Cooperative Pursuit Problem and Some Multi-vehicle Experiments

FL07-15-2
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Outline

Cooperative Pursuit

Cooperative Pursuit

Introduction to Cooperative Pursuit Problem

Motivation

-Some predators hunt as a conjoined group
-Specialized behavior by maintaining a fixed formation during serch and capture of preys

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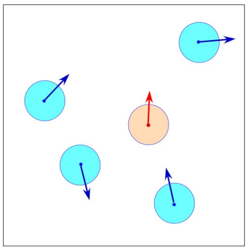
-There may be configurations that are preferred during group hunting

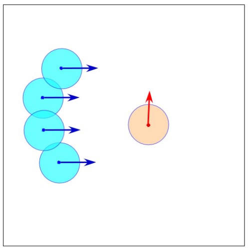
http://www.uoregon.edu/~afalcone/redwolves1.htm

Introduction to Cooperative Pursuit Problem

pursuit problem
: to determine a strategy for a team of pursuers to capture an evader

capture
: evader and some pursuer meet at the same location after a finite time





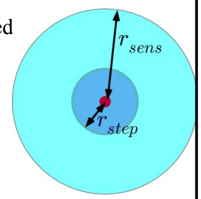
Problem Set-up

- pursuers can communicate the location of a sensed evader and their own position among themselves

- unbounded planar environment : R^2
- finite region $\Omega \in R^2$ where an evader appears with a uniform spatial density
- n pursuers

- evader/pursuers' sensing ability radius : r_{sens}
- evader/pursuers' motion ability radius : r_{step}

$$r_{sens} = \kappa r_{step}, \quad \kappa > 2\sqrt{2}$$



Problem Set-up

- equation of motion

$$\begin{cases} e[t+1] = e[t] + u^e(e[t], \{y^{p_k}[t]\}) \\ p_k[t+1] = p_k[t] + u^{p_k}(e[t], y^e[t+1], p[t]) \end{cases}$$

$e[t]$: absolute positions of the evader

$p_k[t]$: absolute positions of the k^{th} pursuer

$$y^{p_k}[t] = \begin{cases} p_k[t], & \text{if } \|p_k[t] - e[t]\| \le r_{sense} \\ \phi, & \text{otherwise.} \end{cases}$$

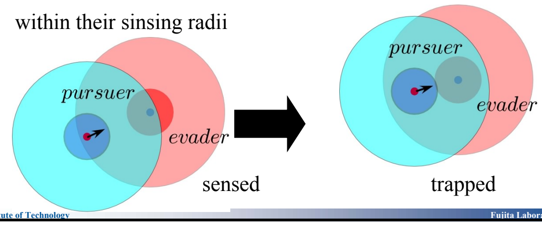
$$y^e[t+1] = \begin{cases} e[t+1], & \text{if for some } k \in \{1,\dots,n\}, \\ & \|p_k[t] - e[t+1]\| \le r_{sense} \\ \phi, & \text{otherwise.} \end{cases}$$

$u^e = 0$ until the evader is sensed by the pursuers for the first time

$\|u^e\|, \|u^{p_k}\| \le r_{step}$

Some Preliminaries

- capture : for any u^e, some pursuer is at the same position as the evader after a finite time
- trap : for any u^e, the motion disc of the evader is completely contained within the union of the sensing discs of the pursuers after finite time
- trapping time t^* : the time taken by the pursuers to trap the evader within their sinsing radii



Some Preliminaries

- trapping chain :

(i) $\{p_1, \dots, p_n\}$ are at the vertices of a convex polygon, and

(ii) for all $k \in \{1, \dots, n-1\}$, $(n \ge 3)$

$$\|p_k - p_{k+1}\| \le 2r_{step}\sqrt{\kappa^2 - 4}$$

- capture region :

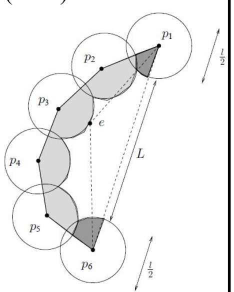
$$S[t] = \bigcup_{k \in \{2,\dots,n-1\}} \beta_{p_k}(r_{sens}) \cap \mathrm{Co}\{p_1, \dots, p_n\}[t]$$

- extended capture region :

$$S^e[t] = \bigcup_{k \in \{1,\dots,n\}} \beta_{p_k}(r_{sens}) \cap \mathrm{Co}\{p_1, \dots, p_n\}[t]$$

$$L = \|p_1 - p_n\| - 2r_{sens}$$

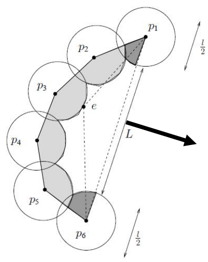
$$l = 4r_{sens}$$



trapping chain

Algorithms for Pursuit(sweep phase)

