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Optimal Search Problem based on Agent System and Sensor Accuracy (Introduction)

エージェントのシステムとセンサ精度に基づく最適探索問題 (Introduction)

FHL07-10-2

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1-1. Search Problem

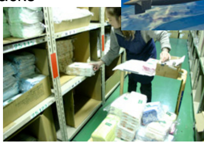
The goal

To maximize the probability of locating the target
(To minimize the time to find the target)
deploying agents with the resources available.

targetを捜し出す確率を最大にするために（発見する時間を最小にするために）
利用可能な資源のもとでagentを展開する.

The work

- protection against submarine attacks
- search and rescue operations
- detecting lost objects
- clearing of land mines
- location parts in a warehouse
- medicines, mining, …etc.



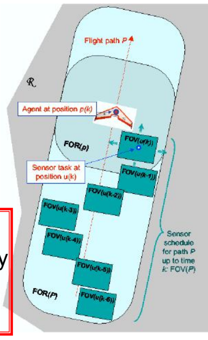
1-2. Method for Search Problem

James R. Riehl, et al., Cooperative Graph-Based Model Predictive Search,2007

cooperative search algorithm
plans paths and sensor schedules
for a team of agents

⇩

in finite time
target detection with probability one



Not consider agent system explicitly.
Concrete method for optimal sensing policy is not proposed.
Not consider sensor accuracy.

2-1. Motivation of this study

The subject

efficient search
- high probability of locating the target in finite horizon
- low cost for movement

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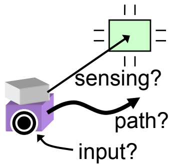
- sensing policy
  based on sensor accuracy
  - distance from agent
  - sensor performance
  - environment factor
- path planning
- input



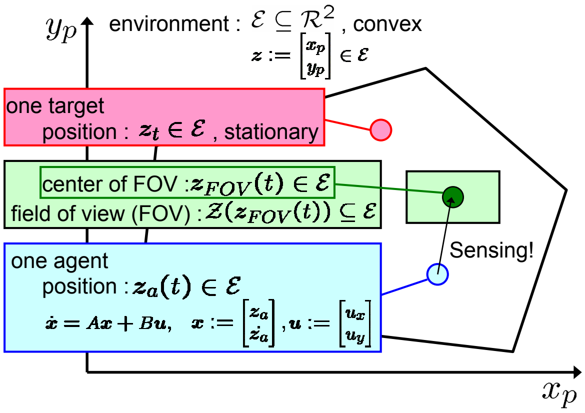
2-2. This Presentation

the concept (direction) of my study

one consequence for optimal search method with sensor accuracy

If a method guarantees a proposed condition,
then the target is found in a finite time.

3-1. Problem Setting

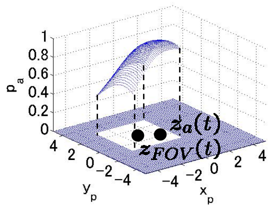


3-2. Sensor Accuracy

probability of sensing $\boldsymbol{z}$ accurately from $z_a(t)$ with $\mathcal{Z}(z_{FOV}(t))$

$p_a(\tilde{z}(t)) \in [0,1], \quad \tilde{z}(t) := [z, z_a(t), z_{FOV}(t)]$

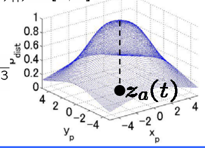
$$p_a(\tilde{z}(t)) := \begin{cases} 0 & \text{if } z \notin \mathcal{Z}(z_{FOV}(t)) \\ p_{dist}(\|z - z_a(t)\|) & \text{if } z \in \mathcal{Z}(z_{FOV}(t)) \end{cases}$$



probability of sensing $\boldsymbol{z}$ accurately depend on the distance $\|z - z_a(t)\|$

$p_{dist}(\|z - z_a(t)\|) \in [0,1]$

e.g. $\frac{1}{1+\frac{1}{10}\|z-z_a(t)\|^3}$



3-3. Sensing

probability of locating the target up to time t_k

$\delta(z)P_k(z) \quad \delta(z) := \begin{cases} 1 & \text{if } z = z_t \\ 0 & \text{if } z \neq z_t \end{cases} \quad P_k(z) \in [0,1]$

