Output Synchronization taking account of nonholonomic constraints and obstacle and collision avoidance

Output Synchronization taking account of nonholonomic constraints and obstacle and collision avoidance

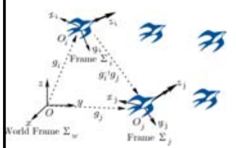
（非ホロノミック制約と衝突・障害物回避を考慮した出力協調）

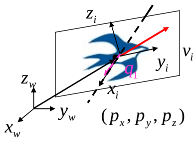
FL07-21-1

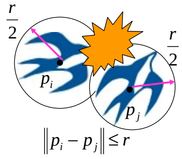
Yuji Igarashi

Outline

1. Previous Results
2. Output Synchronization taking account of Nonholonomic Constraint
3. Output Synchronization taking account of Collision and Obstacle Avoidance
4. Future Works







Previous Results

- **Rigid Body Motion** $(i = 1, \cdots, n)$

$\dot{p}_i = e^{\hat{\xi}_i} v_i$ — $p_i \in \mathcal{R}^3$ position

$\dot{e}^{\hat{\xi}_i} = e^{\hat{\xi}_i} \hat{\omega}_i$ — $e^{\hat{\xi}_i} \in SO(3)$ orientation

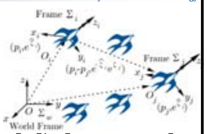
$\varsigma_i = \theta_i \xi_i$ — $v_i \in \mathcal{R}^3$ body velocity

$y_i = \begin{bmatrix} e^{\hat{\xi}_i} & p_i \\ 0 & 1 \end{bmatrix}$ — $\omega_i \in \mathcal{R}^3$ angular velocity

$\theta_i \in \mathcal{R}$ rotation angle

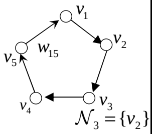
$\xi_i \in \mathcal{R}^3$ rotation axes

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}^\wedge := \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$



- **Using a graph to represent the Intersection topology**

Graph G : Graph consist of a pair $(\mathcal{V}, \mathcal{E}, \mathcal{W})$, where $\mathcal{V}$ is a finite nonempty set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of pair of nodes, called edges and $\mathcal{W}$ is a set of weights over the set of edges.



$G := (\mathcal{V}, \mathcal{E})$: Graph

$\mathcal{V} := \{1, \cdots n\}$: A set of vertices indexed by set of rigid-bodies

$\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$: A set of edges the represent the neighboring relations

$w_{ij}, \mathcal{W}$: A weight on an edge e_{ij} and A set of weights over the set of edges

neighborhood $\mathcal{N}_i$: A set of rigid-bodies whose information is available to rigid-body i

Previous Results

- **Goal** Output Synchronization

$$\lim_{t \to \infty} (y_i - y_j) = 0$$

- **Control Input**

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} e^{-\hat{\xi}_i} & 0 \\ 0 & e^{-\hat{\xi}_i} \end{bmatrix} \begin{bmatrix} v_c(t) \\ e^{\hat{\xi}_c(t)} \omega_c(t) \end{bmatrix} + \sum_{j \in N_i} w_{ij} \begin{bmatrix} k_{pi} I & 0 \\ 0 & k_{ei} I \end{bmatrix} \begin{bmatrix} e^{-\hat{\xi}_i} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} p_j - p_i \\ \mathrm{sk}(e^{-\hat{\xi}_i} e^{\hat{\xi}_j}) \end{bmatrix}$$

$\omega_c(t) := e^{-\hat{\xi}_c(t)} \dot{e}^{\hat{\xi}_c(t)} \quad e^{-\hat{\xi}_c(t)}$, $v_c(t)$ are the same value for all rigid body.

w_{ij} is weight of edge

- **Assumptions**
  - (A1) the rigid-bodies' orientation matrices, $e^{\hat{\xi}_i}$ $\forall i$ are positive definite.
  - (A2) Graph is fixed, strongly connected and each weights are positive , i.e. $w_{ij} > 0 \ \forall i, j$.

- **Potential Function**

$$V := \sum_{i=1}^{n} \gamma_i \left(\frac{1}{2k_{pi}} p_i^T p_i + \frac{1}{k_{ei}} \phi(e^{\hat{\xi}_i}) \right)$$

$$L_w = \{L_{wij}\} := \begin{cases} \sum_{j \in N_i} w_{ij} & \text{if } i = j \\ -w_{ij} & \text{if } j \in N_i \\ 0 & \text{if } j \notin N_i \end{cases} \quad \gamma = \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_n \end{bmatrix}$$

$$\gamma^T L_w = 0^T$$

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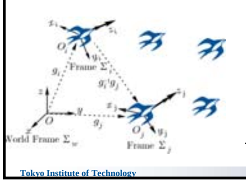
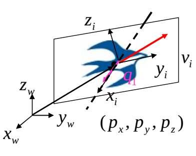
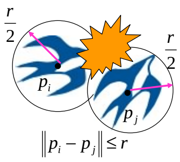






Rigid Body Motion in SE(3)

- **Position and Orientation**

$p_i \in \mathcal{R}^3 \quad e^{\hat{\xi}_i} = e^{\hat{\xi}\theta_i} \in SO(3)$

Rodrigues' formula
Direction of Rotation: $\xi_i \in \mathcal{R}^3$
Angle of Rotation: $\theta_i \in \mathcal{R}$

- **Homogeneous Representation**

$$g_i = \begin{bmatrix} e^{\hat{\xi}_i} & p_i \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{4 \times 4}$$

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}^\wedge := \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

- **Rigid-body Motion in SE(3)** $(i = 1, \cdots, n)$

$\hat{V}_i^b = g_i^{-1} \dot{g}_i$

$V_i^b = \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} \in \mathcal{R}^6$

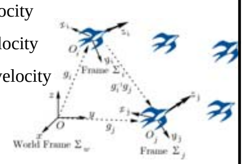
$\hat{V}_i^b = \begin{bmatrix} \hat{\omega}_i & v_i \\ 0 & 0 \end{bmatrix} \in \mathcal{R}^{4 \times 4}$

$\dot{p}_i = e^{\hat{\xi}_i} v_i$ — $V_i^b \in \mathcal{R}^6$ body velocity

$\dot{e}^{\hat{\xi}_i} = e^{\hat{\xi}_i} \hat{\omega}_i$ — $v_i \in \mathcal{R}^3$ linear velocity

$\varsigma_i = \theta_i \xi_i$ — $\omega_i \in \mathcal{R}^3$ angular velocity

$y_i = \begin{bmatrix} e^{\hat{\xi}_i} & p_i \\ 0 & 1 \end{bmatrix}$



- **Simplified Assumptions (SA)**
  - (SA1) The each rigid-bodies' rotation matrices $e^{\hat{\xi}_i}$ $\forall i$ are positive definite.

Note: The rotation matrix $e^{\hat{\xi}_i} = e^{\hat{\xi}\theta_i}$ are positive definite if and only if $|\theta_i| < \frac{\pi}{2}$.

Output Synchronization taking account of nonholonomic constraints and obstacle and collision avoidance

Graph Structure

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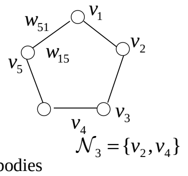
- **Using a graph to represent the Intersection topology**

Graph G : Graph consist of a pair $(\mathcal{V},\mathcal{E},\mathcal{W})$, where $\mathcal{V}$ is a finite nonempty set of nodes, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is a set of pair of nodes, called edges and $\mathcal{W}$ is a set of weights over the set of edges.



