Coverage Control Problem with Anisotropic Sensors – part I

FL07-08-2

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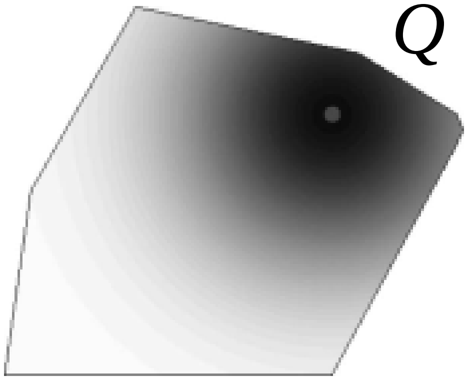
Outline

- Coverage Problem Review (Proof)
- Problem Setting of Anisotropic case
- Anisotropic Voronoi Partition
- Future Works

Coverage Problem - Review

- Objective : Given agents ($p_1,\ldots,p_n$), convex environment Q, achieve optimal coverage.



- Let ϕ be density function
- Let f be sensing performance (non-decreasing)

  $f(\|q-p_i\|)$: how poor p_i to sense q

- Objective function :

$$H(P,W)=\int_Q f(\|q-p_i\|)\phi(q)\,dq$$

$$=\sum_{i=!}^{n}\int_{W_i} f(\|q-p_i\|)\phi(q)\,dq$$

Cortes et al. (2004), IEEE Trans. Robotics and Automation

Coverage Problem - Review

- For $f(\|q-p_i\|)=\|q-p_i\|^2$ and dynamics $\dot{p}_i = u_i$

$$u_i = -k\left(p_i - C_{V_i}\right) \qquad (1)$$

- Assume that $V(P)=\{V_1,\cdots,V_n\}$ is continuously updated

Proposition (Continuous-time Lloyd descent) : For the closed loop system induced by (1), the sensors location converges asymptotically to the set of critical points of H, i.e., the set of centroidal Voronoi configurations on Q . Assuming this set is finite, the sensors location converges to a centroidal Voronoi configuration.

Cortes et al. (2004), IEEE Trans. Robotics and Automation

Proof :

$$\frac{\partial H}{\partial p_i}(P) = \int_{V_i(P)} \frac{\partial}{\partial p_i} f\left(\left\| q - p_i \right\|\right) \phi(q) dq$$

V_i is continuously updated

$$H(P) = -\sum_{i=1}^{n} \int_{V_i(P)} \left\| q - p_i \right\|^2 \phi(q) dq$$

$$\frac{\partial H}{\partial p_i}(P) = 2\int_{V_i(P)} (q - p_i)\phi(q) dq = 2M_{V_i(P)}\left(CM_{V_i(P)} - p_i\right)$$

$$\frac{d}{dt} H(P(t)) = \sum_{i=1}^{n} \frac{\partial H_v}{\partial p_i} \dot{p}_i$$

$$= -2k \sum_{i=1}^{n} M_{V_i(P)} \left\| p_i - CM_{V_i(P)} \right\|^2$$

- By LaSalle’s principle, the sensors location converges to the set of centroid Voronoi configurations.

Coverage Problem - Review

- If the set is a finite collection of points, then P(t) converges to one of them

LaSalle's principle : Let $\Omega \subset D$ be a compact set that it is positively invariant with respect to X. Let $x(0) \in M$ and x_* be an accumulation point of *x(t)*. Then $x_* \in M$ and $\mathrm{dist}(x(t), M) \to 0$ as $t \to \infty$

Corollary : If the set *M* is a finite collection of points, then the limit of *x(t)* exists and equals one of them

Coverage Problem - Review

$$\frac{\partial H(P)}{\partial p_1} = \int_0^{\frac{p_1+p_2}{2}} \frac{\partial}{\partial p_1} \left\| q - p_1 \right\|^2 dq$$

1 dimension 2 agents, Q=[0,1]

