Introduction to Leader Following Problem

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Today's Topics

- Consensus and Leader Following problem
- Definition of Graph
- Distributed consensus algorithms
- Graph connection
- Real-time switching follower
- Average velocity algorithms
- Conclusion
- Future work
- Reference

Consensus and Leader Following problem

- Consensus problem – To reach an agreement regarding a certain quantity of interest that depends on the state of all agents.
- Leader Following problem – To imitate (模倣する) the leader characteristic and hold that state until leader changes.
- Leader Following problem is the special case of Consensus problem.

Definition of Graph

- Vertex (Node) – Agent
- Edge – Information flow

Figure 1

- In Fig.1, the information flows from agent j to agent i.
- Graph
  - Graph is a set of connections (Edges) between objects (Vertices).
  - G is strongly connected graph if there is a directed path connecting any two arbitrary nodes.
  - G is a complete graph if all edges of G connects every pair of vertices.

Definition of Graph

- Adjacency Matrix
  - $A = [a_{ij}]$ is the adjacency matrix and its elements are defined as follows:

$$a_{ij} = \begin{cases} 1 & j \in N_i \\ 0 & otherwise \end{cases}$$

when N_i is neighbor set of agent i

- Degree Matrix
  - D is the degree matrix if $D = diag(d_1,\ldots,d_n)$ and its elements $d_i = \sum_{j \neq i} a_{ij}$ and zero off-diagonal elements.

Distributed consensus algorithms

- Consider the following linear system

$$\dot{x}_i = \sum_{j \in N_i} a_{ij}(x_j(t) - x_i(t))$$

i,j : index of agent which information flow from agent j to agent i

- Note:
  - Graph G must be strongly connected graph, otherwise, some agents cannot track the leader. (The agents didn't get any information about leader so it cannot follow the leader.)
- For leader following problem, when i = 1 (leader), $\dot{x}_1(0) = \ldots = \dot{x}_1(\infty) = c$ (independent to group variable)

Graph Connection

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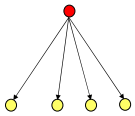
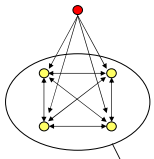
We considered two types of graph connection.

- Unconnected follower
- Connected follower





Complete graph

Simulation

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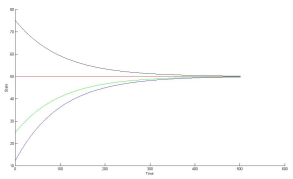
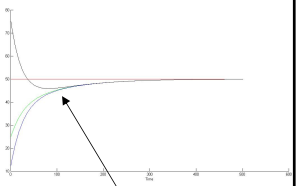
- Unconnected Follower
- Connected Follower





The latter is faster!

Same as consensus case

Think it again

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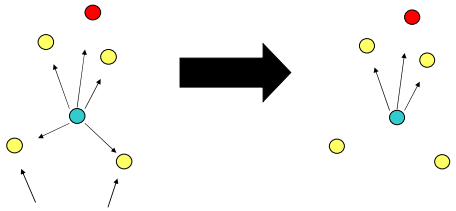
Consider the connected follower type



Do we need the below agents ? Is it better not to connect to them ?

Define it as “Real-time switching follower”

Real-time switching follower

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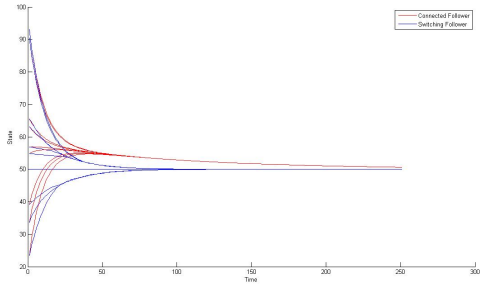

Average velocity algorithms

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- Prof. Vicsek has studied the behavior of birds. He found that when one bird in the flock make a direction mistake, it would make other surrounding birds to make a same mistake.

Average velocity algorithms

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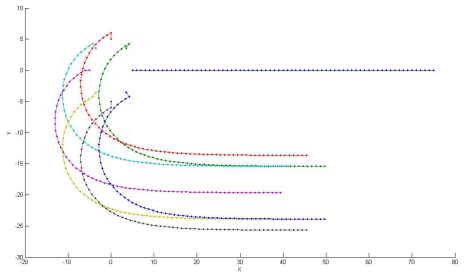
- He proposed a model for that event which update state using average direction. (The velocity is assumed to be the same)

$$\theta_i(t+1) = \frac{1}{d_i}\sum_k \theta_k(t)$$

- In leader follower problem, only follower will be updated.

Average velocity algorithms



Analysis

- From the simulation, we have set the initial condition as there are many agents separated on the circle position. All of them move in the way out of the center of circle.
- From the second step, the follower change the direction to the same way.
- In the end, the follower reach a leader direction.
- Note that, in the real world, robot cannot change direction rapidly. There is a minimum settling time for changing direction.

Conclusion

- We have described the basic of graph theory and the introduction to the leader following problem.
- We have proposed a new way to solve the leader following problem based on Distributed consensus algorithms.
- The good way to reach leader characteristic is to follow the leader and other follower who is nearer to the leader.
- We have also introduced the Average velocity algorithms which is proposed by Prof. Vicsek.

Future work

- Consider the collision avoidance cases both the agent-agent and agent-obstacle.
- Consider the case when the neighbor of the agent is undetected or added.
- Formation Control

Reference

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