An introduction for Synchronization and Reaching Control of Robot Manipulators

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藤田研究室
尹　磊磊

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Control of Robot Manipulator (Passive-Based)

Control Lows for Singular Manipulator

- Takegaki, Arimoto – Position Control

$$\tau = g(q) - k_p(q - q_d) - k_d\dot{q} \qquad q \to q_d, \dot{q} \to 0, (\dot{q}_d = 0) \cdots \quad (1)$$

- Paden, Panja – Tracking Control

$$\tau = M\ddot{q}_d + C\dot{q}_d + g(q) - k_p(q - q_d) - k_d(\dot{q} - \dot{q}_d) \quad q \to q_d, \dot{q} \to \dot{q}_d \cdots (2)$$

- Slotin, Li – Tracking Control

$$\tau = M\underbrace{\left(\ddot{q}_d - \Lambda(\dot{q} - \dot{q}_d)\right)}_{\ddot{q}_r} + C\underbrace{\left(\dot{q} - \Lambda(q - q_d)\right)}_{\dot{q}_r}$$

$$+ g(q) - k_p(q - q_d) - k_d\underbrace{\left((\dot{q} - \dot{q}_d) - \Lambda(q - q_d)\right)}_{s_1}$$

$$q \to q_d, \dot{q} \to \dot{q}_d \qquad \cdots\cdots \quad (3)$$

Introduction -- Synchronization Control

Nijmeijer

- Synchronization of Joint States

$$q_1 = q_2 = \cdots = q_d$$

$$\dot{q}_1 = \dot{q}_2 = \cdots = \dot{q}_d$$

- Use the Synchronization Error to creat Reference Signals

$$\tau_i = M_i \underbrace{\left(\ddot{q}_d - \sum_{j=1, i\neq j}^{p} k_{ca_{ij}} \left(\ddot{q}_i - \ddot{q}_j \right) \right)}_{\ddot{q}_{ri}}$$

$$+ C_i \underbrace{\left(\dot{q}_d - \sum_{j=1, i\neq j}^{p} k_{cv_{ij}} \left(\dot{q}_i - \dot{q}_j \right) \right)}_{\dot{q}_{ri}} + g_i(q_i)$$

$$- k_{pi} \underbrace{\left((q_i - q_d) + \sum_{j=1, i\neq j}^{p} k_{cp_{ij}} \left(q_i - q_j \right) \right)}_{s_i}$$

$$- k_{di} \underbrace{\left((\dot{q}_i - \dot{q}_d) - \sum_{j=1, i\neq j}^{p} k_{cv_{ij}} \left(\dot{q}_i - \dot{q}_j \right) \right)}_{\dot{s}_i} \quad \cdots (4)$$

- Reference Signals $q_{ri} = q_d - \sum_{j=1, i\neq j}^{p} k_{cp_{ij}} \left(q_i - q_j \right)$

Spong

- Output Synchronization

$$y_i = y_j$$

$$\Rightarrow q_i = q_j (i, j = 0, \cdots, p)$$

- Chose a output function in the controller to ignore the acceleration terms

$$\tau_i = M_i \left(-\lambda \dot{q}_i \right) + C_i \left(-\lambda q_i \right) + g_i(q_i)$$

$$+ \sum_{j=1, i\neq j}^{p} k_{ij} \left(\underbrace{\left(\dot{q}_i + \lambda q_i \right)}_{y_i} - \underbrace{\left(\dot{q}_j + \lambda q_j \right)}_{y_j} \right) \quad \cdots\cdots (5)$$

- output function $y_i = \dot{q}_i + \lambda q_i$

Introduction – Reaching Problem

Reaching problem is to move the manipulator from a start point to the destination point in the task coordinates.

