3D Attitude Coordination Problem with Collision Avoidance

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Today's Topics

- Introduction
- Problem formulation
- Proposed controller
- Proof of convergence
- Conclusion
- Future work
- Reference

Introduction

- Multi Agent System
  - Same properties for each agent.
- 3D Attitude Coordination Problem
  - Have the same (face) direction at steady state.
  - 3D dimensions
- Collision Avoidance
  - Each agent doesn't have the same position at the same time.

Problem Formulation (1)

Agent Model

In this work, we will consider agent as a holonomic point-robot which means that the robot can be controlled via linear velocity and angular velocity (body frame) independently.

Note : Relationship between linear and angular velocity

Although we can control those two variables independently, the linear velocity and angular velocity (world frame) both depend on the same rotation matrix.

Problem Formulation (2)

Kinematic Model

The kinematic model for each agent is represented as

$$\dot{q}_i = e^{\hat{\zeta}_i} v_i \quad (1)$$

$$\dot{\zeta}_i = e^{\hat{\zeta}_i} w_i \quad (2)$$

$$\zeta_i = \theta_i \xi_i \quad i = 1, \ldots, n \quad (3)$$

when $e^{\hat{\zeta}_i} \in SO(3)$

$q_i \in R^3$ *is the position in a world frame.*

$v_i \in R^3$ *is the linear velocity in a body frame.*

$w_i \in R^3$ *is the angular velocity in a body frame.*

Problem Formulation (3)

Definition 1

A group of n-agent is said to achieve attitude coordination problem, if

$$\lim_{t \to \infty} \left(e^{\hat{\zeta}_i} - e^{\hat{\zeta}_j} \right) = 0, \forall j \neq i. \quad (4)$$

Definition 2

Two agents are said to be collided, if

$$q_{i,t} = q_{j,t}, \forall j \neq i. \quad (5)$$

Problem Formulation (4)

Sensing Region Set

Sensing region is the sphere space that the agent can detect others. It was defined as $N_i = \{k \mid \|q_i - q_k\| < r_t\}$.

Collision Avoidance Region Set

Collision avoidance region is the sphere space that the agent will try to avoid other agents, which are in the same region. It was defined as $M_i = \{k \mid \|q_i - q_k\| < r_s\}$.

Problem Formulation (5)

Assumption

1. An agent can detect other agents and then form the strongly connected graph for connection.
2. The communication between agent is two-way direction (undirected graph).
3. Sensing region is bigger than collision avoidance region ($r_t > r_s$).

Proposed Controller

Theorem

Consider a space R^3 with n agents by specified agent model. From the assumption, the agent will achieve attitude coordination problem with collision avoidance property using the following controller

$$\begin{aligned} When\ r_s \le \|q_i - q_k\| < r_t\ :\quad v_i &= c \\ w_i &= k_1 \sum_{j \in N_i \setminus M_i} skew(e^{-\hat{\zeta}_i} e^{\hat{\zeta}_j}) \end{aligned}$$

which $k_1 > 0$ (6)

$$\begin{aligned} When\ \|q_i - q_k\| < r_s\ :\quad v_i &= -e^{-\hat{\zeta}_i} \sum_{k \in M_i} \frac{\partial V_{ik}}{\partial q_i} \\ w_i &= 0 \end{aligned} \quad (7)$$

Proof (1)

For collsion avoidance purpose, we use the following potential function:

$$V_{ik} = \begin{cases} \frac{a}{d} & d < r \\ h(d^2 - r_s^2)^2 & r \le d < r_s \\ 0 & d \ge r_s \end{cases}$$

when d is defined as $\|q_i - q_k\|$

a r and h are chosen to guarantee that V_{ik} is continuously differentiable at point $d = r$ and $d = r_s$ which $r_s^2 = 3r$ and $a = 4hr^3$.

Proof (2)

The derivative of V_{ik} are computed as

$$\dot{V}_{ik} = \begin{cases} -\frac{a}{d^3}(q_i - q_k)\dot{q}_i & d < r \\ -4h(r_s^2 - d^2)(q_i - q_k)\dot{q}_i & r \le d < r_s \\ 0 & d \ge r_s \end{cases}$$

when d is defined as $\|q_i - q_k\|$

Now consider a candidate Lyapunov function for multi-agent system:

$$V_i = \frac{1}{2} tr(I - e^{\hat{\zeta}_i}) + \sum_{k \in N_i} V_{ik}$$

Proof (3)

When $d \ge r_s$ (out of collision avoidance region):

$$\begin{aligned} V_i &= \frac{1}{2} tr(I - e^{\hat{\zeta}_i}) \\ \because V &= \sum_{i=1}^{n} V_i = \frac{1}{2} \sum_{i=1}^{n} tr(I - e^{\hat{\zeta}_i}) \\ \therefore \dot{V} &= \sum_{i=1}^{n} skew(e^{\hat{\zeta}_i})^{\vee} w_i \end{aligned}$$