$G:=(\mathcal{V},\mathcal{E})$: Graph

$\mathcal{V}:=\{1,\cdots n\}$: A set of vertices indexed by set of rigid-bodies

$\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$: A set of edges the represent the neighboring relations

$e_{ij}\in\mathcal{E}$: An edge from node i to node j.

$w_{ij},\mathcal{W}$: A weight on an edge e_{ij} and A set of weights over the set of edges

neighborhood $\mathcal{N}_i$: A set of rigid-bodies whose information is available to rigid-body i

- **Simplified Assumptions (SA)**
  - (SA2) The weights are positive and symmetric. i.e. $w_{ij}>0,\ w_{ij}=w_{ji},\ j\in\mathcal{N}_i\ \ \forall i$.
  - (SA3) The graph is undirected, strongly connected, and fixed.

Weighted Graph Laplacian

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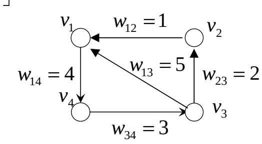
- **Definition**

$$L_w=\{L_{wij}\}:=\begin{cases}\sum_{j\in\mathcal{N}_i} w_{ij} & \text{if } i=j\\ -w_{ij} & \text{if } j\in\mathcal{N}_i\\ 0 & \text{if } j\notin\mathcal{N}_i\end{cases},\quad \gamma=\begin{bmatrix}\gamma_1\\ \vdots\\ \gamma_n\end{bmatrix}\quad \text{s.t.}\quad L_w^T\gamma=0$$

- **Example**

$$L_w=\begin{bmatrix}6&-1&-5&0\\0&2&-2&0\\0&0&3&-3\\-4&0&0&4\end{bmatrix},\quad \gamma=\begin{bmatrix}0.36\\0.18\\0.73\\0.54\end{bmatrix}$$



- **Properties**
  - Eigenvalues have nonnegative real part and its minimum is 0.
  - If the graph is strongly connected, rank L_w is n-1.
  - If (SA2) and (SA3) is satisfied, L_w is symmetric and positive definite.
  - If the graph is balanced (included undirected) and the weights are symmetric, then $1^T L_w=0$[1].

[1] D. J. Lee and M. W. Spong, ''Stable Flocking of Multiple Inertial Agents on Balanced Graphs,'' IEEE *Trans. On Automatics Control*, Vol. 52, No. 8, pp. 1469—1475, 2007.

Nonholonomic Constraint

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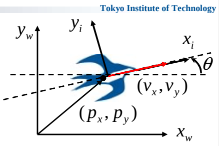
- **2D Plane**

$$\begin{bmatrix}\dot x\\ \dot y\end{bmatrix}=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}v_x\\ v_y\end{bmatrix}\ \Rightarrow\ \begin{bmatrix}\dot x\\ \dot y\end{bmatrix}=v_x\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}$$

$$\dot x\sin\theta-\dot y\cos\theta=0$$

$$\dot x\sin\theta-\dot y\cos\theta=\begin{bmatrix}-\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\dot x\\ \dot y\end{bmatrix}=v_y=0$$



- **3D**

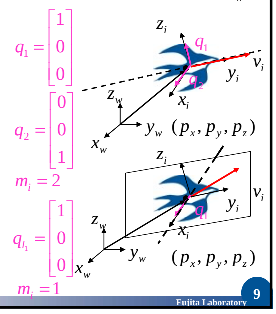
$$\dot p_i=e^{\hat\xi_i}v_i$$

$$q_{l_i}^T v_i=0\quad q_{l_i}^T q_{k_i}=0\quad q_{l_i}\in\mathcal{R}^3$$

$$l_i,k_i=\{0,\cdots,m_i\}\quad m_i=\{0,1,2\}\quad (1)$$
$$l_i\neq k_i$$

Equation (1) means that rigid-body can not move to q_{l_i} .

m_i is a number of nonholonomic constrains.



Output Synchronization

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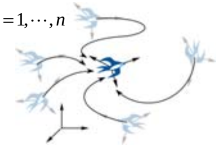
- **Goal** Output Synchronization ([7], pp. 109 Definition1)

In the absence of communication delays, the rigid-bodies are said to output synchronize if

$$\lim_{t\to\infty}\left(y_i(t)-y_j(t)\right)=0\qquad \forall i,j=1,\cdots,n$$

By the definition of the output $y_i=g_i\in SE(3)$, output synchronization implies that both of the position and attitude of all the rigid bodies converge to the same value.



- **Strategy**

Due to nonholonomic constrains it is difficult that positions and orientations of all rigid-bodies converge simultaneously to the same value.

Step1: Velocity Input is considered to converge positions to the same value.

Step2: After the objection of Step1 is achieved, velocity input is switched to converge orientations to the same value without changing positions.

[7] N. Chopra and M. W. Spong, ''Passivity-Based Control of Multi-Agent Systems,'' *in Advance in Robot Control: From Everyday Physics to Human-Like Movements*, S. Kawamura and M. Svnin, eds., pp. 107—134, Springer, 2006.

Rendezvous Problem for Nonholonomic Agents

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Step1: Velocity Input is considered to converge positions to the same value.

- **2D Plane**

[2], D. V. Dimarogonas and K. J. Kyriakopoulos, ''On the Rendezvous Problem for Multiple Nonholonomic Agents,'' *IEEE Trans. on Automatic Control*, vol. 52, no. 5, pp. 916—922, 2007.

$$\begin{cases}\begin{bmatrix}\dot x_i\\ \dot y_i\end{bmatrix}=\begin{bmatrix}\cos\theta_i & -\sin\theta_i\\ \sin\theta_i & \cos\theta_i\end{bmatrix}\begin{bmatrix}v_{xi}\\ v_{yi}\end{bmatrix}\\ \dot\theta_i=\omega_i\end{cases}$$

$$\dot x_i\sin\theta_i-\dot y_i\cos\theta_i=0$$

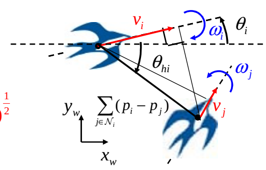
$$v_{xi}=-\mathrm{sgn}(\delta_{xi}\cos\theta_i+\delta_{yi}\sin\theta_i)\cdot(\delta_{xi}^2+\delta_{yi}^2)^{\frac{1}{2}}$$

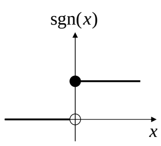
$$\delta_{xi}=\sum_{j\in\mathcal{N}_i}(p_{xi}-p_{xj})$$

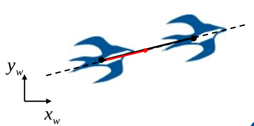
$$\delta_{yi}=\sum_{j\in\mathcal{N}_i}(p_{yi}-p_{yj})$$

$$\mathrm{sgn}(x):=\begin{cases}1 & \text{if } x\geq 0\\ 0 & \text{if } x<0\end{cases}$$

$$\omega_i=-(\theta_i-\theta_{hi})$$







Velocity Input for position convergence

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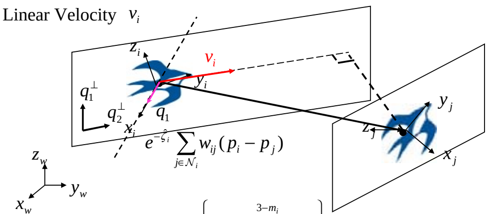
Step1: Velocity Input is considered to converge positions to the same value.