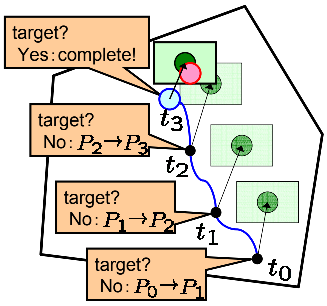
$P_{k+1}(z) = 1 - (1 - P_k(z))(1 - p_a(\tilde{z}(t_k)))$

⇨ $P_{k+1}(z) \geq P_k(z)$

The subject

$\delta(z_t)P_k(z_t) = 1$

But, z_t is unknown to agent



4. Theorem 1

Theorem 1

If a method (algorithm, procedure) guarantees

$\inf_{z \in \mathcal{E}} P_k(z) \to 1$ as $t \to t_k$ ------ (1)

or

$\inf_{z \in \mathcal{E}} P_k(z) = 1$ at time t_k, ------ (2)

then the target is found up to time t_k.

proof

(1) $\Rightarrow \quad P_k(z_t) \to 1 \quad \Rightarrow \quad \delta(z_t)P_k(z_t) \to 1$

$(\because P_k(z) \leq 1) \qquad (\because \delta(z_t) = 1)$

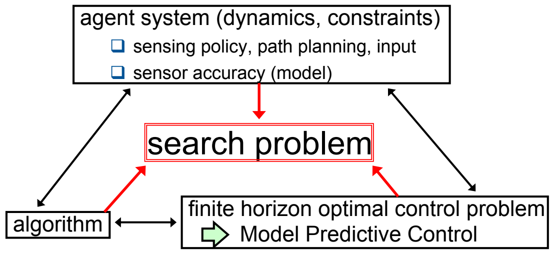
(2) $\Rightarrow \quad P_k(z_t) = 1 \quad \Rightarrow \quad \delta(z_t)P_k(z_t) = 1$

$(\because P_k(z) \leq 1) \qquad (\because \delta(z_t) = 1)$

Therefore, the target is found up to time t_k. □

5. Conclusions and Future works

the concept (direction) of my study



one consequence for optimal search method with sensor accuracy

If a method guarantees a proposed condition, then the target is found in a finite time.

6-1-1. Example (Problem Setting)

$p_{dist}(\|z - z_a(t)\|) = \frac{1}{1+\|z-z_a(t)\|^3}$

environment : $\mathcal{E} \subseteq \mathcal{R}^2$

discretize → $\mathcal{E}_d = \{1, 2, \cdots, 10\}^2$

$z_{FOV}(t) = z_a(t_k), t \in [t_k, t_{k+1})$

$\mathcal{Z}(z_a(t_k)) = \{z \in \mathcal{E}_d \,|\, \|z_a(t_k) - z\| < 1.5\}$

one agent

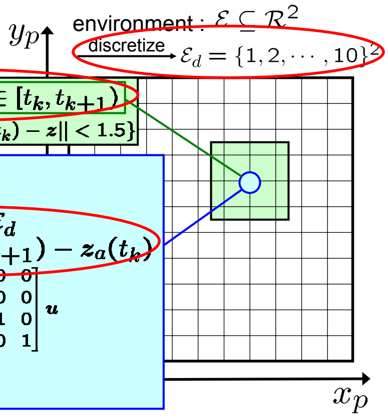
position : $z_a(t) \in \mathcal{E}$

$z_a(t_k) \in \mathcal{E}_d$

$\dot{z}_a(t_{k+1}) = z_a(t_{k+1}) - z_a(t_k)$

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u$$

$$x := \begin{bmatrix} z_a \\ \dot{z}_a \end{bmatrix}, \quad u := \begin{bmatrix} u_x \\ u_y \end{bmatrix}$$



6-1-2. Optimal Control Problem

optimal control problem

at time t_k

optimal $z_{FOV}(t)$, $x(t)$, $u(t)$, $t \in [t_k, t_{k+1}]$

sensing policy, path planning, input

⇩ in this example

given $x(t_k) = [z_a(t_k), \dot{z}_a(t_k)]^T, P_k(z)$

find $u(t), t \in [t_k, t_{k+1}]$, $x(t_{k+1}) = [z_a(t_{k+1}), z_a(t_{k+1}) - z_a(t_k)]^T$