Arimoto - PD Controller for Robot Manipulator

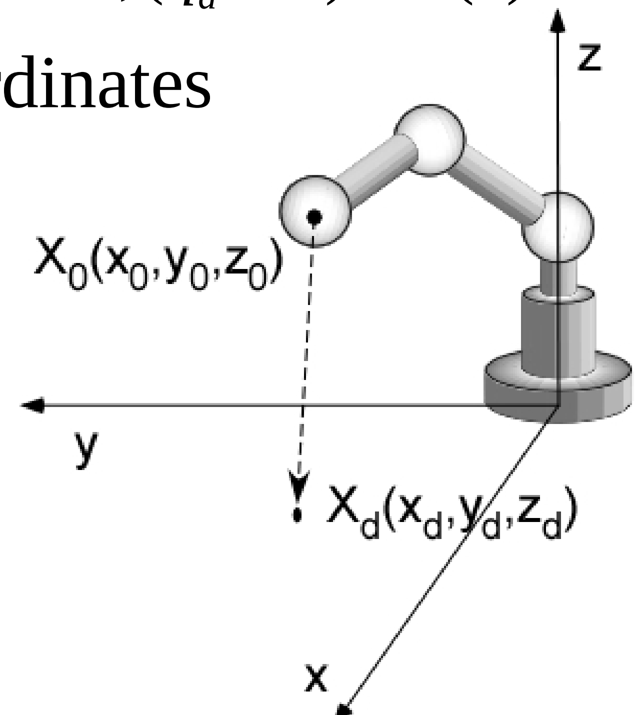
- Feedback PD-Control in joint coordinates

$$\tau = g(q) - \underline{k_p(q - q_d)} - k_d \dot{q} \qquad q \to q_d, \dot{q} \to 0, (\dot{q}_d = 0) \cdots \quad (1)$$

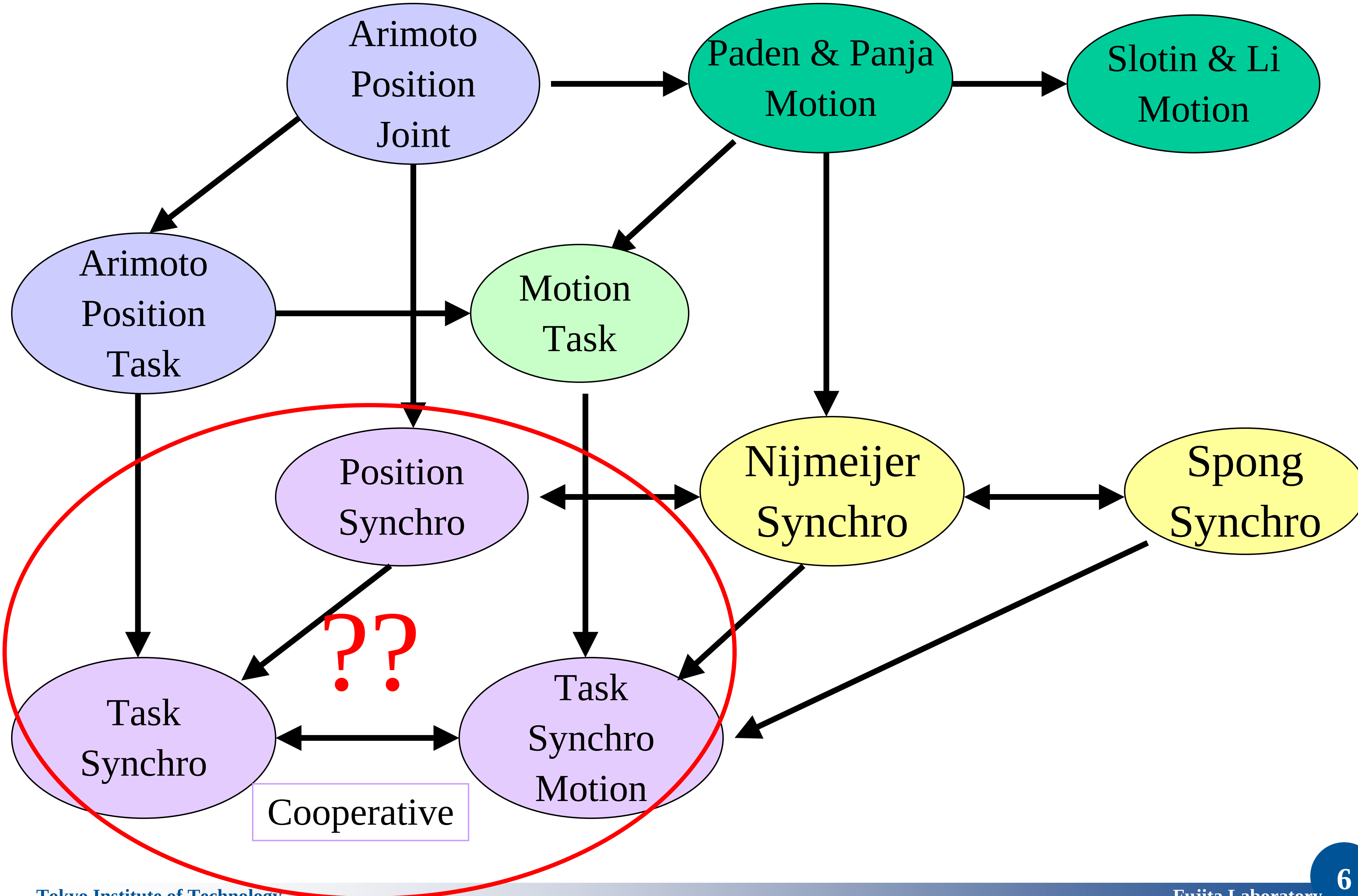
- Feedback PD-Control in task coordinates