- **In this presentation**

Linear Velocity v_i



$$v_i=\min_{v\in S}\|v-\alpha_i\|\qquad S:=\left\{q\mid q=\sum_{l_i=1}^{3-m_i}a_{l_i}q_{l_i}^\perp\right\}$$

$$v_i=-k_{pi}\sum_{l_i=1}^{3-m_i}\left(\alpha_i^T q_{l_i}^\perp\right)q_{l_i}^\perp\qquad \alpha_i:=e^{-\hat\xi_i}\sum_{j\in\mathcal{N}_i}w_{ij}(p_i-p_j)\qquad k_{pi}>0\qquad (1)$$

$$=-k_{pi}\sum_{l_i=1}^{3-m_i}\left(q_{l_i}^\perp(q_{l_i}^\perp)^T\right)\alpha_i\qquad (q_{l_i}^\perp)^T q_k=0\quad (q_{l_i}^\perp)^T q_{k_i}^\perp=0\quad l_i\neq k_i\quad \|q_{l_i}^\perp\|=1$$

Output Synchronization taking account of nonholonomic constraints and obstacle and collision avoidance

Velocity Input for position convergence

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Step1: Velocity Input is considered to converge positions to the same value.

- **In this presentation**

  Angular Velocity ω_i

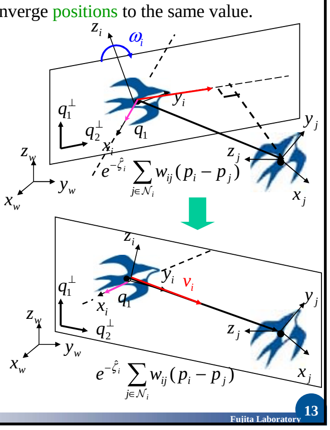
$$\omega_i = k_{ei}\mathrm{sk}(e^{-\hat{\beta}_i}) \quad \beta_i := \xi_i \theta_i \qquad (2)$$

$$\xi_i := \frac{q_i^{\perp} \times e^{-\hat{\xi}_i} \sum_{j\in\mathcal{N}_i}(p_j - p_i)}{\left\| q_i^{\perp} \times e^{-\hat{\xi}_i} \sum_{j\in\mathcal{N}_i}(p_j - p_i) \right\|}$$

$$\theta_i := \cos^{-1}\left((q_i^{\perp})^T \frac{e^{-\hat{\xi}_i}\sum_{j\in\mathcal{N}_i}(p_j - p_i)}{\left\| e^{-\hat{\xi}_i}\sum_{j\in\mathcal{N}_i}(p_j - p_i) \right\|} \right)$$

$$q_i^{\perp} := \sum_{l_i=1}^{3-m_i} a_{l_i} q_{l_i}^{\perp} \qquad \times : \text{outer product}$$

$$\left(\frac{e^{-\hat{\xi}_i}\sum_{j\in\mathcal{N}_i}(p_j - p_i)}{\left\| e^{-\hat{\xi}_i}\sum_{j\in\mathcal{N}_i}(p_j - p_i) \right\|} = e^{\hat{\beta}_i} q_i^{\perp} \right)$$



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Convergence

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- **Convergence of positions**

  Define the potential function as the following function

$$V := \frac{1}{2} p^T L_w p$$

$$= \frac{1}{4}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i} w_{ij}(p_i - p_j)^2 \ .$$

(c.f. If there exists no noholonomic constraint, the potential function is

$$V := \sum_{i=1}^{n} \gamma_i \left(\frac{1}{2k_{pi}} p_i^T p_i + \frac{1}{k_{ei}} \phi(e^{\hat{\xi}_i}) \right)$$
)

Differentiating this potential function yields

$$\dot{V} = -p^T L_w \begin{bmatrix} e^{\hat{\xi}_1} & & \\ & \ddots & \\ & & e^{\hat{\xi}_n} \end{bmatrix} \begin{bmatrix} k_1 \sum_{l_1=1}^{3-m_i} q_{l_1}^{\perp}(q_{l_1}^{\perp})^T & & \\ & \ddots & \\ & & k_n \sum_{l_n=1}^{3-m_n} q_{l_n}^{\perp}(q_{l_n}^{\perp})^T \end{bmatrix} \begin{bmatrix} e^{-\hat{\xi}_1} & & \\ & \ddots & \\ & & e^{-\hat{\xi}_n} \end{bmatrix} L_w p$$

≤ 0 . (Please see appendix in detail.)

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Convergence

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- **Convergence of positions** (LaSalle's Invariance Principle)

  When $\dot{V} = 0$, it is possible to occur the following cases.

  - $\sum_{j\in\mathcal{N}_i}(p_i - p_j) = 0$

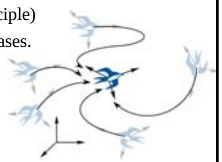
    We can show objective is achieved in this case.

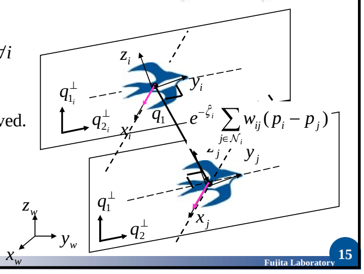
    i.e. $p_i - p_j = 0 \quad \forall i, j$

  - $(q_{l_i}^{\perp})^T \sum_{j\in\mathcal{N}_i} e^{-\hat{\xi}_i}(p_i - p_j) = 0$, $\sum_{j\in\mathcal{N}_i}(p_i - p_j) \neq 0 \quad \forall i$

    In this case objective isn't achieved.

    Next this case isn't invariant set.





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Convergence

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- **Convergence of positions** (LaSalle's Invariance Principle)

  In order to show the later case isn't invariant, we show that

$$(q_{l_i}^{\perp})^T \sum_{j\in\mathcal{N}_i} e^{-\hat{\xi}_i}(p_i - p_j) = 0 \quad \sum_{j\in\mathcal{N}_i}(p_i - p_j) \neq 0 \quad \forall i$$

$$\Rightarrow \quad \frac{d}{dt}\left((q_{l_i}^{\perp})^T \sum_{j\in\mathcal{N}_i} e^{-\hat{\xi}_i}(p_i - p_j) \right) \neq 0 \quad \exists i \ .$$

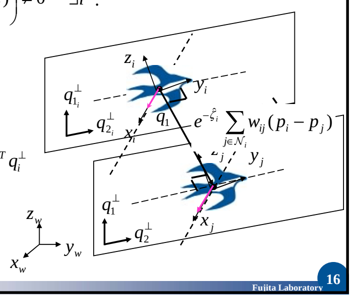
In fact

$$\frac{d}{dt}\left((q_{l_i}^{\perp})^T \sum_{j\in\mathcal{N}_i} e^{-\hat{\xi}_i}(p_i - p_j) \right)$$

$$= -k_{ei}\left\| e^{-\hat{\xi}_i}\sum_{j\in\mathcal{N}_i}(p_j - p_i) \right\| (q_{l_i}^{\perp})^T q_i^{\perp}$$

$$\neq 0$$

(Please see appendix in detail.)



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Velocity Input for position convergence

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Step2: After the objection of Step1 is achieved, velocity input is switched to converge orientations to the same value without changing positions.