Proof :

$$\frac{\partial H(P)}{\partial p_1} = \frac{\partial}{\partial p_1} \int_0^{\frac{p_1+p_2}{2}} \left\| q - p_1 \right\|^2 dq + \frac{\partial}{\partial p_1} \int_{\frac{p_1+p_2}{2}}^{1} \left\| q - p_2 \right\|^2 dq$$

$$\frac{\partial}{\partial p_1} \int_0^{\frac{p_1+p_2}{2}} \left\| q - p_1 \right\|^2 dq = \lim_{h\to 0} \left(\int_0^{\frac{p_1+h+p_2}{2}} \left\| q - p_1 + h \right\|^2 dq - \int_0^{\frac{p_1+p_2}{2}} \left\| q - p_1 \right\|^2 dq \right)$$

$$= \lim_{h\to 0} \frac{1}{h} \left(\int_{\frac{p_1+p_2}{2}}^{\frac{p_1+h+p_2}{2}} \left\| q - p_1 + h \right\|^2 dq + \int_0^{\frac{p_1+p_2}{2}} \left\| q - p_1 + h \right\|^2 - \left\| q - p_1 \right\|^2 dq \right)$$

$$= \lim_{h\to 0} \frac{1}{h} \int_{\frac{p_1+p_2}{2}}^{\frac{p_1+h+p_2}{2}} \left\| q - p_1 + h \right\|^2 dq + \int_0^{\frac{p_1+p_2}{2}} \frac{\partial}{\partial p_1} \left\| q - p_1 \right\|^2 dq$$

Mean value theorem

$$= \lim_{h\to 0} \frac{1}{h} \left\| c - p_1 + h \right\|^2 \left(\frac{p_1+h+p_2}{2} - \frac{p_1+p_2}{2} \right), \exists c \in \left[\frac{p_1+p_2}{2}, \frac{p_1+p_2+h}{2} \right]$$

$$= \lim_{h\to 0} \frac{1}{2} \left\| c - p_1 + h \right\|^2, \exists c \in \left[\frac{p_1+p_2}{2}, \frac{p_1+p_2+h}{2} \right]$$

Coverage Problem - Review

$$= \frac{1}{2}\left\|\frac{p_2 - p_1}{2}\right\|^2 + \int_0^{\frac{p_1+p_2}{2}} \frac{\partial}{\partial p_1}\left\|q - p_1\right\|^2 dq$$

$$\frac{\partial}{\partial p_1}\int_{\frac{p_1+p_2}{2}}^{1}\left\|q - p_2\right\|^2 dq = \lim_{h\to 0}\frac{1}{h}\left(\int_{\frac{p_1+h+p_2}{2}}^{1}\left\|q - p_2\right\|^2 dq - \int_{\frac{p_1+p_2}{2}}^{1}\left\|q - p_2\right\|^2 dq\right)$$

$$= -\lim_{h\to 0}\frac{1}{h}\left(\int_{\frac{p_1+p_2}{2}}^{\frac{p_1+h+p_2}{2}}\left\|q - p_2\right\|^2 dq\right)$$

