leq 0, \forall j \neq i \right\}$$

Joint Detection Probability

Let us think for more realistic anisotropic model e.g camera.
Voronoi approach : has to construct the anisotropic Voronoi
➡ too complicated !! Another approach ?

Voronoi approach : one point is sensed by one agent.

Joint detection approach (Wei et.al, CDC 2005) : one point is sensed by some agents. ➡ no need hard partitions

Problem Setting :

$R(x) \geq 0, \int_\Omega R(x) < \infty$: density function

Ω : region of interest (ROI)

$s = (s_1, s_2, ..., s_N)$: position of mobile sensors

$p_i(x)$: probability that sensor i detects the event at x. monotonically decreasing differentiable function

Joint Detection Probability

The joint probability that an event is observed by the sensor :

$$P(x,s) = 1 - \prod_{i=1}^{N} [1 - p_i(x)]$$

The optimal coverage is formulated by :

$$\max_{s} \underbrace{\int_\Omega R(x) P(x,s) dx}_{F(s)}$$

$$\frac{\partial F}{\partial s_i} = \int_\Omega R(x) \frac{\partial P(x,s)}{\partial s_i} dx$$

$$\frac{\partial F}{\partial s_i} = \int_\Omega R(x) \prod_{k=1, k \neq i}^{N} [1 - p_k(x)] \frac{dp_i(x)}{dd_i(x)} \frac{s_i - x}{d_i(x)} dx, \quad d_i(x) = \|x - s_i\|$$

➡ requires global information

Joint Detection Probability

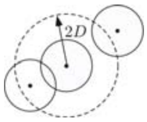
Assumption : each sensor has sensing radius D

$$p_i(x) = 0, \frac{dp_i(x)}{dd_i(x)} = 0 \text{ for all } x \text{ s.t. } d_i(x) \geq D$$

$\Omega_i = \{x : d_i(x) \leq D\}$: sensor i's region of coverage

Neighbors of sensor i (N_i) :

$$N_i = \{k : \|s_i - s_k\| < 2D, k = 1, ..., N, k \neq i\}$$



$$\frac{\partial F}{\partial s_i} = \int_{\Omega_i} R(x) \prod_{k \in N_i} [1 - p_k(x)] \frac{dp_i(x)}{dd_i(x)} \frac{s_i - x}{d_i(x)} dx$$ ➡ Requires only local information

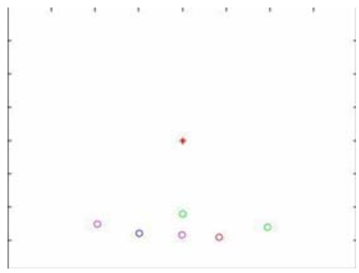
$$s_i^{k+1} = s_i^k + \alpha_k \frac{\partial F}{\partial s_i^k}$$

Joint Detection Probability

Simulation :

$\Omega = 40 \times 40$ $x_0 = [0,20]$ $N = 6, D = 5$

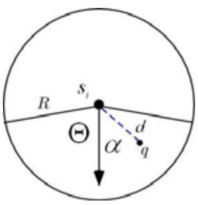
$R(x) = 3 - 0.1\|x - x_0\|$ $p_i(x) = e^{-\|x - s_i\|}$



Joint Detection Probability

Isotropic ➡ Anisotropic (limited sensing range)

Sensor model :



Assumptions :

$$\Omega_i = \{x : d_i(x) \leq R \cap \alpha \leq \Theta\}$$

$$\alpha = \cos^{-1}\left(\frac{(x - p_i)^T (\cos\theta_i, \sin\theta_i)}{\|x - s_i\|}\right)$$

$$d_i(x) = \|x - s_i\|$$

θ_i : orientation of sensor

$p_i(x)$: detection probability, depends on distance and angle from the point to be sensed

e.g.

$$p_i(x) = \frac{(d_i - R)^2 (\alpha - \Theta)^2}{R^2 \Theta^2}$$

Future Works

Voronoi approach :

- Simulations to validate the result
- Consider orientation as optimization variable

Joint detection approach :

- Derive control law for anisotropic case
- Simulations to validate the result
- Consider communication cost
- Extend into 3D coverage problem

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