$$\tau = g(q) - k_d \dot{q} - \underline{J_x^T(q) k_p (x - x_d)}$$

$$x \to x_d, \dot{q} \to 0, (\dot{q}_d = 0) \cdots \quad (6)$$



Introduction of Reaching Problem for Multiple Manipulator



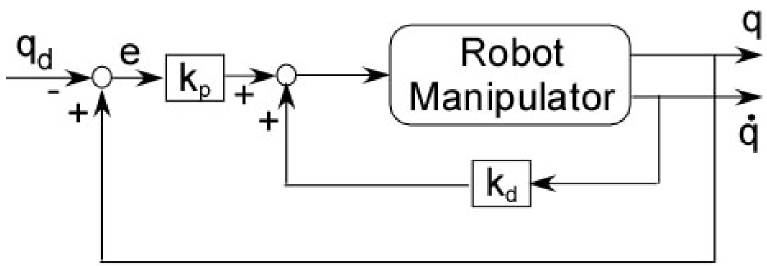
Arimoto Method for Robot Manipulator Control

Control Object

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)=\tau \quad \cdots\cdots \quad (7)$$

Control Law in Joint Coordinates

$$\tau = g(q)-k_p(q-q_d)-k_d\dot{q} \quad \cdots\cdots \quad (8)$$



Closed-loop System

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+k_d\dot{q}+k_p(q-q_d)=0\cdots(9)$$

Theorem1

上記制御対象(7)、コントローラ(8)によって構成された閉ループ系(9)について考える。ゲイン行列 k_p, k_d が正定の時、システムが平衡点 $(q,\dot{q})=(q_d,0)$ が大域的漸近安定となる。

Theorem1

If the gain matrix k_p, k_d are chosen to be positive definite, tehn the controller(8) results in the equilibrium state $(q,\dot{q})=(q_d,0)$ of the closed-loop system(9) being globally asymptotically stable

Proof of Theorem 1

Storage Function

$$V=\frac{1}{2}\dot{q}^T M(q)\dot{q}+\frac{1}{2}e^T k_p e \quad (e=q-q_d)\cdots\cdots \qquad (10)$$