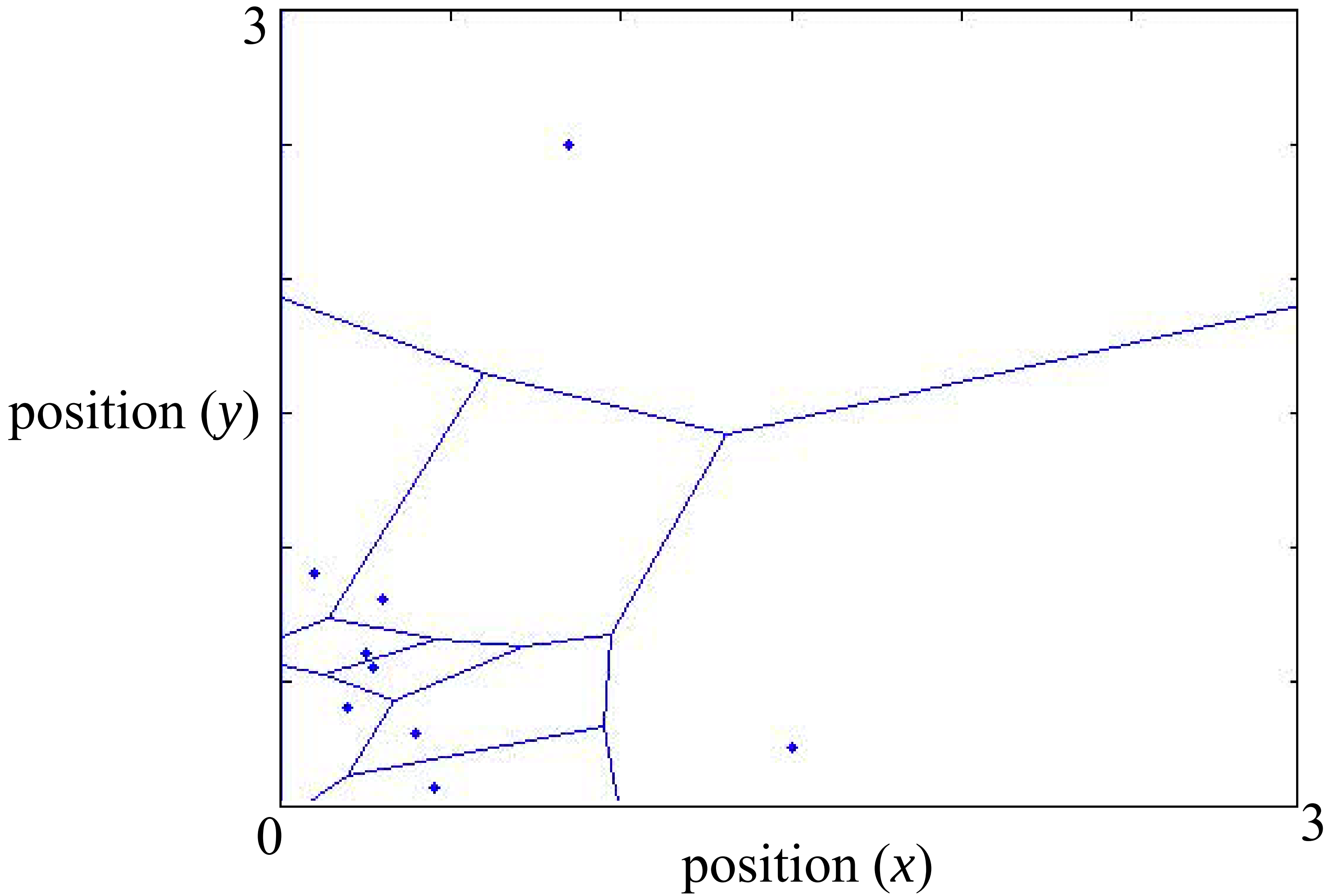
$$= -\frac{1}{2}\lim_{h\to 0}\left\|c - p_2\right\|^2, \exists c \in \left[\frac{p_1+p_2+h}{2}, \frac{p_1+p_2}{2}\right]$$

$$= -\frac{1}{2}\left\|\frac{p_2 - p_1}{2}\right\|^2$$

$$\therefore \frac{\partial H(P)}{\partial p_1} = \int_0^{\frac{p_1+p_2}{2}} \frac{\partial}{\partial p_1}\left\|q - p_1\right\|^2 dq$$

