Toward the Adoption of A Nonlinear Observer into The Cooperative Control Systems
FL_07_17_1

Naoto Kobayashi

Outline

- Background
- A Nonlinear Observer to Estimate The Relative Rigid Body Motion
- Experiment
- Simulation
- Future Works

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Background

■Output Synchronization in SE(3)

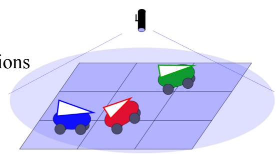
●information of neighbors' relative positions and orientations is necessary

■Experiment Systems

●birds eye camera

→ all agents' positions and orientations can be sensed

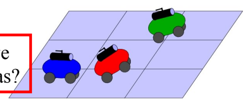
→ but not distributed



●mounted camera (local camera)

→ each agent can have its own "eyes"

→ distributed

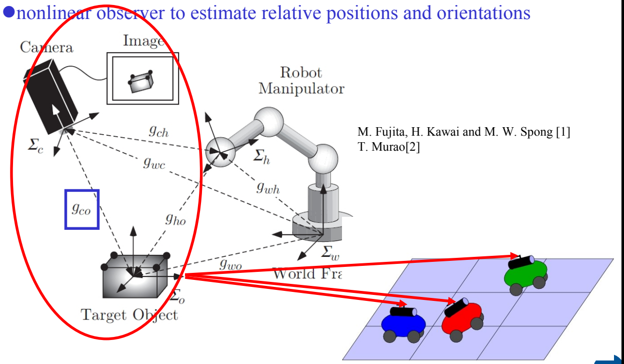


How can agents know their neighbors' relative positions and orientations by mounted cameras?

Background

■Visual Feedback

●nonlinear observer to estimate relative positions and orientations



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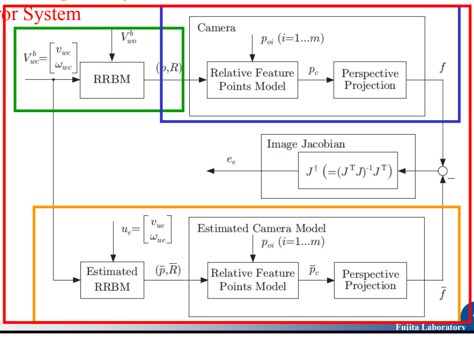
Nonlinear Observer -Outline-

■Outline

- ●Relative Rigid Body Motion
- ●Camera Model
- ●Estimated Relative Rigid Body Motion
- ●Estimation Error System



Nonlinear Observer -RRBM-

■Relative Rigid Body Motion

Σ_w :world frame

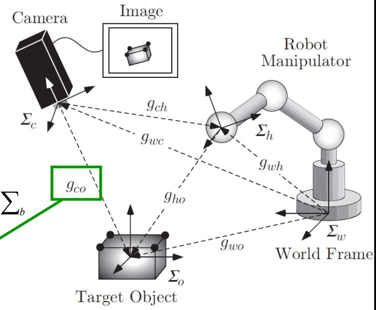
Σ_c :camera frame

Σ_o :object frame

$g_{ab} = (p_{ab}, e^{\hat{\xi}_{ab}})$

$= \begin{bmatrix} e^{\hat{\xi}_{ab}} & p_{ab} \\ 0 & 1 \end{bmatrix} \in SE(3)$

:configuration of a frame Σ_b relative to a frame Σ_a



$$g_{co} = g_{wc}^{-1} g_{wo}$$

$$\dot{g}_{co} = \dot{g}_{wc}^{-1} g_{wo} + g_{wc}^{-1} \dot{g}_{wo}$$

$$= -g_{wc}^{-1} \dot{g}_{wc} g_{wc}^{-1} g_{wo} + g_{wc}^{-1} \dot{g}_{wo} \quad ...(1)$$