Derivative of Eq(1)

$$\frac{d}{dt}V=\frac{d}{dt}\left(\frac{1}{2}\dot{q}^T M(q)\dot{q}+\frac{1}{2}e^T k_p e\right)$$

$$=\underline{\dot{q}^T M(q)\ddot{q}}+\frac{1}{2}\dot{q}^T\dot{M}(q)\dot{q}+e^T k_p\dot{e}$$

$$=\underline{-\dot{q}^T\left(C(q,\dot{q})\dot{q}+k_d\dot{q}+k_p e\right)}+\frac{1}{2}\dot{q}^T\dot{M}(q)\dot{q}+e^T k_p\dot{e}$$

$$=-\dot{q}^T C(q,\dot{q})\dot{q}-\dot{q}^T k_d\dot{q}-\dot{q}^T k_p e+\frac{1}{2}\dot{q}^T\dot{M}(q)\dot{q}+e^T k_p\dot{e}$$

$$=\frac{1}{2}\underbrace{\dot{q}^T\left(\dot{M}(q)-2C\right)\dot{q}}_{=0\because Skew-symmetric}-\dot{q}^T k_d\dot{q}\underbrace{-\dot{q}^T k_p e+e^T k_p\dot{e}}_{=0\because \dot{e}=\dot{q}-\dot{q}_d=\dot{q}}$$

$$=-\dot{q}^T k_d\dot{q}\le 0$$

Proof of Theorem 1

Hence, the differential of the storage function $\dot{V} = -\dot{q}^T k_d \dot{q} \le 0$ is just Negative Semi-Definite, so the system stability can not be guaranteed

But, LaSalle's theorem can be used to proof it

So, Lets proof in $\left\{ \begin{bmatrix} q^T & \dot{q}^T \end{bmatrix} \middle| \dot{V} = 0 \right\} = \left\{ \begin{bmatrix} q^T & \dot{q}^T \end{bmatrix} \middle| \dot{q} = 0 \right\}$ the Largest invarian set is $\begin{bmatrix} q_d^T & 0 \end{bmatrix}^T$

from $\dot{V} = -\dot{q}^T k_d \dot{q} \le 0$,we know if and only if $\dot{q} = 0$, $\dot{V} = 0$

If $\dot{V} = 0$, then $\ddot{q} = -M^{-1} k_p (q - q_d)$, because of the closed-loop(9)

Assume that $\dot{V} = 0, q \ne q_d$, then, $\ddot{q} \ne 0$

$\therefore \dot{q} \ne 0, \dot{V} \ne 0$, thus $q = q_d$

Hence, $\begin{bmatrix} q_d^T & 0 \end{bmatrix}^T$ is the only invariant point in the set $\left\{ \begin{bmatrix} q^T & \dot{q}^T \end{bmatrix} \middle| \dot{V} = 0 \right\}$

Thus, use the Lasalle's Theorem, we can proof the Position Control method of Robot Manipulator by Arimoto is asymptotically stable

Control Law in Task Coordinates

$$\tau = g(q) - k_d\dot{q} - \underline{J_x^T(q)k_p\left(x - x_d\right)} \quad \cdots\cdots \quad (11)$$

Closed-loop System

$$M\left(q\right)\ddot{q} + C\left(q,\dot{q}\right)\dot{q} + k_d\dot{q} + J_x^T(q)k_p\Delta x = 0 \quad \cdots \quad (12)$$

There $\Delta x = x - x_d$

Take the Storage Function as

$$V = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2}\Delta x^T k_p \Delta x \quad \cdots\cdots \quad (13)$$

$$\frac{d}{dt}V = \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \Delta x^T k_p \Delta\dot{x}$$

$$= \frac{1}{2}\underbrace{\dot{q}^T\left(\dot{M}(q) - 2C\right)\dot{q}}_{=0\ \because Skew-symmetric} - \dot{q}^T k_d\dot{q}\ \underbrace{\underbrace{-\dot{q}^T J_x^T(q)k_p\Delta x}_{=-\Delta\dot{x}^T k_p\Delta x(\because \dot{q}=J_x^{-1}\Delta\dot{x})} + \Delta x^T k_p\Delta\dot{x}}_{=0}$$

$$= -\dot{q}^T k_d\dot{q} \le 0$$

the proof of asymptotically stable is the same as in joint coordinates

Nijmeijer Mutual synchronization control

- Based on the sigular manipulator motion control by Paden, Panja

- Control Object : n-link Robot Manipulators

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i \qquad \cdots \quad (14)$$