Velocity Input

$$\begin{cases} v_i = 0 \\ \omega_i = k_{ei}\sum_{j\in\mathcal{N}_i}\mathrm{sk}(e^{-\hat{\xi}_i}e^{\hat{\xi}_j}) \end{cases} \quad \forall i$$

We can show orientations of all rigid-bodies converge to the same orientation using The following potential function

$$V_A := \sum_{i=1}^{n}\phi(e^{\hat{\xi}_i}) \ .$$

- **Problem**
  - Each rigid-body doesn't always know all rigid-bodies' positions are the same. Note: Each rigid-body can get the information about only neighborhoods.
  - Convergence of step1 and step2 is shown separately.

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✓ 2. Output Synchronization taking account of Nonholonomic Constraint

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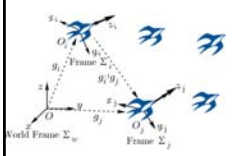

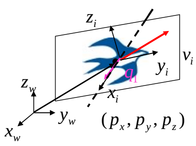

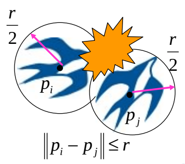

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Output Synchronization taking account of nonholonomic constraints and obstacle and collision avoidance

Rigid Body Motion in SE(3)

- **Position and Orientation**

$p_i \in \mathcal{R}^3 \quad e^{\hat{\xi}_i} = e^{\hat{\xi}\theta_i} \in SO(3)$

Rodrigues' formula
Direction of Rotation: $\xi_i \in \mathcal{R}^3$
Angle of Rotation: $\theta_i \in \mathcal{R}$

- **Homogeneous Representation**

$$g_i = \begin{bmatrix} e^{\hat{\xi}_i} & p_i \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{4\times4}$$

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}^{\wedge} := \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

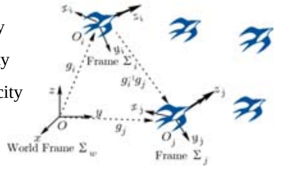
- **Rigid-body Motion in SE(3)** $(i = 1, \cdots, n)$

$\hat{V}_i^b = g_i^{-1}\dot{g}_i$ $\quad V_i^b \in \mathcal{R}^6$ body velocity

$V_i^b = \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} \in \mathcal{R}^6$ $\quad v_i \in \mathcal{R}^3$ linear velocity

$\omega_i \in \mathcal{R}^3$ angular velocity

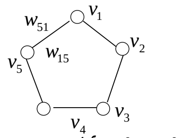
$$\hat{V}_i^b = \begin{bmatrix} \hat{\omega}_i & v_i \\ 0 & 0 \end{bmatrix} \in \mathcal{R}^{4\times4}$$



Graph Structure

- **Using a graph to represent the Intersection topology**

Graph G : Graph consist of a pair $(\mathcal{V}, \mathcal{E}, \mathcal{W})$, where $\mathcal{V}$ is a finite nonempty set of nodes, $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ is a set of pair of nodes, called edges and $\mathcal{W}$ is a set of weights over the set of edges.



$G := (\mathcal{V}, \mathcal{E})$: Graph
$\mathcal{V} := \{1, \cdots n\}$: A set of vertices indexed by set of rigid-bodies
$\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$: A set of edges the represent the neighboring relations
$e_{ij} \in \mathcal{E}$: An edge from node *i* to node *j*.
$w_{ij}, \mathcal{W}$: A weight on an edge e_{ij} and A set of weights over the set of edges
neighborhood $\mathcal{N}_i$: A set of rigid-bodies whose information is available to rigid-body *i*

- **Simplified Assumptions (SA)**
  - (SA2) The weights are positive and symmetric. i.e. $w_{ij} > 0$, $w_{ij} = w_{ji}$, $j \in \mathcal{N}_i$ $\forall i$.
  - (SA3) The graph is undirected, strongly connected and fixed.

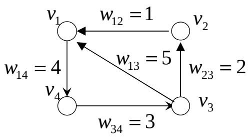
Weighted Graph Laplacian

- **Definition**

$$L_w = \{L_{wij}\} := \begin{cases} \sum_{j\in\mathcal{N}_i} w_{ij} \text{ if } i = j \\ -w_{ij} \text{ if } j \in \mathcal{N}_i \\ 0 \text{ if } j \notin \mathcal{N}_i \end{cases}, \quad \gamma = \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_n \end{bmatrix} \quad \text{s.t.} \quad L_w^T \gamma = 0$$

- **Example**

$$L_w = \begin{bmatrix} 6 & -1 & -5 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 3 & -3 \\ -4 & 0 & 0 & 4 \end{bmatrix}, \quad \gamma = \begin{bmatrix} 0.36 \\ 0.18 \\ 0.73 \\ 0.54 \end{bmatrix}$$



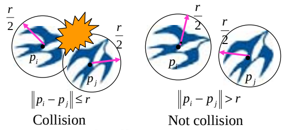
- **Properties**
  - Eigenvalues have nonnegative real part and its minimum is 0.
  - If the graph is strongly connected, rank L_w is n-1.
  - If (SA2) and (SA3) is satisfied, L_w is symmetric and positive definite.
  - If the graph is balanced (included undirected) and the weights are symmetric, then $1^T L_w = 0$[1].

[1] D. J. Lee and M. W. Spong, "Stable Flocking of Multiple Inertial Agents on Balanced Graphs," IEEE *Trans. On Automatics Control*, Vol. 52, No. 8, pp. 1469—1475, 2007.

Collision and Output Synchronization

- **Definition (Collision)**

Collision between rigid-body *i* and *j* occurs if $\|p_i - p_j\| \le r \quad r > 0$.

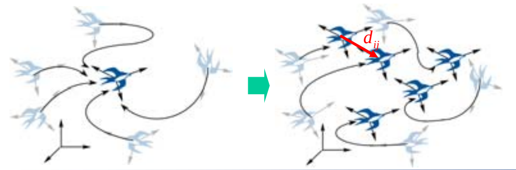


- **Output Synchronization**

Since $y_i = \begin{bmatrix} e^{\hat{\xi}_i} & p_i \\ 0 & 1 \end{bmatrix}$, output synchronization and collision avoidance are satisfied simultaneously.

If $y_i = \begin{bmatrix} e^{\hat{\xi}_i} & p_i + d_i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} e^{\hat{\xi}_i} & z_i \\ 0 & 1 \end{bmatrix}$ output synchronization means

$$\lim_{t\to\infty}(p_i - p_j) = d_{ij} \quad \text{and} \quad \lim_{t\to\infty}(e^{\hat{\xi}_i} - e^{\hat{\xi}_j}) = 0 .$$



$d_{ij} := d_i - d_j$
$z_i = p_i + d_i$
$\|d_i - d_j\| \ge R$

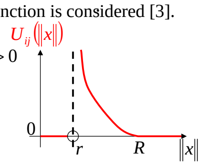
Potential Function

- **Potential Function for Collision Avoidance**

Due to collision avoidance the following potential function is considered [3].

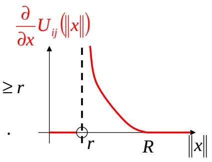
$$U_{ij}(\|p_i - p_j\|) = \left(\min\left(0, \frac{\|p_i - p_j\|^2 - R^2}{\|p_i - p_j\|^2 - r^2}\right)\right)^2 \quad R > r > 0$$

If $\|p_i - p_j\| = r$, then $U_{ij}(\|p_i - p_j\|) = \infty$.
If $\|p_i - p_j\| \ge R$, then $U_{ij}(\|p_i - p_j\|) = 0$.