- three phases : sweep, pursuit and capture
- sweep phase : to sense an evader within the capture region of the trapping chain



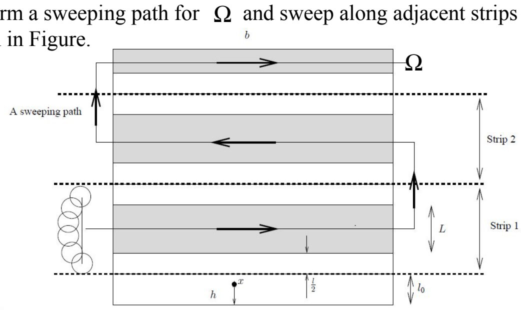
sweep Ω in the direction of the outward normal to p_1p_n ,with respect to the convex hull of the pursuers

Algorithms for Pursuit(sweep phase)

- algorithm SWEEP

(i) Choose the first rectangular strip at random distance, l_0, from one edge of Ω and sweep it lengh-wise. The distance l_0 is a uniform random variable taking values in the interval $[-\frac{l}{2}, L + \frac{l}{2}]$.

(ii) Form a sweeping path for Ω and sweep along adjacent strips as shown in Figure.

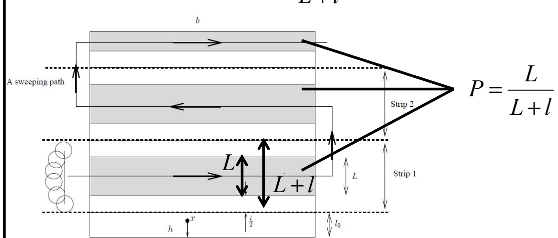


Algorithms for Pursuit(sweep phase)

Theorem (sweep property)

: For an evader located anywhere in Ω , the probability, P , of detecting it inside S for a group of pursuers in a trapping chain, using the sweeping policy, is given by

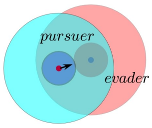
$$P = \frac{L}{L+l}$$



Algorithms for Pursuit(pursuit phase)

- pursuit phase : algorithm TRAP

: Once an evader has been detected within the capture region at time t_0 ,the pursuers need to ensure that the evader is trapped within their sensing ranges.



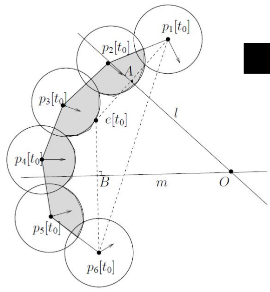
algorithm TRAP

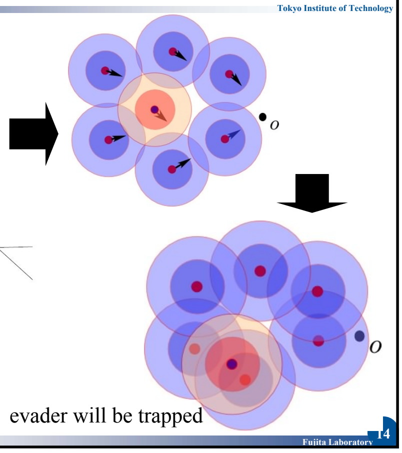
(i) While $e[t+1] \notin S^e[t]$, the pursuers move towards the circumcenter O of $\Delta p_1[t_0]e[t_0]p_n[t_0]$ with maximum step size.

(ii) Otherwise, one of the pursuers which senses the evader directly, moves with maximum step towards the evader and the others move parallel to that pursuer with the maximum step size.

Algorithms for Pursuit(pursuit phase)

algorithm TRAP





evader will be trapped

Algorithms for Pursuit(pursuit phase)

Theorem (TRAP)

: Starting from a trapping chain formation , the pursuers trap the evader within their sensing radii using algorithm TRAP, if $e[t_0] \in S[t_0]$. The trapping time t^* using algorithm TRAP satisfies,

$$t^* \leq \max_{k \in \{1,\dots,n\}} \left\lceil \frac{\|p_k[t_0] - O\|}{r_{step}} \right\rceil$$

where O is the circumcenter of $\Delta p_1[t_0]e[t_0]p_n[t_0]$.

Corollary

: At the end of algorithm TRAP, the evader is inside the convex hull of the pursuers.

Algorithms for Pursuit(capture phase)

- capture phase

: Now, pursuers have accsess to the next position of the evader at the present time instant. So the problem reduces to one having unlimited sensing capabilities for the pursuers.

