Probabilistic Design of Robust Dual-Mode MPC without Set Invariance

FL07-09-2
Tatsuya Miyano

Outline

1. Introduction
2. Feasibility Region
3. Dual-Mode MPC Algorithm
4. Numerical Simulations
5. Conclusions and Future Works

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Previous Works

・MPC has been utilized in slow systems (e.g. chemical processes).

・There are some attempts to apply MPC to fast systems.
 ・Multi-parametric (MP) techniques: e.g. Bemporad et.al (2000)

These techniques enable us to trade off on-line complexity with off-line computation.

⇒ These techniques require to solve NP-hard MP program !

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These techniques enable us to trade off on-line complexity with off-line computation.

⇒ These techniques require to solve NP-hard MP program !

 ・Robust dual mode MPC based on ellipsoids: e.g. Imsland et.al (2006)

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Problem Formulation

Tokyo Institute of Technology

$$x(t+1) = A(\Delta)x(t) + B(\Delta)u(t)$$

State and input constraints: $x(t) \in \mathcal{X}, u(t) \in U, \forall t \in \mathcal{Z}_+$

$\Delta \in \mathcal{D}$ is a random parameter on the probability space $(D, \mathcal{D}, \mathrm{Prob}_\Delta)$.

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$u(t+k \mid t) = -Kx(t+k \mid t) + c(k)$ Mode 1

Prediction time Current time

if $k \in \{0, \ldots, n_c - 1\}$

$u(t+k \mid t) = -Kx(t+k \mid t)$ Mode 2

if $k \geq n_c$

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$u(t+k \mid t) = -Kx(t+k \mid t)$ if $k \geq n_c$ Mode 2

K minimizes the infinite horizon LQ cost, J_{LQ}, with weights $Q > 0, R > 0$ in the absence of system constraints.

Feasibility Region

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Prediction Model

$$x(t+k+1 \mid t) = \Phi(\Delta)x(t+k \mid t) + B(\Delta)c(k)$$

$$u(t+k \mid t) = -Kx(t+k \mid t) + c(k)$$

$$c(k) = 0 \quad \forall k \geq n_c \qquad \Phi(\Delta) := A(\Delta) - B(\Delta)K$$

The following augmented model is equivalent to the above prediction model.

$$\xi(k+1) = \tilde{A}(\Delta)\xi(k) \qquad \xi(k) := [x^{\mathrm{T}}(t+k \mid t) \quad \tilde{c}^{\mathrm{T}}(k)]^{\mathrm{T}}$$

$$u(k) = \tilde{C}\xi(k) \qquad \tilde{c}(k) := [c^{\mathrm{T}}(k) \quad c^{\mathrm{T}}(k+1) \quad . \quad . \quad . \quad c^{\mathrm{T}}(k+n_c-1)]^{\mathrm{T}}$$

$$\tilde{A}(\Delta) := \begin{bmatrix} A(\Delta) - B(\Delta)K & B(\Delta)E \\ 0 & I_L \end{bmatrix} \quad \tilde{C} := [-K \quad E] \quad E := [I \quad 0 \quad . \quad . \quad . \quad 0]$$

I_L :nilpotent matrix

Augmented Model

Feasibility Region

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Definition:Feasibility Region (FR)

A set $\mathcal{F}$ is said to be an FR if its any element $\xi(0) \in \mathcal{F}$ satisfies $\xi(k) \in \mathcal{X} \times U \quad \forall \Delta \in D, k \in \mathcal{Z}_+$.

We employ an ellipsoidal feasibility region (EFR) to reduce on-line complexity.Then we obtain EFR $\Omega(Q_P)$.

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$$\Omega(Q_P) := \{\xi \mid \xi^{\mathrm{T}} Q_P^{-1} \xi \leq 1\}$$

State and input constraints: $M_0(\Delta)(i,:)Q_P M_0^{\mathrm{T}}(\Delta)(i,:) \leq 1$

$$\forall \Delta \in D \quad M_0(\Delta) := \begin{bmatrix} M_0(\Delta) \\ [M_x \quad 0]\tilde{A}^k(\Delta) \\ M_u \tilde{C}\tilde{A}^k(\Delta) \end{bmatrix}$$

State constraints

Input constraints

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Optimization Problem

We define a cost $J(\xi) := \xi^{T} P \xi$. Then an optimization problem is as follows.