Derivative of potential function is

$$\frac{\partial}{\partial x}U_{ij}(\|x\|) = \begin{cases} 0 & \text{if } \|x\| \ge R \\ 4\frac{(R^2 - r^2)(\|x\|^2 - R^2)}{(\|x\|^2 - r^2)^3}x & \text{if } R \ge \|x\| \ge r \\ 0 & \text{if } r > \|x\| . \end{cases}$$



[1] D. J. Lee and M. W. Spong, "Stable Flocking of Multiple Inertial Agents on Balanced Graphs," IEEE *Trans. On Automatics Control*, Vol. 52, No. 8, pp. 1469—1475, 2007.

Potential Function

- **Potential Function for Collision Avoidance**

Sum of the potential functions

$$U_c := \sum_{i=1}^{n}\sum_{j=1}^{n} U_{ij}(\|p_i - p_j\|) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\min\left(0, \frac{\|p_i - p_j\|^2 - R^2}{\|p_i - p_j\|^2 - r^2}\right)\right)^2$$

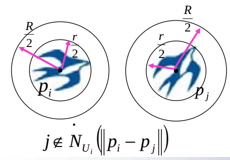
Derivative of sum of the potential functions is

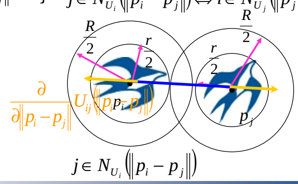
$$\frac{d}{dt}U_c := \sum_{i=1}^{n}\sum_{j\in N_{U_i}(\|p_i - p_j\|)}\frac{\partial}{\partial\|p_i - p_j\|}U_{ij}(\|p_i - p_j\|)^T(\dot{p}_i - \dot{p}_j)$$

$$N_{U_i}(\|p_i - p_j\|) := \{j \mid r < \|p_i - p_j\| < R\}$$

From definition

$$j \in N_{U_i}(\|p_i - p_j\|) \Leftrightarrow i \in N_{U_j}(\|p_j - p_i\|)$$





Output Synchronization taking account of nonholonomic constraints and obstacle and collision avoidance

Obstacle Avoidance

- **Obstacle**

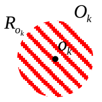
  In this presentation we assume that obstacles are sphere and don't move.

  Collision between rigid-body i and obstacle k if

  $$\|p_i - \tilde{p}_{ik}\| < r \quad r > 0,$$

  where

  $$\tilde{p}_{ik} := \mu_{ik} p_i + (1-\mu_{ik}) o_k \qquad \mu_{ik} = \frac{R_{o_k}}{\|p_i - o_k\|}.$$



o_k : center
R_{o_k} : radius
O_k : a set of obstacle

$$O_k := \{p \mid \|p - o_k\| \le R_{o_k}\}$$

- **Properties**

$$\tilde{p}_{ik} = \inf_{p \in O_k} (\|p_i - p\|)$$

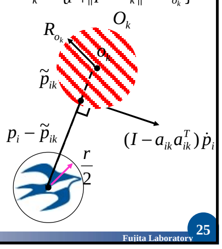
$$\dot{\tilde{p}}_{ik} = \mu_{ik}(I - a_{ik}a_{ik}^T)\dot{p}_i \qquad a_{ik} := \frac{p_i - o_k}{\|p_i - o_k\|}$$

$$(p_i - \tilde{p}_{ik})^T \dot{\tilde{p}}_i = 0$$

- **Potential Function for Obstacle Avoidance**

$$U_{ij}'(\|p_i - \tilde{p}_{ik}\|) = \left(\min\left(0, \frac{\|p_i - \tilde{p}_{ik}\|^2 - R^2}{\|p_i - \tilde{p}_{ik}\|^2 - r^2}\right)\right)^2$$

$$U_o = \sum_{i=1}^n \sum_{j=1}^n \left(\min\left(0, \frac{\|p_i - \tilde{p}_{ik}\|^2 - R^2}{\|p_i - \tilde{p}_{ik}\|^2 - r^2}\right)\right)^2$$



Velocity Input for Collision and Obstacle Avoidance

- **Velocity Input** — For output synchronization — For collision avoidance

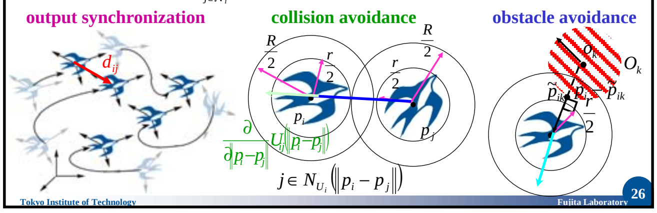
$$v_i = e^{-\hat{\xi}_i}\left(v_c - K_i \sum_{j \in \mathcal{N}_i} w_{ij}(z_i - z_j) - K_i \sum_{i=1}^n \sum_{j \in \mathcal{N}_{U_i}(\|p_i - p_j\|)} \frac{(R^2 - r^2)(\|p_i - p_j\|^2 - R^2)}{(\|p_i - p_j\|^2 - r^2)^3}(p_i - p_j)\right.$$

$$\left. - K_i \sum_{i=1}^n \sum_{j \in \mathcal{N}_{U_i}(\|p_i - \tilde{p}_{ik}\|)} \frac{(R^2 - r^2)(\|p_i - \tilde{p}_{ik}\|^2 - R^2)}{(\|p_i - \tilde{p}_{ik}\|^2 - r^2)^3}(p_i - \tilde{p}_{ik})\right)$$

For obstacle avoidance

$$K_i \in \mathcal{R}^{3\times 3} \quad (K_i + K_i^T) > 0$$

$$\omega_i = e^{-\hat{\xi}_i} e^{\hat{\xi}_c} \omega_c + k_{ei} \sum_{j \in \mathcal{N}_i} \mathrm{sk}(e^{-\hat{\xi}_i} e^{\hat{\xi}_j}) \qquad k_{ei} > 0 \quad \forall i$$



Velocity Input for Collision and Obstacle Avoidance

- **Velocity Input** — For output synchronization — For collision avoidance

$$v_i = e^{-\hat{\xi}_i}\left(v_c - K_i \sum_{j \in \mathcal{N}_i} w_{ij}(z_i - z_j) - K_i \sum_{i=1}^n \sum_{j \in \mathcal{N}_{U_i}(\|p_i - p_j\|)} \frac{(R^2 - r^2)(\|p_i - p_j\|^2 - R^2)}{(\|p_i - p_j\|^2 - r^2)^3}(p_i - p_j)\right.$$

$$\left. - K_i \sum_{i=1}^n \sum_{j \in \mathcal{N}_{U_i}(\|p_i - \tilde{p}_{ik}\|)} \frac{(R^2 - r^2)(\|p_i - \tilde{p}_{ik}\|^2 - R^2)}{(\|p_i - \tilde{p}_{ik}\|^2 - r^2)^3}(p_i - \tilde{p}_{ik})\right)$$

For obstacle avoidance

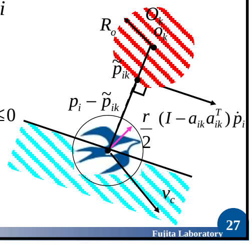
$$K_i \in \mathcal{R}^{3\times 3} \quad (K_i + K_i^T) > 0$$

$$\omega_i = e^{-\hat{\xi}_i} e^{\hat{\xi}_c} \omega_c + k_{ei} \sum_{j \in \mathcal{N}_i} \mathrm{sk}(e^{-\hat{\xi}_i} e^{\hat{\xi}_j}) \qquad k_{ei} > 0 \quad \forall i$$

- **Simplified Assumptions (SA)**
  - (SA4)

$$\left(\sum_{i=1}^n \sum_{j \in \mathcal{N}_{U_i}(\|p_i - \tilde{p}_{ik}\|)} \frac{(R^2 - r^2)(\|p_i - \tilde{p}_{ik}\|^2 - R^2)}{(\|p_i - \tilde{p}_{ik}\|^2 - r^2)^3}(p_i - \tilde{p}_{ik})\right)^T v_c \le 0$$

  - (SA1) $\bar{e}^{\hat{\xi}_i} := e^{-\hat{\xi}_i} e^{\hat{\xi}_c} \ \forall i$ are positive definite.