We use algorithm SPHERES to caputure an evader.

algorithm SPHERES

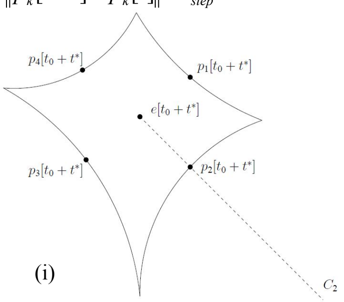
(i) Each pursuers p_k initially selects a point C_k such that,

- $p_k[t_o + t^*]$ lies on the line segment $C_k e[t_o + t^*]$ and
- The connected component of $R^2 \setminus \bigcup_{k=1}^{n} B_{C_k}(\|C_k - p_k[t_0 + t^*]\|)$ that contains $e[t_o + t^*]$ is bounded.

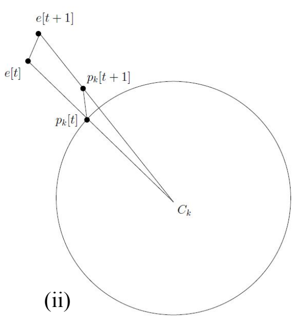
$t_o + t^*$: the time at the end of the pursuit phase

Algorithms for Pursuit(capture phase)

(ii) For every pursuer, choose $p_k[t+1]$ on line joining $e[t+1]$ and C_k such that $\|p_k[t+1] - e[t+1]\|$ is minimized, subject to $\|p_k[t+1] - p_k[t]\| \leq r_{step}$



(i)



(ii)

Algorithms for Pursuit(capture phase)

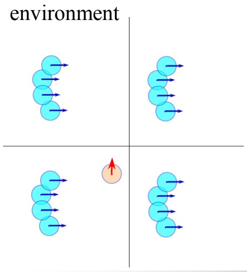
Theorem (SPHERES)

: Assume the evader lies within the convex hull of the pursuers. If every pursuer follows the algorithm SPHERES, then the evader will be captured in finite number of steps.

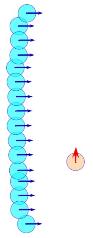
Allocate or Conjoin

Which is more efficient?

•form k groups of n pursuers each
•assign each group to a part of the environment



•form a single chain of kn pursuers



Allocate or Conjoin

- limiting trapping chain

(i) for all $k \in \{1,\dots,n-1\}$,

$$\|p_k - p_{k+1}\| = 2r_{step}\sqrt{\kappa^2 - 4} := d$$

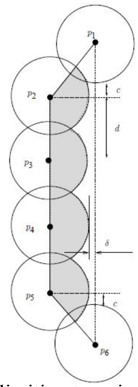
(ii) for all $k \in \{2,\dots,n-1\}$,

$$\mathrm{dist}(p_k, \overline{p_1 p_n}) = r_{sens} + \delta$$

proposition(limiting trapping chain property)
: For a limiting trapping chain,

$$P(n) = \frac{nd - (3d - 2c)}{nd - (3d - l - 2c)}$$

$P(n)$: probability of successful evader detection in the sweep phase



limiting trapping chain

Allocate or Conjoin

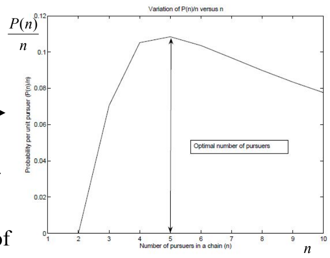
- probability of capturing the evader per pursuer : $\frac{P(N)}{N}$

$N := kn$: total number of pursuers
k : number of groups

$$\frac{P(N,k)}{N} = \frac{Nd - (3d - 2c)k}{N(Nd - (3d - l - 2c)k)}$$

•maximum when $k = 1$
• $N = n$

$$\frac{P(n)}{n} = \frac{nd - (3d - 2c)}{n(nd - (3d - l - 2c))}$$



There exists an optimal number of pursuers in a single chain.

Proof (TRAP)

Theorem (TRAP)
: Starting from a trapping chain formation , the pursuers trap the evader within their sensing radii using algorithm TRAP, if $e[t_0] \in S[t_0]$. The trapping time t^* using algorithm TRAP satisfies,

$$t^* \le \max_{k\in\{1,\dots,n\}} \left\lceil \frac{\|p_k[t_0] - O\|}{r_{step}} \right\rceil$$

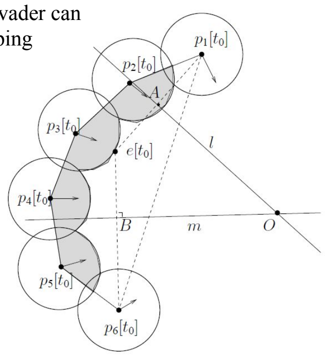
where O is the circumcenter of $\Delta p_1[t_0]e[t_0]p_n[t_0]$.

