Experimental Study on Coverage Control with Anisotropic Sensors

Fujita Lab, Dept. of Control and System Engineering,
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David Asikin

Outline

Introduction

1. **Coverage Control:**
   - Definition
   - Application
   - FAQ
2. **Review:**
   - Voronoi
   - Lloyd's Algorithm
   - Objective Function

Coverage Control

- **Definition:**
  Deployment of mobile sensor nodes in the region of interest, where interesting events might happen and the corresponding detection mechanism is required.
- **Application:**
  Search and rescue, surveillance robots, planet exploration, environmental monitoring, military and defense, etc.

Coverage Control

- **Why multiple robots?**
  Robustness, improve reliability & probability of finding events, able to handle complex task, deepen understanding of nature, etc.
- **What is the goal in coverage control?**
  Optimum placement of sensors.
- **Why is that the goal?**
  Maximum possible utilization of the sensors: enhance network coverage, extend the system lifetime.

Review

- Objective function:

$$H(p,W)=\int_W f(\|q-p\|)\phi(q)dq$$

$f(\|q-p\|)$: Sensing performance
(f=big → poor sensing)
$\phi(q)$: Density function
p = agent position q = object W = partition

- By minimizing H, we get optimum coverage. Why?
  When H = min, agents move to the area with the highest occurrence possibility.

Review

- We assume: $f\left(\|q-p\|\right)=\|q-p\|^2$
  Simplify the objective function using parallel axis theorem.

$$H(p,W)=\int_W f\left(\|q-p\|\right)\phi(q)dq$$

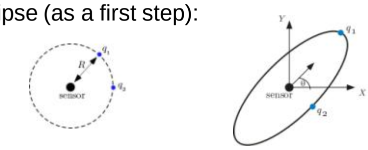
↓

$$H(p,W)=H(c_W,W)+M_W\|p-c_W\|^2$$
$$M_W=\int_W \phi(q)dq \quad : mass$$

- To minimize this, $p=C_W$ (=centroid of W partition)
- Therefore, use this as an input to make agents go to centroid of the partition.

Anisotropic Sensors

- Definition and Difference with Isotropic sensors?
- Ellipse (as a first step):



- Sensing performance: position & orientation
- Why Anisotropic? (Research Objective?)
  More realistic. Most sensors (i.e. cameras, directional microphones, radars) are anisotropic.

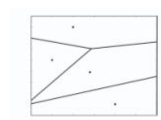
Anisotropic Sensors

- Isotropic: $f\left(\|q-p\|\right)=\|q-p\|^2=\left\{(q-p)^T(q-p)\right\}^2$
- Anisotropic:

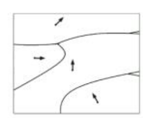
$$f\left(\|q-p\|_{L_i}\right)=\|q-p\|_{L_i}=(q-p)^T L_i (q-p)$$
$$L_i=F_i^T F_i$$
$$F_i=\left[\begin{pmatrix}\frac{c}{a} & 0\\ 0 & \frac{c}{b}\end{pmatrix}\begin{pmatrix}\cos\theta_i & \sin\theta_i\\ -\sin\theta_i & \cos\theta_i\end{pmatrix}\right] \qquad a,b,c>0$$

θ_i : orientation of the i-th sensor
a, b : length of major and minor axis of the ellipse
p : agent position $\qquad f$: sensing performance
q : object

Anisotropic Sensors



- Isotropic: $V_i=\left\{q\in Q \,\middle|\, \|q-p_i\|\le\|q-p_j\|, \forall j\ne i\right\}$
  - Needs information only from neighbors.
  - Distributed control.
  - Voronoi partitions are connected.



- Anisotropic: $V_i^*=\left\{q\in Q \,\middle|\, \|q-p_i\|_{L_i}\le\|q-p_j\|_{L_j}, \forall j\ne i\right\}$
  - Needs information from all agents.
  - Non-distributed control.
  - Possibility of '**orphan**' areas.

Boundary of two adjacent Anisotropic Voronoi partitions is a quadratic curve.

Anisotropic Sensors

- We have understood the difference between Isotropic and Anisotropic sensors.
- Now, the goal is to:

1. Find the optimal configuration:

$$\min_{P,\Theta,W} H$$

2. Find the input u_i that drives the agents to the optimal configuration:

$$\dot{p}_i=u_i$$

Assumptions and Equations

Assumption:
Orientation of all agents are equal and fixed over time.