Analysis

- **Analysis**

  We show collision doesn't occur.

  Define the potential function as the following function

$$V_C = z^T L_w z + \sum_{i=1}^n \phi(\bar{e}^{\hat{\xi}_i}) + \frac{1}{4}\sum_{i=1}^n \sum_{j=1}^n U_{ij}(\|p_i - p_j\|) + \frac{1}{4}\sum_{i=1}^n \sum_{j=1}^n U_{ij}(\|p_i - \tilde{p}_{ik}\|)$$

For output synchronization — For collision avoidance — For obstacle avoidance

$$= \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} w_{ij}(z_i - z_j)^T (z_i - z_j) + \sum_{i=1}^n \phi(\bar{e}^{\hat{\xi}_i}) + \frac{1}{4}\sum_{i=1}^n \sum_{j=1}^n \left(\min\left(0, \frac{\|p_i - p_j\|^2 - R^2}{\|p_i - p_j\|^2 - r^2}\right)\right)^2$$

$$+ \frac{1}{4}\sum_{i=1}^n \sum_{j=1}^n \left(\min\left(0, \frac{\|p_i - \tilde{p}_{ik}\|^2 - R^2}{\|p_i - \tilde{p}_{ik}\|^2 - r^2}\right)\right)^2.$$

From definition of potential function $V_C = \infty$ if and only if collision happens.

Analysis

- **Analysis**

  Differentiating this potential function yields

$$\dot{V}_C = \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} w_{ij} \frac{d}{dt}(z_i - z_j)^T (z_i - z_j) + \sum_{i=1}^n \dot{\phi}(\bar{e}^{\hat{\xi}_i}) + \frac{1}{4}\sum_{i=1}^n \sum_{j=1}^n \frac{d}{dt}\left(\min\left(0, \frac{\|p_i - p_j\|^2 - R^2}{\|p_i - p_j\|^2 - r^2}\right)\right)^2$$

$$+ \frac{1}{4}\sum_{i=1}^n \sum_{j=1}^n \frac{d}{dt}\left(\min\left(0, \frac{\|p_i - \tilde{p}_{ik}\|^2 - R^2}{\|p_i - \tilde{p}_{ik}\|^2 - r^2}\right)\right)^2$$

$$\le \sum_{i=1}^n \left(\sum_{j \in \mathcal{N}_i} w_{ij}(z_i - z_j) + \sum_{j \in N_{U_i}(\|p_i - p_j\|)} \frac{(R^2 - r^2)(\|p_i - p_j\|^2 - R^2)}{(\|p_i - p_j\|^2 - r^2)^3}(p_i - p_j)\right.$$

$$\left. + \sum_{j \in N_{U_i}(\|p_i - \tilde{p}_{ik}\|)} \frac{(R^2 - r^2)(\|p_i - \tilde{p}_{ik}\|^2 - R^2)}{(\|p_i - \tilde{p}_{ik}\|^2 - r^2)^3}(p_i - \tilde{p}_{ik})\right)^T K_i \left(\sum_{j \in \mathcal{N}_i} w_{ij}(z_i - z_j)\right.$$

$$\left. + \sum_{j \in N_{U_i}(\|p_i - p_j\|)} \frac{(R^2 - r^2)(\|p_i - p_j\|^2 - R^2)}{(\|p_i - p_j\|^2 - r^2)^3}(p_i - p_j) + \sum_{j \in N_{U_i}(\|p_i - \tilde{p}_{ik}\|)} \frac{(R^2 - r^2)(\|p_i - \tilde{p}_{ik}\|^2 - R^2)}{(\|p_i - \tilde{p}_{ik}\|^2 - r^2)^3}(p_i - \tilde{p}_{ik})\right)$$

$$+ \left(\sum_{i=1}^n \sum_{j \in \mathcal{N}_{U_i}(\|p_i - \tilde{p}_{ik}\|)} \frac{(R^2 - r^2)(\|p_i - \tilde{p}_{ik}\|^2 - R^2)}{(\|p_i - \tilde{p}_{ik}\|^2 - r^2)^3}(p_i - \tilde{p}_{ik})\right)^T v_c - \frac{1}{2}\sum_{i=1}^n \sum_{j \in \mathcal{N}_i} w_{ij} \lambda_{\min}(e^{\hat{\xi}_i} + e^{-\hat{\xi}_i}) \phi(e^{-\hat{\xi}_i} e^{\hat{\xi}_j})$$

$$\le 0 \qquad (4)$$

Analysis

- **Analysis**

  If there is no collision in initial condition, V_C is bounded. And from (4) V_C doesn't go to infinity. This means collisions don't occurs.

  Note: We show collisions don't occurs. However we can't guarantee achievement of output synchronization. Rigid-bodies converge to a condition satisfied

$$\left(\sum_{j \in \mathcal{N}_i} w_{ij}(z_i - z_j) + \sum_{j \in N_{U_i}(\|p_i - p_j\|)} \frac{(R^2 - r^2)(\|p_i - p_j\|^2 - R^2)}{(\|p_i - p_j\|^2 - r^2)^3}(p_i - p_j)\right.$$

$$\left. + \sum_{j \in N_{U_i}(\|p_i - \tilde{p}_{ik}\|)} \frac{(R^2 - r^2)(\|p_i - \tilde{p}_{ik}\|^2 - R^2)}{(\|p_i - \tilde{p}_{ik}\|^2 - r^2)^3}(p_i - \tilde{p}_{ik})\right) = 0$$

$$\lim_{t \to \infty}(e^{\hat{\xi}_i} - e^{\hat{\xi}_j}) = 0 \quad \forall i, j$$

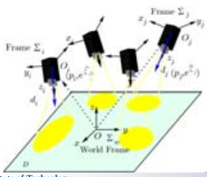
Output Synchronization taking account of nonholonomic constraints and obstacle and collision avoidance

Future Works

1. Experiments
2. Extension to Visual Attitude Coordination
3. Extension of Effective Coverage Control using Visual Sensor

⋮

and so on.