Off-line

T : projection matrix to x - space

$$\min_{Q_P, P} \ln \det(TQ_PT^{T})^{-1} + \mathrm{tr}(P)$$

$$M_0(\Delta)(i,:)Q_P M_0^{T}(\Delta)(i,:) \le 1 \quad P - \tilde{A}^{T}(\Delta)P\tilde{A}(\Delta) > \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix} + \tilde{C}^{T}R\tilde{C}$$

$\forall \Delta \in D$

On-line

$$\min_{\tilde{c}} J(x(t), \tilde{c}(t)) \quad \text{subject to } [x^{T}(t) \quad \tilde{c}^{T}(t)]^{T} \in \Omega(Q_P)$$

$$\tilde{c}(t) = \begin{cases} c_0 & \text{if } [x^{T}(t) \quad c_0^{T}]^{T} \in \Omega(Q_P) \\ c_a & \text{otherwise} \end{cases}$$

c_0 is an analytical solution in the absence of system constraints.
c_a is an analytical solution with system constraints.

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In case

i) $[x^{T}(t+1) \quad \tilde{c}^{T}(t+1)]^{T} \notin \Omega(Q_P)$ — Infeasible

ii) $J(\xi(t+1)) > J(x(t+1), [c^{T}(t+1) \quad c^{T}(t+2) \quad . \ . \ . \quad c^{T}(t+n_c-1) \quad 0]^{T})$ — Asymptotic stability is not proved

$$\tilde{c}(t+1) = [c^{T}(t+1) \quad c^{T}(t+2) \quad . \ . \ . \quad c^{T}(t+n_c-1) \quad 0]^{T}$$

Scenario Approach

Off-line

$$\min_{Q_P, P} \ln \det(TQ_PT^{T})^{-1} + \mathrm{tr}(P)$$

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$\forall \Delta \in D$

The off-line computation requires the solution of an NP-hard robust convex program.

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The probability that P and Q_P do not satisfy constraints with probability greater than $1-\varepsilon$ is less than δ.

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Numerical Simulations

We apply a dual-mode MPC strategy to SICE-DD ARM.

The model is as follows. It has uncertain physical parameters.

$$\dot{x}(t)=\begin{bmatrix}0 & 1\\ 0 & -\frac{B}{M+2R}\end{bmatrix}x(t)+\begin{bmatrix}0\\ \frac{1}{M+2R}\end{bmatrix}u(t) \qquad x(t):=\begin{bmatrix}\theta_1\\ \dot{\theta}_1\end{bmatrix} \quad u(t):=\tau_1$$

$M := I_1 + I_2 + m_1 r_1^2 + m_2(l_1^2 + r_2^2)$

$R := m_2 l_1 r_2$

B : viscosity resistance of link1

m_i : mass of link$i$

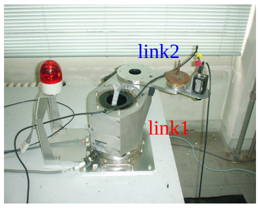
I_i : moment of inertia of link$i$

l_i : length of link$i$

r_i : length to center of gravity of link$i$

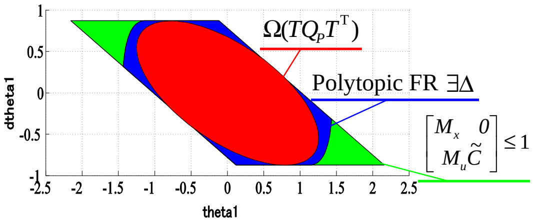
θ_i : angle of link$i$ τ_i : torgue of link$i$

$\dot{\theta}_i$: angular velocity of link$i$



Numerical Simulations

We tried to demonstrate the effectiveness of our method through the numerical simulations. But in fail.



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Conclusions and Future Works

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・Future works

- ・More meaningful numerical simulations
- ・Extension to exact-linearized or linearized model