=

$$\theta_i(t)=\theta_j(t), \forall i\ne j$$
$$t\ge 0$$

- **Why?**
  - To make the controller distributed.
  - As a first step to understanding of a more complicated anisotropic problem.

Assumptions and Equations

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- The result of the assumption:

1. Anisotropic Voronoi's boundaries become straight lines (similar to isotropic). (App. E & F)
2. Because optimal partition = Anisotropic Voronoi and the orientation of the agents are fixed, we only need to optimize agents' position.
3. Objective function becomes: (App. G)

$$H_{V^*}(P)=\sum_{i=1}^{n}\int_{V_i^*}\left\|F(q-p_i)\right\|^2\phi(q)\,dq$$

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Assumptions and Equations

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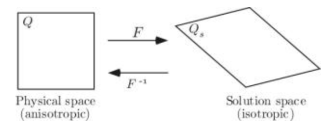
$$H_{V^*}(P)=\sum_{i=1}^{n}\int_{V_i^*}\left\|F(q-p_i)\right\|^2\phi(q)\,dq$$

$\bar{q}=Fq$ $z_i=Fp_i$



$$\bar{V}_i=\left\{\bar{q}\in Q_s\ \middle|\ \left\|\bar{q}-z_i\right\|\le\left\|\bar{q}-z_j\right\|,\forall j\neq i\right\}$$

$$H_{\bar{V}}(Z)=\sum_{i=1}^{n}\int_{\bar{V}_i}\left\|\bar{q}-z_i\right\|^2\phi(\bar{q})\left|\det\left(F^{-1}\right)\right|dq$$

Can be treated as an isotropic problem (Cortes et. al)

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Assumptions and Equations

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$$H_{\bar{V}}(Z)=\sum_{i=1}^{n}\int_{\bar{V}_i}\left\|\bar{q}-z_i\right\|^2\phi(\bar{q})\left|\det\left(F^{-1}\right)\right|dq$$

parallel axis theorem ↓ Goal 1: $\min_P H_{V^*}$

$$\frac{\partial H_{\bar{V}}}{\partial z_i}(Z)=2\left|\det\left(F^{-1}\right)\right|M_{\bar{V}_i}\left(z_i-C_{\bar{V}_i}\right)$$

- Therefore, **the local minimum points are the centroids of the Anisotropic Voronoi** partition because

$$C_{V_i^*}=F^{-1}C_{\bar{V}_i}$$

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Assumptions and Equations

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- Based on the result that $\min_P H_{V^*}$ is achieved when $z_i=C_{\bar{V}_i}$, where $C_{V_i^*}=F^{-1}C_{\bar{V}_i}$,

we can make a controller that optimizes configuration, such as:

$$u_i=-k\left(p_i-C_{V_i^*}\right)$$

- Take $H_{\bar{V}}$ as Lyapunov function,

$\dot{H}_{\bar{V}}\le 0$ + LaSalle's principle

↓

Converge to Centroids of Anisotropic Voronoi (App. H)

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Stability and Simulation

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- Simulation on a square region *Q* (5x5):
  - Number of agents = 5
  - *a*,*b*,*c*,θ = 2,1,1,-π/3
  - $\phi(q)=1,\forall q$
  - $p_i\neq p_j,\forall i\neq j$

- Simulation:
  1. Random p_i
  2. Transform to z_i
  3. Calculate $\bar{V}_i, C_{\bar{V}_i}$
  4. Transform to $V_i^*, C_{V_i^*}$
  5. p_i becomes $C_{V_i^*}$
  6. Back to 1.

Note:
○ = initial position
■ = final position
-- = trajectory

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Stability and Simulation

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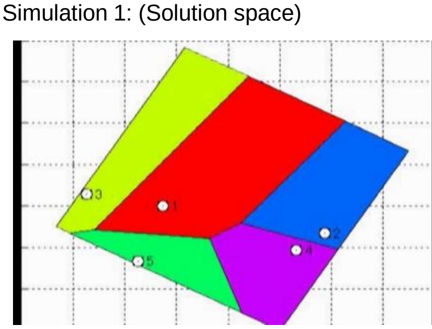
- Simulation 1: (Solution space)