Appendix

1. Property of $e^{\hat\xi\theta}$
2. Derivative of Potential Function(Nonholonomic Case)
3. Calculation of Invariant Set
4. Properties of $\tilde{p}_{ik}$

Property of $e^{\hat\xi\theta}$

- Property of $e^{\hat\xi\theta}$

We show

$$\frac{e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)}{\left\|e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|}=e^{\hat\beta_i}q_i^\perp$$

From

$$e^{\hat\xi\theta}=I+\sin\theta\hat\xi+(1-\cos\theta)\hat\xi^2$$

thus

$$e^{-\hat\beta_i}q_i^\perp=\left(I+\sin\theta_i\hat\xi_i+(1-\cos\theta_i)\hat\xi_i^2\right)q_i^\perp$$

$$\xi_i^Tq_i^\perp=\left(\frac{q_i^\perp\times e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)}{\left\|q_i^\perp\times e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|}\right)^Tq_i^\perp=0$$

$$=\left(I+\sin\theta_i\hat\xi_i+(1-\cos\theta_i)(\xi_i\xi_i^T-I)\right)q_i^\perp$$

$$=q_i^\perp+\sin\theta_i\hat\xi_iq_i^\perp+(1-\cos\theta_i)(\xi_i\xi_i^T-I)q_i^\perp$$

Property of $e^{\hat\xi\theta}$

- Property of $e^{\hat\xi\theta}$

$$\hat\xi_iq_i^\perp=\left(\frac{q_i^\perp\times e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)}{\left\|q_i^\perp\times e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|}\right)\times q_i^\perp$$

$$=\left((q_i^\perp)^Tq_i^\perp\right)\frac{e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)}{\left\|q_i^\perp\times e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|}-\left((q_i^\perp)^T\frac{e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)}{\left\|q_i^\perp\times e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|}\right)q_i^\perp$$

$$(\because (b\times c)\times a=(a^Tb)c-(a^Tc)b)$$

$$=\frac{e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)-\left((q_i^\perp)^Te^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right)q_i^\perp}{\left\|q_i^\perp\times e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|}$$

$$=\frac{e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)}{\left\|q_i^\perp\times e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|}-\left((q_i^\perp)^T\frac{e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)}{\left\|q_i^\perp\times e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|}\right)q_i^\perp$$

Property of $e^{\hat\xi\theta}$

- Property of $e^{\hat\xi\theta}$

$$e^{-\hat\beta_i}q_i^\perp=\left(I+\sin\theta_i\hat\xi_i+(1-\cos\theta_i)\hat\xi_i^2\right)q_i^\perp$$

$$=\left(\frac{e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)-\left((q_i^\perp)^Te^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right)q_i^\perp}{\left\|e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|}\right)+\cos\theta_iq_i^\perp$$

$$=\left(\frac{e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)-\left((q_i^\perp)^Te^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right)q_i^\perp}{\left\|e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|}\right)+\left((q_i^\perp)^T\frac{e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)}{\left\|e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|}\right)q_i^\perp$$

$$=\frac{e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)}{\left\|e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|}$$

Property of $e^{\hat\xi\theta}$

- Property of $e^{\hat\xi\theta}$

$$e^{-\hat\beta_i}q_i^\perp=\left(I+\sin\theta_i\hat\xi_i+(1-\cos\theta_i)\hat\xi_i^2\right)q_i^\perp$$

$$=q_i^\perp+\sin\theta_i\hat\xi_iq_i^\perp+(1-\cos\theta_i)(\xi_i\xi_i^T-I)q_i^\perp$$

$$=q_i^\perp+\sin\theta_i\left(\frac{e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)-\left((q_i^\perp)^Te^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right)q_i^\perp}{\left\|q_i^\perp\times e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|}\right)-(1-\cos\theta_i)q_i^\perp$$

$$=\sin\theta_i\left(\frac{e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)-\left((q_i^\perp)^Te^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right)q_i^\perp}{\left\|q_i^\perp\times e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|}\right)+\cos\theta_iq_i^\perp$$

$$=\sin\theta_i\left(\frac{e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)-\left((q_i^\perp)^Te^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right)q_i^\perp}{\left\|e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|\sin\theta_i}\right)+\cos\theta_iq_i^\perp$$

$$=\left(\frac{e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)-\left((q_i^\perp)^Te^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right)q_i^\perp}{\left\|e^{-\hat\xi_i}\sum_{j\in N_i}(p_j-p_i)\right\|}\right)+\cos\theta_iq_i^\perp$$

Output Synchronization taking account of nonholonomic constraints and obstacle and collision avoidance

Derivative of Potential Function(Nonholonomic Case)

Tokyo Institute of Technology

- **Convergence of positions**

Define the potential function as the following function

$$V := \frac{1}{2} p^T L_w p \;= \frac{1}{4} \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} w_{ij} (p_i - p_j)^2$$

Differentiating this potential function yields

$$\dot{V} := p^T L_w \dot{p}$$

$$= p^T L_w \begin{bmatrix} e^{\hat{\zeta}_1} & & \\ & \ddots & \\ & & e^{\hat{\zeta}_n} \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

$$= -p^T L_w \begin{bmatrix} e^{\hat{\zeta}_1} & & \\ & \ddots & \\ & & e^{\hat{\zeta}_n} \end{bmatrix} \begin{bmatrix} k_1 \sum_{l_1=1}^{3-m_1} \left(q_{l_1}^{\perp} (q_{l_1}^{\perp})^T \right) \alpha_1 \\ \vdots \\ k_n \sum_{l_n=1}^{3-m_n} \left(q_{l_n}^{\perp} (q_{l_n}^{\perp})^T \right) \alpha_n \end{bmatrix} \qquad \alpha_i := e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} w_{ij} (p_i - p_j)$$

Derivative of Potential Function(Nonholonomic Case)

Tokyo Institute of Technology

$$\dot{V} := p^T L_w \dot{p}$$

$$= -p^T L_w \begin{bmatrix} e^{\hat{\zeta}_1} & & \\ & \ddots & \\ & & e^{\hat{\zeta}_n} \end{bmatrix} \begin{bmatrix} k_1 \sum_{l_1=1}^{3-m_1} \left(q_{l_1}^{\perp} (q_{l_1}^{\perp})^T \right) e^{-\hat{\zeta}_1} \sum_{j \in \mathcal{N}_1} w_{1j} (p_1 - p_j) \\ \vdots \\ k_n \sum_{l_n=1}^{3-m_n} \left(q_{l_n}^{\perp} (q_{l_n}^{\perp})^T \right) e^{-\hat{\zeta}_n} \sum_{j \in \mathcal{N}_n} w_{nj} (p_n - p_j) \end{bmatrix}$$

$$= -p^T L_w \begin{bmatrix} e^{\hat{\zeta}_1} & & \\ & \ddots & \\ & & e^{\hat{\zeta}_n} \end{bmatrix} \begin{bmatrix} k_1 \sum_{l_1=1}^{3-m_1} \left(q_{l_1}^{\perp} (q_{l_1}^{\perp})^T \right) & & \\ & \ddots & \\ & & k_n \sum_{l_n=1}^{3-m_n} \left(q_{l_n}^{\perp} (q_{l_n}^{\perp})^T \right) \end{bmatrix} \begin{bmatrix} e^{-\hat{\zeta}_1} & & \\ & \ddots & \\ & & e^{-\hat{\zeta}_n} \end{bmatrix} \begin{bmatrix} \sum_{j \in \mathcal{N}_1} w_{1j} (p_1 - p_j) \\ \vdots \\ \sum_{j \in \mathcal{N}_n} w_{nj} (p_n - p_j) \end{bmatrix}$$

$$= -p^T L_w \begin{bmatrix} e^{\hat{\zeta}_1} & & \\ & \ddots & \\ & & e^{\hat{\zeta}_n} \end{bmatrix} \begin{bmatrix} k_1 \sum_{l_1=1}^{3-m_1} \left(q_{l_1}^{\perp} (q_{l_1}^{\perp})^T \right) & & \\ & \ddots & \\ & & k_n \sum_{l_n=1}^{3-m_n} \left(q_{l_n}^{\perp} (q_{l_n}^{\perp})^T \right) \end{bmatrix} \begin{bmatrix} e^{-\hat{\zeta}_1} & & \\ & \ddots & \\ & & e^{-\hat{\zeta}_n} \end{bmatrix} L_w p$$