Nonlinear Observer -RRBM-

■Relative Rigid Body Motion

$$\hat{V}_{ab}^b = g_{ab}^{-1} \dot{g}_{ab} = \begin{bmatrix} e^{-\hat{\xi}_{ab}} \dot{e}^{\hat{\xi}_{ab}} & e^{-\hat{\xi}_{ab}} \dot{p}_{ab} \\ 0 & 0 \end{bmatrix}$$

$$V_{ab}^b = \begin{bmatrix} v_{ab}^b \\ \omega_{ab}^b \end{bmatrix} = \begin{bmatrix} e^{-\hat{\xi}_{ab}} \dot{p}_{ab} \\ (e^{-\hat{\xi}_{ab}} \dot{e}^{\hat{\xi}_{ab}})^{\vee} \end{bmatrix}$$

$\hat{V}_{ab}^b \in se(3)$

$V_{ab}^b \in R^6$:body velocity

$v_{ab}^b \in R^3$:body translation velocity

$\omega_{ab}^b \in R^3$:body angular velocity

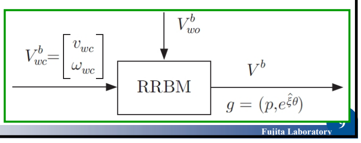
$$\dot{g}_{co} = -g_{wc}^{-1} \dot{g}_{wc} g_{wc}^{-1} g_{wo} + g_{wc}^{-1} \dot{g}_{wo} \quad ...(1)$$

$$= -\hat{V}_{wc}^b g_{co} + g_{co} \hat{V}_{wo}^b$$

$$\hat{V}_{co}^b = g_{co}^{-1} \dot{g}_{co} = -g_{co}^{-1} \hat{V}_{wc}^b g_{co} + \hat{V}_{wo}^b$$

$$V_{co}^b = -Ad_{(g_{co}^{-1})} V_{wc}^b + V_{wo}^b \text{ :RRBM model}$$

$$\mathrm{Ad}_{(g_{ab})} = \begin{bmatrix} e^{\hat{\xi}_{ab}} & \hat{p}_{ab} e^{\hat{\xi}_{ab}} \\ 0 & e^{\hat{\xi}_{ab}} \end{bmatrix}$$



Nonlinear Observer -Camera Model-

■Camera Model

$p_{oi} \in R^3$:position vector of the i-th feature point relative to Σ_o

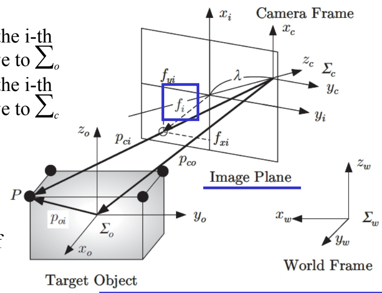
$p_{ci} \in R^3$:position vector of the i-th feature point relative to Σ_c

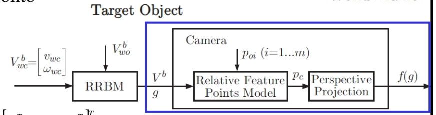
$$p_{ci} := [x_{ci} \quad y_{ci} \quad z_{ci}] = g_{co} p_{oi}$$

$f_i := [f_{xi} \quad f_{yi}]^T$

:**perspective projection** of the i-th feature point onto the **image plane**

$$f_i = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix}$$





$p_c = [p_{c1}^T \ \cdots \ p_{cm}^T]^T$, $f = [f_1^T \ \cdots \ f_m^T]^T$

Nonlinear Observer -EsRRBM-

■Estimated Relative Rigid Body Motion

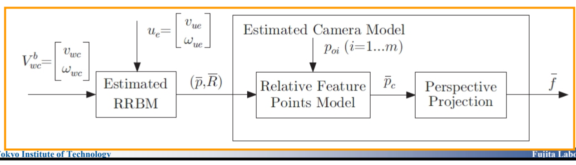
$$\bar{V}_{co}^b = -Ad_{(\bar{g}_{co}^{-1})} V_{wc}^b + u_e \text{ :EsRRBM model}$$

u_e :input to EsRRBM

$$\bar{p}_{ci} := [\bar{x}_{ci} \quad \bar{y}_{ci} \quad \bar{z}_{ci}] = \bar{g}_{co} p_{oi}$$

$$\bar{f}_i = \frac{\lambda}{\bar{z}_{ci}} \begin{bmatrix} \bar{x}_{ci} \\ \bar{y}_{ci} \end{bmatrix}$$