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Stability and Simulation

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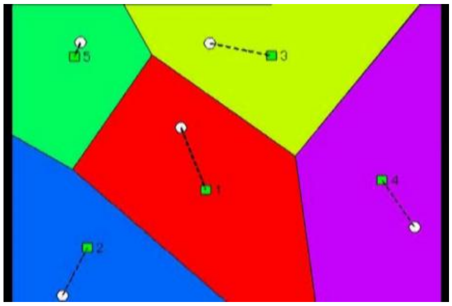
- Simulation 1: (Physical space)



Stability and Simulation

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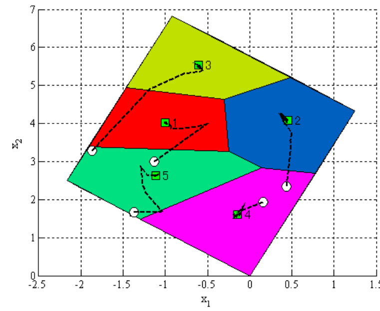
- Trajectory (Solution):



Stability and Simulation

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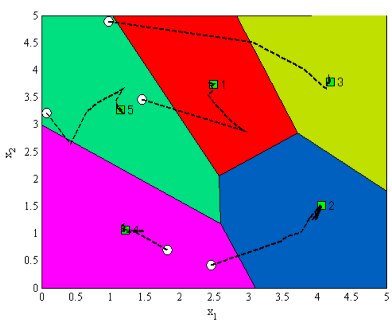
- Trajectory (Physical):



Conclusion

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- We consider coverage using Anisotropic sensors.
- Assumption: fixed & equal sensors' orientation θ.
- Derived controller for Anisotropic problem (from isotropic).
- Proven stable by LaSalle's principle.

Experiment Plan

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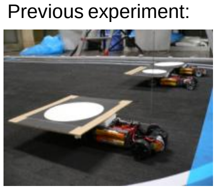
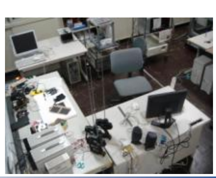
- Previous experiment:



Experiment Plan

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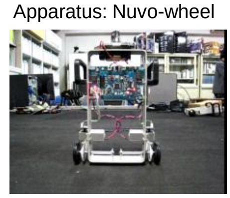
- Apparatus: Nuvo-wheel




- Objective:
- Stabilize Nuvo-wheel while moving
- Isotropic Voronoi 1D rectification
- Anisotropic Voronoi 2D (**Why** 2D?)

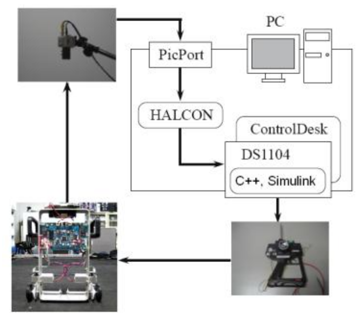
Experiment Plan

- **WHY** Nuvo-wheel?
- 4-wheels vehicle cannot rotate smoothly.
- No backward problem as in 4-wheels vehicle.
- More interesting than intrinsically stable 4-wheels vehicle.

- Status:
- Interpreting C++ code
- Simulink: for simulation only (to examine gain: *K*)
- Wire → Wireless

Experiment Plan

- Tentative diagram:



Future Work

- Make Nuvo-wheel wireless.
- Lloyd's Algorithm 1D experiment revision.
- Anisotropic Voronoi 2D.

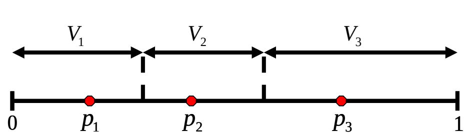
- **My Goal:**
- **Early Dec:**
  Wireless Nuvo-wheel, complete mastering of the codes.
- **Mid Dec:**
  Working prototype of 1D Voronoi revision.
- **End of Dec:**
  Realization of 2D Anisotropic Voronoi exp.

Any question?

Appendix A

- **Voronoi partition:**
  The set of all points *q* whose distance from p_i is less than or equal to the distances from all other p_j

$$V_i = \{ q : (\forall j \neq i) \|q - p_i\| \leq \|q - p_j\| \}$$



Appendix B

- **Lloyd's Algorithm:**
  A method for evenly distributing points over an unknown area.

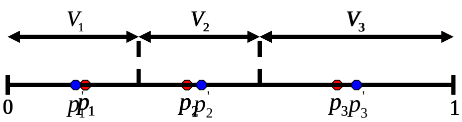
- **The steps:**
  Step 0: Start with a random area, {*Wi*}, and random points, {*pi*}.
  Step 1: Construct Voronoi partition {*Vi*}, generated by {*pi*}.
  Step 2: Update pi to be the centroid of *Vi*. Return to Step 1.