- Synchronization Control Law

$$\tau_i = M_i \underbrace{\left(\ddot{q}_d - \sum_{j=1,i\neq j}^{p} k_{ca_{ij}}\left(\ddot{q}_i - \ddot{q}_j\right)\right)}_{\ddot{q}_{ri}} + C_i \underbrace{\left(\dot{q}_d - \sum_{j=1,i\neq j}^{p} k_{cv_{ij}}\left(\dot{q}_i - \dot{q}_j\right)\right)}_{\dot{q}_{ri}} + g_i(q_i)$$

$$-k_{pi}\underbrace{\left((q_i - q_d) + \sum_{j=1,i\neq j}^{p} k_{cp_{ij}}\left(q_i - q_j\right)\right)}_{s_i} - k_{di}\underbrace{\left((\dot{q}_i - \dot{q}_d) - \sum_{j=1,i\neq j}^{p} k_{cv_{ij}}\left(\dot{q}_i - \dot{q}_j\right)\right)}_{\dot{s}_i} \cdots \quad (15)$$

- Closed-Loop

$$M_i\ddot{s}_i + C_i\dot{s}_i + k_{di}\dot{s}_i + k_{pi} = 0 \qquad \cdots \quad (16)$$

$$s_i = (q_i - q_d) + \sum_{j=1,i\neq j}^{p} k_{cp_{ij}}\left(q_i - q_j\right)$$

Theorem2

制御対象(14)、コントローラ(15)によって構成された閉ループ系(16)について考える。もしゲイン行列 k_{pi}, k_{di} が正定であれば、Synchronization Error $s_i, \dot{s}_i,\ i=1,\dots p$ に対して、大域的に漸近安定である。

Theorem2

Consider the closed loop system formed by the controller (15), and the robot danamics(14). Then the synchronization errors $s_i, \dot{s}_i,\ i=1,\dots p$ are globlly asymptotically stable if the control gains k_{pi}, k_{di} are positive definite

Storage Function

$$V=\sum_{i=1}^{p}\left(\frac{1}{2}\dot{s}_i^T M(q)\dot{s}_i+\frac{1}{2}s_i^T k_{p,i} s_i\right) \quad \cdots\cdots \quad (17)$$

Proof of the Stability

Derivative of Storage Function (17)

$$\frac{d}{dt}V = \sum_{i=1}^{p}\left(\dot{s}_i^T \underline{M_i \ddot{s}_i} + \frac{1}{2}\dot{s}_i^T \dot{M}_i \dot{s}_i + s_i^T k_{p,i}\dot{s}_i \right)$$

$$= \sum_{i=1}^{p}\left(\underbrace{\frac{1}{2}\dot{s}_i^T\left(\dot{M}_i - 2C_i\right)\dot{s}_i}_{=0} - \dot{s}_i^T k_{d,i}\dot{s}_i \underbrace{- \dot{s}_i^T k_{p,i} s_i + s_i^T k_{p,i}\dot{s}_i}_{=0} \right)$$

$$= \sum_{i=1}^{p}\left(-\dot{s}_i^T k_{d,i}\dot{s}_i\right) \le 0 \qquad \therefore \dot{V} : \text{Positive Semi-Definite}$$

But, the system can not be guranteed to be stable, but Barbalat's Lemma can be used to proof the stability of the synchro error $\left(s_i, \dot{s}_i\right)$

- V is Finity Limit $\because V > 0, \dot{V} \le 0$
- $\dot{V}$ is Uniformly Continuous $\because \dot{s}_i, \ddot{s}_i$ is bounded, $\therefore \ddot{V} = -2\sum_{i=1}^{p}\left(\ddot{s}_i^T k_{d,i}\dot{s}_i\right)$ bounded