minimize

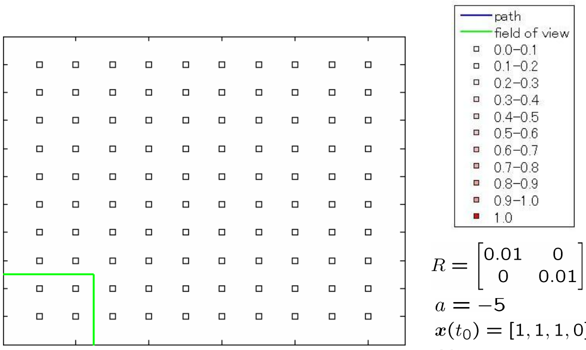
$$J(z_a(t_{k+1})) = \int_{t_k}^{t_{k+1}} u^T(t)Ru(t)dt + P_k(z_a(t_{k+1}))$$

$$= \phi(x(t_k), x(t_{k+1})) + P_k(z_a(t_{k+1}))$$

→ $\mathcal{O}(n_z)$

the number of lattice point $z \in \mathcal{E}_d$

6-2-1. Simulation 1



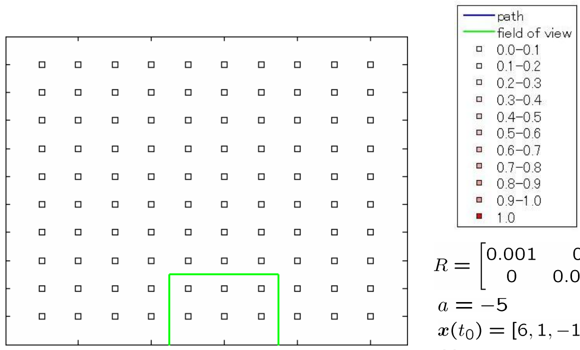
$R = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$

$a = -5$

$\boldsymbol{x}(t_0) = [1, 1, 1, 0]^T$

60 steps

6-2-2. Simulation 2



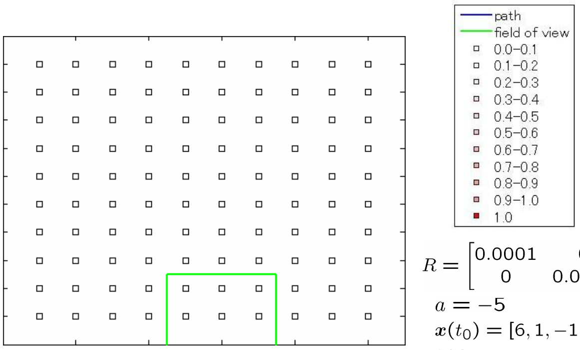
$R = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix}$

$a = -5$

$\boldsymbol{x}(t_0) = [6, 1, -1, 0]^T$

60 steps

6-2-3. Simulation 3



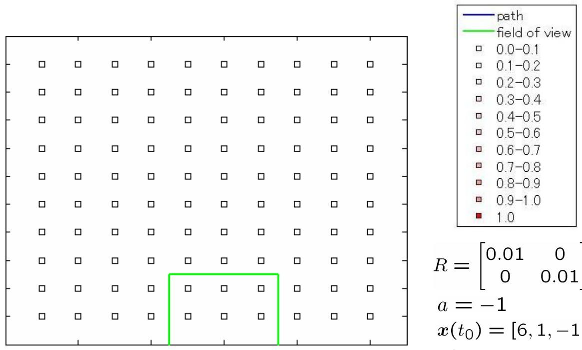
$R = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix}$

$a = -5$

$\boldsymbol{x}(t_0) = [6, 1, -1, 0]^T$

100 steps

6-2-4. Simulation 4



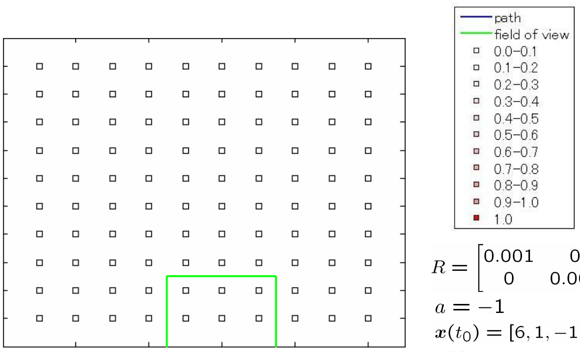
$R = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$

$a = -1$

$\boldsymbol{x}(t_0) = [6, 1, -1, 0]^T$

40 steps

6-2-5. Simulation 5



$R = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix}$

$a = -1$

$\boldsymbol{x}(t_0) = [6, 1, -1, 0]^T$

100 steps