Calculation of Invariant Set

Tokyo Institute of Technology

- **Convergence of positions** (LaSalle's Invariance Principle)

In order to show the later case isn't invariant, we show that

$$(q_{l_i}^{\perp})^T \sum_{j \in \mathcal{N}_i} e^{-\hat{\zeta}_i} (p_i - p_j) = 0 \quad \sum_{j \in \mathcal{N}_i} (p_i - p_j) \neq 0 \quad \forall i$$

$$\Rightarrow \quad \frac{d}{dt} \left((q_{l_i}^{\perp})^T \sum_{j \in \mathcal{N}_i} e^{-\hat{\zeta}_i} (p_i - p_j) \right) \neq 0 \quad \exists i \;.$$

$$\frac{d}{dt} \left((q_{l_i}^{\perp})^T \sum_{j \in \mathcal{N}_i} e^{-\hat{\zeta}_i} (p_i - p_j) \right) = (q_{l_i}^{\perp})^T \sum_{j \in \mathcal{N}_i} \dot{e}^{-\hat{\zeta}_i} (p_i - p_j) + (q_{l_i}^{\perp})^T \sum_{j \in \mathcal{N}_i} e^{-\hat{\zeta}_i} (\dot{p}_i - \dot{p}_j)$$

$$= (q_{l_i}^{\perp})^T \sum_{j \in \mathcal{N}_i} \dot{e}^{-\hat{\zeta}_i} (p_i - p_j)$$

$$= (q_{l_i}^{\perp})^T \sum_{j \in \mathcal{N}_i} (e^{\hat{\zeta}_i} \hat{\omega}_i)^T (p_i - p_j)$$

$$= -(q_{l_i}^{\perp})^T \sum_{j \in \mathcal{N}_i} \hat{\omega}_i e^{-\hat{\zeta}_i} (p_i - p_j)$$

$$= -(q_{l_i}^{\perp})^T \hat{\omega}_i \sum_{j \in \mathcal{N}_i} e^{-\hat{\zeta}_i} (p_i - p_j)$$

Calculation of Invariant Set

Tokyo Institute of Technology

- **Convergence of positions** (LaSalle's Invariance Principle)

$$\omega_i = k_{ei} \mathrm{sk}(e^{-\hat{\beta}_i}) \qquad \beta_i := \xi_i \theta_i$$

$$\xi_i := \frac{q_i^{\perp} \times e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i)}{\left\| q_i^{\perp} \times e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i) \right\|}$$

$$\theta_i := \cos^{-1} \left((q_i^{\perp})^T \frac{e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i)}{\left\| e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i) \right\|} \right)$$

$$= k_{ei} \sin \theta_i \xi_i$$

$$= k_{ei} \sin \theta_i \frac{q_i^{\perp} \times e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i)}{\left\| q_i^{\perp} \times e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i) \right\|}$$

$$= k_{ei} \sin \theta_i \frac{q_i^{\perp} \times e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i)}{\sin \theta_i \left\| e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i) \right\|}$$

$$= k_{ei} \frac{q_i^{\perp} \times e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i)}{\left\| e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i) \right\|}$$

$$= k_{ei} \left(q_i^{\perp} \times \frac{e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i)}{\left\| e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i) \right\|} \right)$$

Calculation of Invariant Set

Tokyo Institute of Technology

- **Convergence of positions** (LaSalle's Invariance Principle)

$$\frac{d}{dt} \left((q_{l_i}^{\perp})^T \sum_{j \in \mathcal{N}_i} e^{-\hat{\zeta}_i} (p_i - p_j) \right)$$

$$= -(q_{l_i}^{\perp})^T \hat{\omega}_i \sum_{j \in \mathcal{N}_i} e^{-\hat{\zeta}_i} (p_i - p_j)$$

$$= -k_{ei} (q_{l_i}^{\perp})^T \left(q_i^{\perp} \times \frac{e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i)}{\left\| e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i) \right\|} \right) \times \sum_{j \in \mathcal{N}_i} e^{-\hat{\zeta}_i} (p_i - p_j)$$

$$= -k_{ei} (q_{l_i}^{\perp})^T \left\{ \left(\frac{e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i)}{\left\| e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i) \right\|} \right)^T \left(\sum_{j \in \mathcal{N}_i} e^{-\hat{\zeta}_i} (p_i - p_j) \right) q_i^{\perp} + \left(\frac{e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i)}{\left\| e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i) \right\|} \right)^T q_i^{\perp} \sum_{j \in \mathcal{N}_i} e^{-\hat{\zeta}_i} (p_i - p_j) \right\}$$

$$= -k_{ei} (q_{l_i}^{\perp})^T \left\{ \left(\frac{e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i)}{\left\| e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i) \right\|} \right)^T \left(\sum_{j \in \mathcal{N}_i} e^{-\hat{\zeta}_i} (p_i - p_j) \right) q_i^{\perp} \right\}$$

$$= -k_{ei} \left\| e^{-\hat{\zeta}_i} \sum_{j \in \mathcal{N}_i} (p_j - p_i) \right\| (q_{l_i}^{\perp})^T q_i^{\perp} \neq 0$$

Properties of $\tilde{p}_{ik}$

Tokyo Institute of Technology

- $\dot{\tilde{p}}_{ik} = \mu_{ik} (I - a_{ik} a_{ik}^T) \dot{p}_i$

$$\mu_{ik} = \frac{R_{o_k}}{\| p_i - o_k \|}$$

$$\dot{\tilde{p}}_{ik} = \mu_{ik} \dot{p}_i + \dot{\mu}_{ik} p_i - \dot{\mu}_{ik} o_k$$

$$= -\mu_{ik} \frac{(p_i - o_k)^T}{\| p_i - o_k \|^2} \dot{p}_i (p_i - o_k) + \mu_{ik} \dot{p}_i$$

$$= \mu_{ik} \frac{(p_i - o_k)(p_i - o_k)^T}{\| p_i - o_k \|^2} \dot{p}_i + \mu_{ik} \dot{p}_i$$

$$= \mu_{ik} \left(I - \frac{(p_i - o_k)(p_i - o_k)^T}{\| p_i - o_k \|^2} \right) \dot{p}_i = \mu_{ik} \left(I - a_{ik} (a_{ik})^T \right) \dot{p}_i \qquad a_{ik} := \frac{p_i - o_k}{\| p_i - o_k \|}$$

- $(p_i - \tilde{p}_{ik})^T \dot{\tilde{p}}_i = 0$

$$(p_i - \tilde{p}_{ik})^T \dot{\tilde{p}}_i = (p_i - \mu_{ik} p_i - (1 - \mu_{ik}) o_k)^T \mu_{ik} \left(I - a_{ik} (a_{ik})^T \right) \dot{p}_i$$

$$= (1 - \mu_{ik})(p_i - o_k)^T \mu_{ik} \left(I - a_{ik} (a_{ik})^T \right) \dot{p}_i$$

$$= (1 - \mu_{ik}) \mu_{ik} \| (p_i - o_k) \| a_{ik}^T \left(I - a_{ik} (a_{ik})^T \right) \dot{p}_i$$

$$= (1 - \mu_{ik}) \mu_{ik} \| (p_i - o_k) \| \left(a_{ik}^T - a_{ik}^T a_{ik} (a_{ik})^T \right) \dot{p}_i$$

$$= (1 - \mu_{ik}) \mu_{ik} \| (p_i - o_k) \| \left(a_{ik}^T - a_{ik}^T \right) \dot{p}_i \; = 0$$