Simulation-1

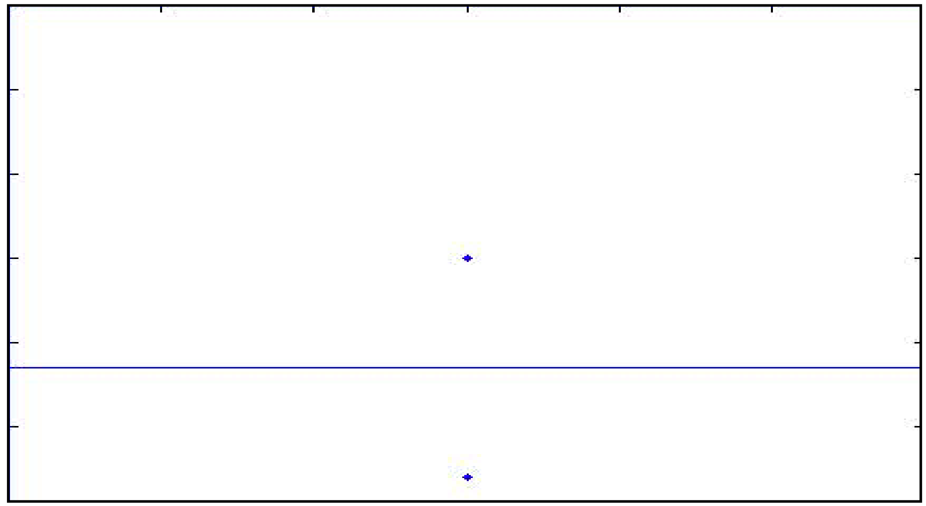
- Given 9 agents distributed in a square area of density Φ=1



Simulation - 2

Case 1

position (*y*)

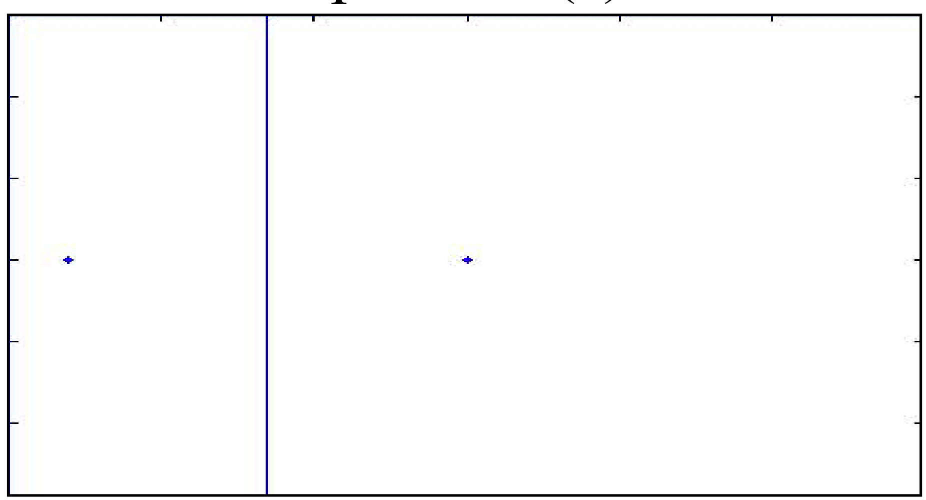


position (*x*)

$$H_1 > H_2$$

Case 2

position (*y*)



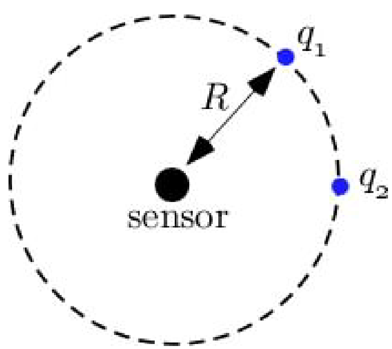
position (*x*)

Coverage - Anisotropic sensor

Goal : 3D Coverage Problem

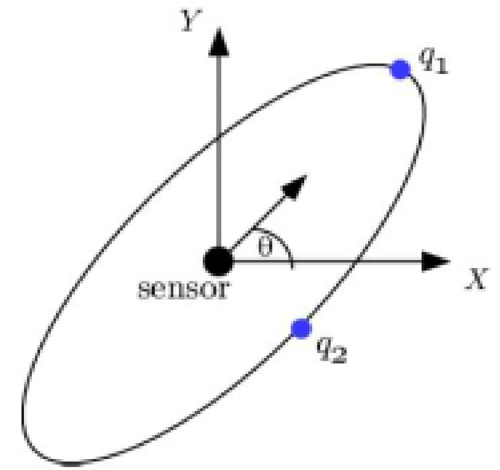
First step : 2D Coverage Problem with Orientation of robots