Proof (4)

Apply controller in (6);

$$\dot{V} = \sum_{i=1}^{n} skew(e^{\hat{\zeta}_i})^\vee (e^{-\hat{\zeta}_i} e^{\hat{\zeta}_j})^\vee$$

From the idea in [Igarashi, 2007]; We get

$$\begin{aligned} \dot{V} &\leq -k_1 \sum_{i=1}^{n} \sum_{j \in N_i} \frac{1}{2} \lambda_{\min}(e^{\hat{\zeta}_i} + e^{-\hat{\zeta}_i}) tr(I - e^{-\hat{\zeta}_i} e^{\hat{\zeta}_j}) \\ &\leq 0 \end{aligned}$$

Using Lasalle's invariance principle, we can prove that all agents reach the same attitude at steady state.

Proof (5)

When $r \leq d < r_s$:

$$\begin{aligned} V_i &= \frac{1}{2} tr(I - e^{\hat{\zeta}_i}) + \sum_{k \in M_i} h(d^2 - r_s^2)^2 \\ \dot{V} &= \sum_{i=1}^{n} \left(skew(e^{\hat{\zeta}_i})^\vee w_i - \left(\sum_{k \in M_i} 4h(r_s^2 - d^2)(q_i - q_k)^T \right) e^{\hat{\zeta}_i} v_i \right) \end{aligned}$$

From (2), apply the controller in (7)

$$\dot{V} = \sum_{i=1}^{n} \left\{ 0 - \| \sum_{k \in M_i} 4h(r_s^2 - d^2)(q_i - q_k) \|^2 \right\}$$

Proof (6)

$$\begin{aligned} \dot{V} &= -16h^2 \sum_{i=1}^{n} \sum_{k \in M_i} (r_s^2 - d^2)^2 d^2 \\ &< 0 \end{aligned} \qquad (8)$$

And then consider when $d < r$:

$$\begin{aligned} V &= \sum_{i=1}^{n} \left(\frac{1}{2} tr(I - e^{\hat{\zeta}_i}) + \sum_{k \in M_i} \left(\frac{a}{d} \right) \right) \\ \dot{V} &= \sum_{i=1}^{n} \left\{ skew(e^{\hat{\zeta}_i})^\vee w_i - \sum_{k \in M_i} \left(\frac{a}{d^3} (q_i - q_k)^T e^{\hat{\zeta}_i} v_i \right) \right\} \end{aligned}$$

Proof (7)

Apply the controller in (7) :

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{n} \left\{ 0 - \| \sum_{k \in M_i} \frac{a}{d^3} (q_i - q_k) \|^2 \right\} \\ &= -\sum_{i=1}^{n} \sum_{k \in M_i} \frac{a^2}{d^4} \\ &< 0 \end{aligned} \qquad (9)$$

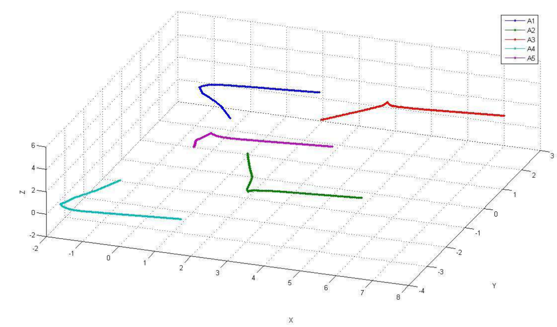
Proof (8)

From the fact that $0 < tr(I - e^{\hat{\zeta}_i}) < 2$ and assume that at the initial state, the collsion didn't occur, therefore, $V(0, q) < \infty$. From (8),(9), so $V(t, q) < V(0, q) < \infty$ and thus collision avoidance was proved.

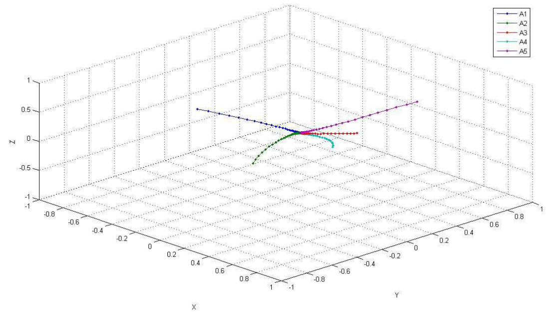
Note: Collsion condition

$$V(t, q) \rightarrow \infty \Leftrightarrow \|q_i - q_k\| \rightarrow 0$$

Simulation : Position



Simulation : ζ



Conclusion and Future works

Conclusion

In this presentation, we have proposed the controller for achieving the flocking problem with the property of collision avoidance. It shows that the system will make each agent avoids the collision and converges to the same direction.

Future works

Develop the controller to handle both linear and angular velocity at the same time so the agents converge to last value faster.

References

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