$\bar{p}_c = [\bar{p}_{c1}^T \ \cdots \ \bar{p}_{cm}^T]^T$, $\bar{f} = [\bar{f}_1^T \ \cdots \ \bar{f}_m^T]^T$



Nonlinear Observer -Estimation Error System-

■Estimation Error System

$g_{ee} = (p_{ee}, e^{\hat{\xi}_{ee}})$

$= \bar{g}_{co}^{-1} g_{co}$:estimation error

$p_{ee} = \bar{e}^{-\hat{\xi}_{co}} (p_{co} - \bar{p}_{co})$

$e^{\hat{\xi}_{ee}} = \bar{e}^{-\hat{\xi}_{co}} e^{\hat{\xi}_{co}}$

$e_e = [p_{ee}^T \quad e_R^T(e^{\hat{\xi}_{ee}})]^T$:estimation error vector

$* \ e_R(e^{\hat{\xi}_{ab}}) = \frac{1}{2}(e^{\hat{\xi}_{ab}} - e^{-\hat{\xi}_{ab}})^{\vee}$

$f - \bar{f} = J(\bar{g}_{co}) e_e$:estimation error of feature point

$$J_i(\bar{g}_{co}) = \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} \bar{e}^{\hat{\xi}_{co}} [I \quad -\hat{\bar{p}}_{oi}] \quad \text{:image jacobian}$$

$$J(\bar{g}_{co}) = [J_1^T(\bar{g}_{co}) \ \cdots \ J_m^T(\bar{g}_{co})]^T$$

Nonlinear Observer -Estimation Error System-

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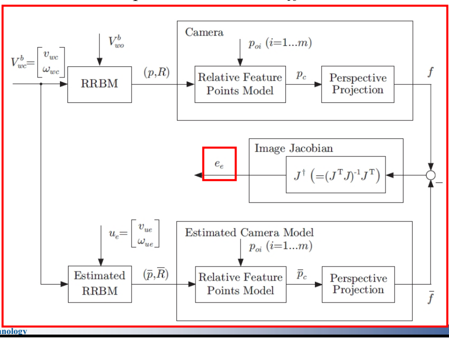
■Estimation Error System

$e_e = J^{+}(\bar{g}_{co})\,(f-\bar{f})$

*take 3 or more feature points so that $J(\bar{g}_{co})$ be column full rank



Nonlinear Observer -Estimation Error System-

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■Estimation Error System

$$V_{ee}^b = (g_{ee}^{-1}\dot{g}_{ee})^{\vee} = \begin{bmatrix} e^{-\hat{\xi}_{ee}}\dot{p}_{ee} \\ (e^{-\hat{\xi}_{ee}}\dot{e}^{-\hat{\xi}_{ee}})^{\vee} \end{bmatrix}$$

$V_{ee}^b = -\mathrm{Ad}_{(\bar{g}_{ee}^{-1})}u_e + V_{wo}^b$:estimation error motion model

■Passivity of the Estimation Error System

If $V_{wo}^b = 0$, then estimation error system satisfies $\int_o^T u_e^T\,(-e_e)dt \ge -\beta_e$

$*\ \beta_e > 0$

proof : Consider the following positive definite function

$$\mathcal{V} = \frac{1}{2}\|\, p_{ee}\|^2 + \phi(e^{\hat{\xi}_{ee}}) \qquad *\ \phi(e^{\hat{\xi}_{ab}}) = \frac{1}{2}\mathrm{tr}(I - e^{\hat{\xi}_{ab}})$$

differentiating ✞ with respect to time yields

$$\dot{\mathcal{V}} = p_{ee}^T\dot{p}_{ee} + e_R^T(e^{\hat{\xi}_{ee}})\omega_{ee}^s$$

$= \ldots.$ ...(2)

Nonlinear Observer -Estimation Error System-

Tokyo Institute of Technology

■Passivity of the Estimation Error System

$$\dot{\mathcal{V}} = p_{ee}^T\dot{p}_{ee} + e_R^T(e^{\hat{\xi}_{ee}})\omega_{ee}^s \quad ...(2)$$
$$= \ldots.$$
$$= u_e^T(-e_e)$$

integrating ✞ from 0 to T, we obtain

$$\int_0^T u_e^T(-e_e)\,d\tau = \int_0^T \dot{\mathcal{V}}d\tau$$
$$= \mathcal{V}(T) - \mathcal{V}(0) \ge -\mathcal{V}(0) \ge -\beta_e \quad ■$$