Uniform (isotropic) sensor



sensing degradation only depends on distance

Non-uniform (anisotropic) sensor

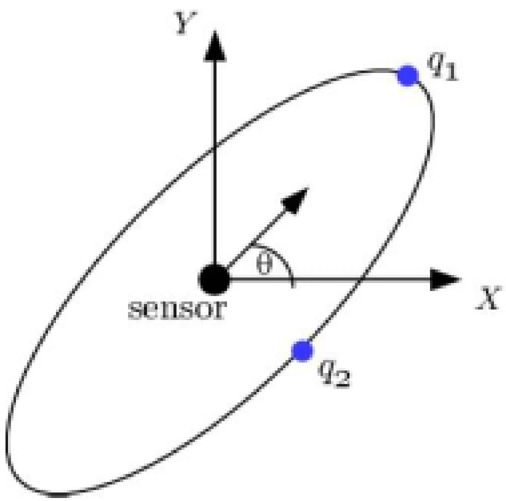


sensing degradation depends on distance and orientation

Cortes et.al

Today's talk

Coverage – Anisotropic sensor



$$\|q-p_i\|_L = (q-p_i)^T L (q-p_i)$$

$$L = \begin{pmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{pmatrix}^T \begin{pmatrix} \frac{c^2}{a^2} & 0 \\ 0 & \frac{c^2}{b^2} \end{pmatrix} \begin{pmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{pmatrix}$$

θ_i : orientation of *i*-th sensor

a, b : length of major and minor axis of the ellipse

Note:

If $L = I$ ➡ $\|q-p_i\| = (q-p_i)^T (q-p_i)$

(Cortes et.al)

Problem Setting

Find P, Θ, W that minimize the function :

$$H(P,\Theta,W) = \sum_{i=1}^{n} \int_{W_i} f\left(\left\| q - p_i \right\|_L \Phi(q) dq\right)$$

Questions:

1. What is the Optimum Partition ?
2. How do we optimize ? One after another or at the same time ?

Anisotropic Voronoi Partition

Q: What is the optimum partition for a fixed sensors' position and orientation ?

- Anisotropic Voronoi partition :

$$V_i^* = \{ q \in Q \mid \|q - p_i\|_L \leq \|q - p_j\|_L, \forall j \neq i \}.$$

- Determined by sensor's position and Orientation

Lemma : the boundary between two adjacent V_i^* and V_j^* is a quadratic curve

Anisotropic Voronoi Partition

Proof :

Any point q in $V_i^* \cap V_j^*$ satisfies $\|q - p_i\|_L = \|q - p_j\|_L$ i.e $(q - p_i)^T L_i (q - p_i) = (q - p_j)^T L_j (q - p_j)$. It is clear that this equation is quadratic in q. (Q.E.D)

Remark :

$$Ax^2 + By^2 + Cxy + Dx + Ey + K = 0,$$

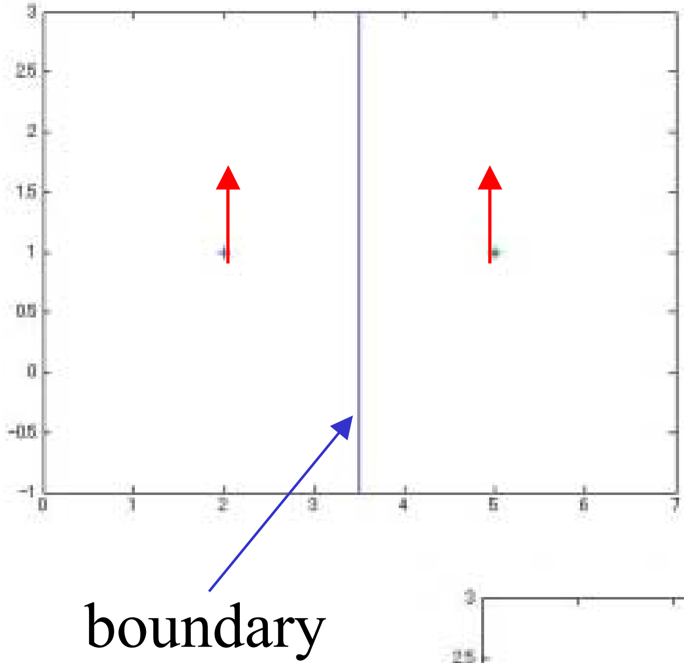
$$A = b^2(\cos^2\theta_i - \cos^2\theta_j) + a^2(\sin^2\theta_i - \sin^2\theta_j)$$