Appendix C

Step 0: Start with a random area, {*Wi*}, and random points, {*pi*}.

Step 1: Construct Voronoi partition {*Vi*}, generated by {*pi*}.

Step 2: Update pi to be the centroid of *Vi*. Return to Step 1.



Appendix D

- Why is the boundary a quadratic curve?

$$V_i^* = \left\{ q \in Q \middle| \|q - p_i\|_{L_i} \le \|q - p_j\|_{L_j}, \forall j \ne i \right\}$$

$$\|q - p_i\|_{L_i} = \|q - p_j\|_{L_j}$$

q = object

p = agent

$$(q - p_i)^T L_i (q - p_i) = (q - p_j)^T L_j (q - p_j)$$

$$q = (x, y) \rightarrow Ax^2 + By^2 + Cxy + Dx + Ey + K = 0$$

$$A = b^2(\cos^2\theta_i - \cos^2\theta_j) + a^2(\sin^2\theta_i - \sin^2\theta_j)$$

$$B = a^2(\cos^2\theta_i - \cos^2\theta_j) + b^2(\sin^2\theta_i - \sin^2\theta_j) \qquad D, E \ne 0$$

$$C = (a^2 - b^2)(\sin 2\theta_i - \sin 2\theta_j)$$

Appendix E

- What happens if $\theta_i = \theta_j = \theta$?

1. $F_i = \left[\begin{pmatrix} \frac{c}{a} & 0 \\ 0 & \frac{c}{b} \end{pmatrix} \begin{pmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{pmatrix} \right]$

   $F_i(t) = F_j(t) = F \qquad L_i(t) = L_j(t) = L$

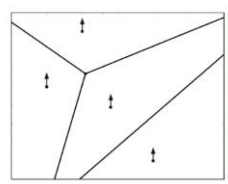
2. $Ax^2 + By^2 + Cxy + Dx + Ey + K = 0$

   $A = B = C = 0$

Appendix F

- What happens if $\theta_i = \theta_j = \theta$?

3. Partition's boundary becomes straight lines.



Appendix G

- Goal 1: Find the optimal configuration.

$$H(P, \Theta, W) = \sum_{i=1}^{n} \int_{W_i} f\left(\|q - p\|_{L_i}\right) \phi(q) dq$$

$$\min_{P,\Theta,W} H = \min_{P} H_{V^*}$$

$$f\left(\|q - p\|_{L_i}\right) = \|q - p_i\|_L^2$$

$$H_{V^*}(P) = \sum_{i=1}^{n} \int_{V_i^*} \|q - p_i\|_L^2 \phi(q) dq$$

$$\begin{aligned} \|q - p_i\|_L^2 &= (q - p_i)^T L_i (q - p_i) \\ &= (q - p_i)^T F_i^T F_i (q - p_i) \\ &= \{F_i (q - p_i)\}^T \{F_i (q - p_i)\} \\ &= \|F(q - p_i)\|^2 \end{aligned}$$

$$H_{V^*}(P) = \sum_{i=1}^{n} \int_{V_i^*} \|F(q - p_i)\|^2 \phi(q) dq$$

Appendix H

- Check the stability of the controller with LaSalle's Principle:

$$\dot{V}(Z) = \frac{d}{dt} H_{V^*} = \frac{dH_{\bar{V}}}{dt} = \sum_i \frac{\partial H_{\bar{V}}}{\partial z_i} \dot{z}_i$$

$$\begin{aligned} \dot{z}_i &= \bar{u}_i \\ &= -k\left(z_i - C_{\bar{V}_i}\right) \\ &= -kF\left(p_i - C_{V_i^*}\right) \end{aligned}$$

$$\begin{aligned} &= \sum_i \left\{ 2\left|\det(F^{-1})\right| M_{\bar{V}_i} \left(z_i - C_{\bar{V}_i}\right) \right\} \left\{ -k\left(z_i - C_{\bar{V}_i}\right) \right\} \\ &= -\sum_i 2k \left|\det(F^{-1})\right| M_{\bar{V}_i} \left(z_i - C_{\bar{V}_i}\right)^2 \\ &= -\sum_i 2k \left|\det(F^{-1})\right| M_{\bar{V}_i} F^2 \left(p_i - C_{V_i^*}\right)^2 \le 0 \end{aligned}$$