$u_e = Ke_e \qquad K > 0$

$\Rightarrow \dot{\mathcal{V}} < 0$

$\Rightarrow e_e \left(= \begin{bmatrix} p_{ee} \\ e_R(e^{\hat{\xi}_{ee}}) \end{bmatrix} \right) \to 0$

Nonlinear Observer -Estimation Error System-

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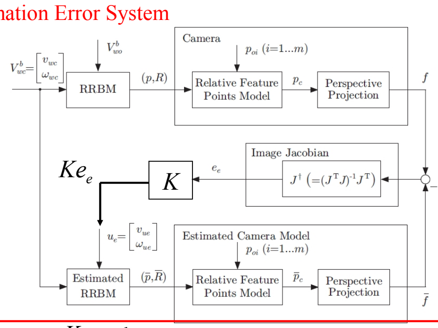
■Estimation Error System



γ :L2-gain

Choose $u_e = Ke_e$, then

- •Estimation error is asymptotically stable if $V_{wo}^b = 0$.
- •Estimation error is L2-gain stable if $V_{wo}^b \neq 0$, $K - \frac{1}{2\gamma^2}I - \frac{1}{2}I \ge 0$.

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Experiment

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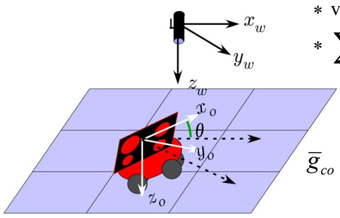
■Experiment of a nonlinear observer

- ●construct a nonlinear observer with simulink
- ●compare estimated position/orientation with measured real position/orientation



$*$ vehicle moves in R^2

$*\ \sum_w = \sum_c$

$$\bar{g}_{co} = \begin{bmatrix} c\bar{\theta}_{wo} & -s\bar{\theta}_{wo} & 0 & \bar{x}_{wo} \\ s\bar{\theta}_{wo} & c\bar{\theta}_{wo} & 0 & \bar{y}_{wo} \\ 0 & 0 & 1 & \bar{z}_{wo} \\ & 0 & & 1 \end{bmatrix}$$

Experiment

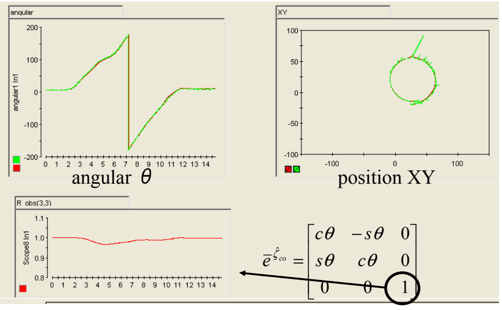
■Experiment of a nonlinear observer

●case1

360° slow rotation

red : estimated trajectory

green : measured trajectory



$$\bar{e}^{\hat{\xi}_{co}} = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Experiment

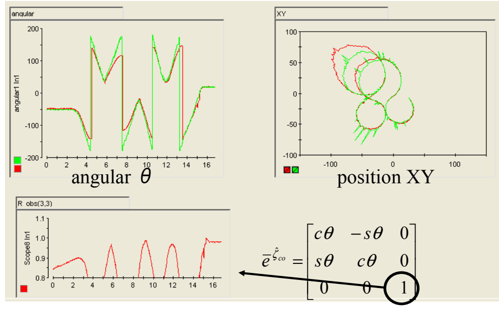
■Experiment of a nonlinear observer

●case2

random fast rotations

red : estimated trajectory

green : measured trajectory



$$\bar{e}^{\hat{\xi}_{co}} = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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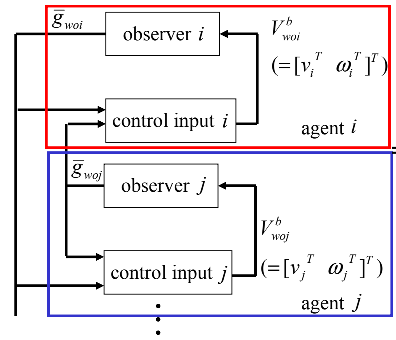
Simulation