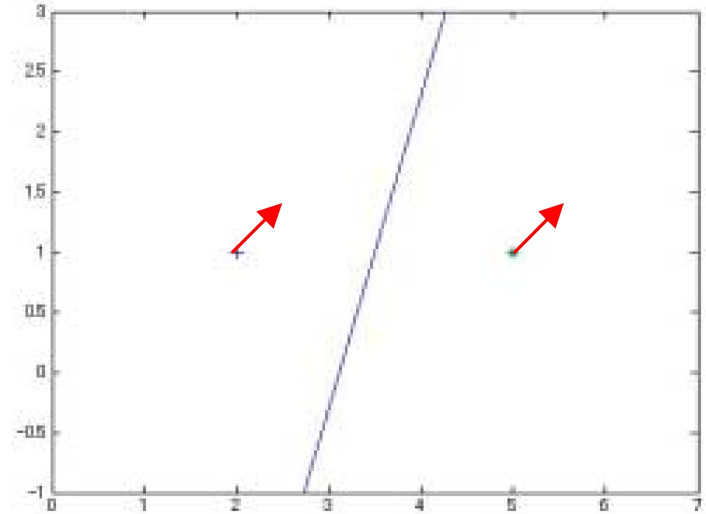
$$B = a^2(\cos^2\theta_i - \cos^2\theta_j) + b^2(\sin^2\theta_i - \sin^2\theta_j)$$

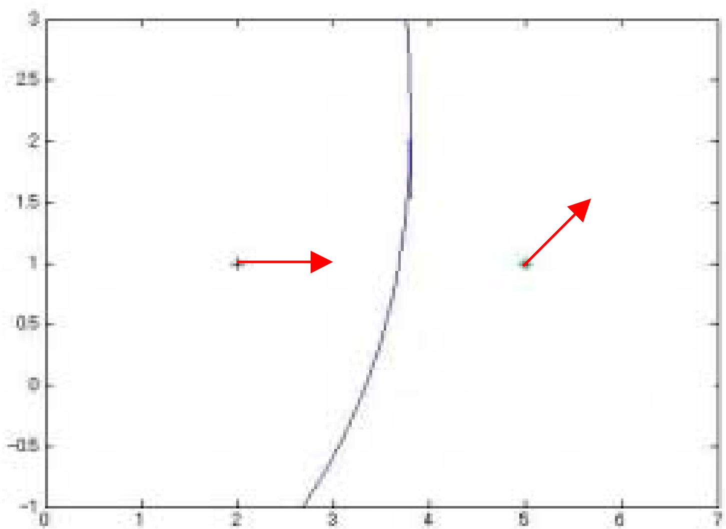
$$C = (a^2 - b^2)(\sin 2\theta_i - \sin 2\theta_j)$$

If the sensors have the same orientation , the boundary will be a line

Anisotropic Voronoi - Examples







Future Works

The optimization function :

$$H(P,\Theta) = \sum_{i=1}^{n} \int_{V_i^*} f\left(\|q - p_i\|_L \Phi(q) dq\right)$$

- Fixed orientation case :

  1. what is the optimum position ? (centroid of AVT ?)

  2. Does $\frac{\partial H(P)}{\partial p_i} = \int_{V_i(P)} \frac{\partial}{\partial p_i} \|q - p_i\|^2 dq$ still hold ?

- For fixed position case, what is the optimum orientation ?

- Can we do fixed orientation opt. $\rightarrow$ fixed position opt. ?