Proof
: (i) the evader cannot step outside $\mathrm{Co}\{p_1[t],\dots,p_n[t]\}$ by crossing $\overline{p_k[t]p_{k+1}[t]}$ because $\|p_k - p_{k+1}\| \le 2r_{step}\sqrt{\kappa^2 - 4}$

Proof (TRAP)

(ii) The best path for the evader is to move along $O - e[t_0]$ with maximum step size. Since $r_{sens} > r_{step}$, the sensing discs of pursuers p_1 and p_n overlap before the evader can reach O , thus closing the trapping chain around the evader.

The bound of t^* is the time taken by the furthest pursuer to reach O .

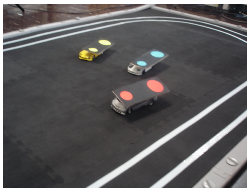


Reference

S.D.Bopardikar, F.Bullo and J.Hespanha, “Cooperative Pursuit with Sensing Limitations, ”Proc. of the 26th American Control Conference, pp. 5394-5399, 2007.

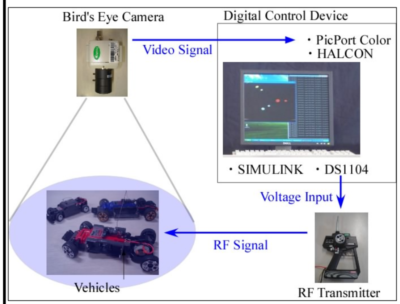
Experimental System Schmatic

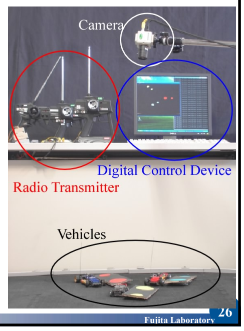
- vehicle : Mini-z (Kyosho)
- image processing : Halcon (MVTec Software GmbH)
- D/A transmitter , real-time workshop : DS1104 (dSPACE)
- model programing : SIMULINK (The Math Works)



Experimental System Schmatic

— Halcon, SIMULINK, Control Desk
— vehicle, RF transmitter, camera





Problem Set-up

- graph :

- control input :

$$\omega_1 = K_1\{e_R(e^{\hat{\xi}_{12}\theta_{12}}) + e_R(e^{\hat{\xi}_{10}\theta_{10}})\}$$

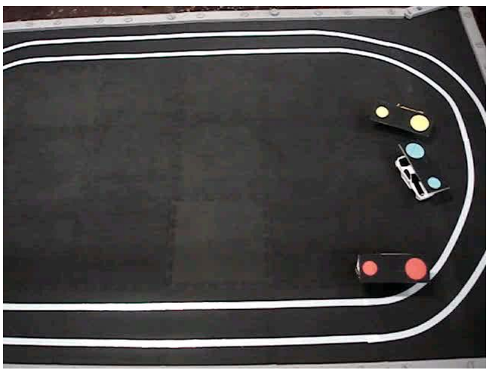
$$\omega_2 = K_2\{e_R(e^{\hat{\xi}_{21}\theta_{21}})\}$$

Experiments

- Leader Following 1

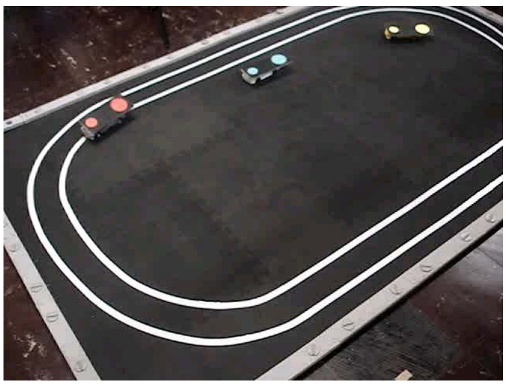
Experiments

- Leader Following 2



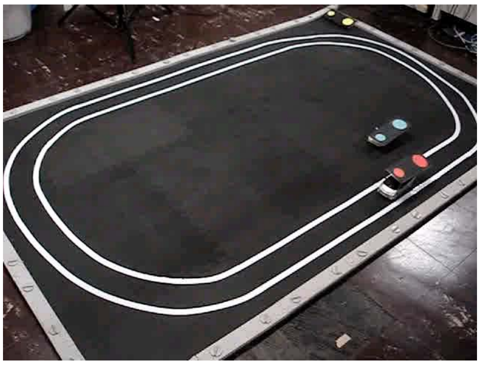
Experiments

- Leader Following 3



Experiments

- White-line + Leader Following



Future Works

- COE
- read the literatures of Coverage, MPC and so on.
- find an interesting theme for my study
- coverage experiments