■Simulation of Output Sunchronization with a Nonlinear Observer

●block diagram



$SO(3)$ ~~$SE(3)$~~ output synchronization

control input :

~~$v_i = \sum_{j \in N_i} \bar{e}^{\hat{\xi}_i}(p_j - p_i)$~~ constant

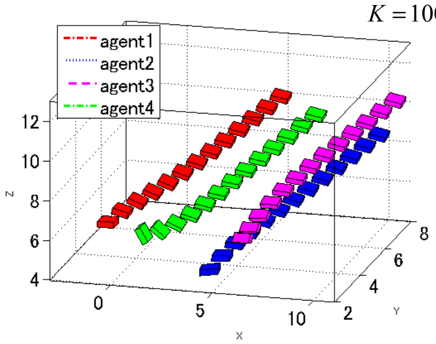
$\omega_i = \sum_{j \in N_i} \bar{e}^{-\hat{\xi}_i} \mathrm{sk}(\bar{e}^{-\hat{\xi}_i}\bar{e}^{\hat{\xi}_j})$

Simulation

■Simulation of Output Sunchronization with a Nonlinear Observer

●SO(3) Output Synchronization

$K = 100$



$u_e = K e_e$

K must be sufficiently large.

Otherwise, synchronization is not achieved.

Simulation

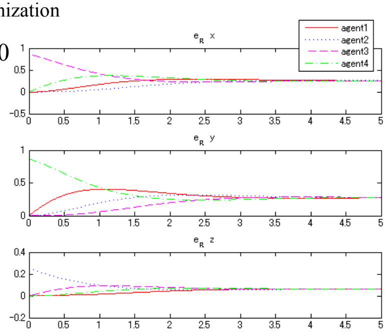
■Simulation of Output Sunchronization with Nonlinear Observer

●SO(3) Output Synchronization

$K = 100$

attitude error vector :

$\frac{1}{2}(e^{\hat{\xi}_i} - e^{-\hat{\xi}_i})^{\vee}$



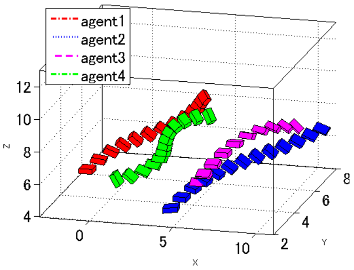
Simulation

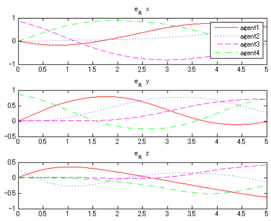
■Simulation of Output Sunchronization with Nonlinear Observer

●SO(3) Output Synchronization

$$u_e = Ke_e$$

$$K = 1$$





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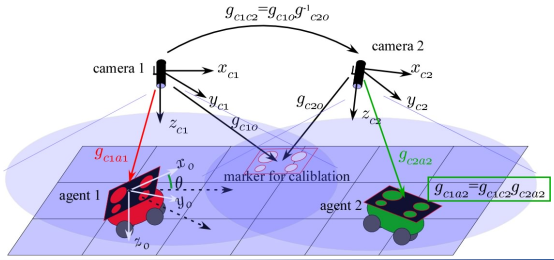
Future Works

■Future Works

●gain tuning
●singular point analysis
●adoption of a nonlinear observer into cooperative control experiment systems
●1 birds eye camera ⟶ 2 birds eye cameras



Future Works

■Future Works

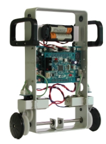
●4 vehicles ⟶ 8 vehicles（石野君？）
●birds eye camera ⟶ mounted camera(石野君？)

●consider the theory of the combination of cooperative control and nonlinear observer

$$V=\sum_{i=1}^{n}\left(\frac{1}{2}\| p_{eei}\|^2+\phi(e^{\hat{\xi}_{ee}})+\|\bar{p}_i\|^2+\phi(\bar{e}^{\hat{\xi}_i})\right) \quad \text{etc}$$

$$\dot{V}=\dots$$ now computing ...

●e-nuvo wheel（伊吹君）
●test bed setup
●open campus
●rengo
●KAIST



e-nuvo wheel

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