Then, storage function V can be take as Barbalat's function

$\therefore \lim_{t\to\infty}\dot{V}\left(s_i, \dot{s}_i\right) = 0$, then, $\lim_{t\to\infty}\dot{s}_i = 0$ Also,we can proof that $\lim_{t\to\infty} s_i = 0$

Hence, $\left(s_i, \dot{s}_i\right)$ is asymptotically stable

Proof of the Stability 2 (Synchronization)

Next, take the coming Lemma to show the Synchronization

Lemma

次の対角優勢行列(Diagonally Doninant Matrix) $M_c(k_{i,j})$ をロボット間のCoupling Matrixとして考える。$M_c(k_{i,j})$ は全て準正定な対角行列 $k_{i,j}, i, j = 1, \dots, p$ に対して正則である。また、全ての準正定な対角行列 $k_{i,j}, i, j = 1, \dots, p$ に対して次の関係が成り立つ。

$$M_c(k_{i,j}) \begin{bmatrix} q_1 \\ \vdots \\ q_p \end{bmatrix} = \begin{bmatrix} q_d \\ \vdots \\ q_d \end{bmatrix} \Leftrightarrow \begin{bmatrix} q_1 \\ \vdots \\ q_p \end{bmatrix} = \begin{bmatrix} q_d \\ \vdots \\ q_d \end{bmatrix}$$

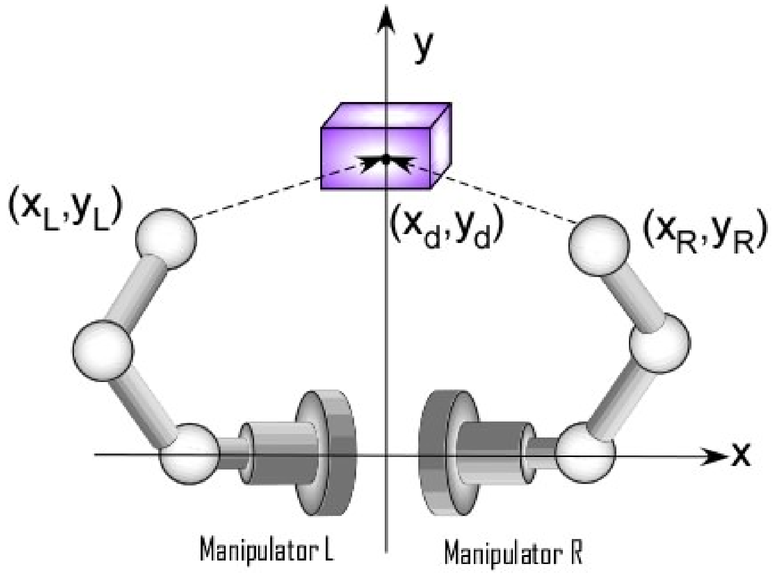
$$M_c(k_{i,j}) = \begin{bmatrix} \left(I_n + \sum_{j=1, j\neq i}^{p} k_{1,j}\right) & -k_{1,2} & \cdots & -k_{1,p} \\ -k_{2,1} & \left(I_n + \sum_{j=1, j\neq i}^{p} k_{2,j}\right) & \cdots & -k_{2,p} \\ \vdots & \vdots & \ddots & \vdots \\ -k_{p,1} & -k_{p,2} & \cdots & \left(I_n + \sum_{j=1, j\neq i}^{p} k_{p,j}\right) \end{bmatrix}$$

Problem Establishment

Cooperative control of Multiple Robot Manipulators

Dynamics of the Control Objects

$$\begin{cases} M_L\ddot{q}_L + C_L\dot{q}_L + g_L = \tau_L \\ M_R\ddot{q}_R + C_R\dot{q}_R + g_R = \tau_R \end{cases}$$



Goal

Use a synchronization method to control two manipulators from a start point move to the desired poit in the task coordinates

First

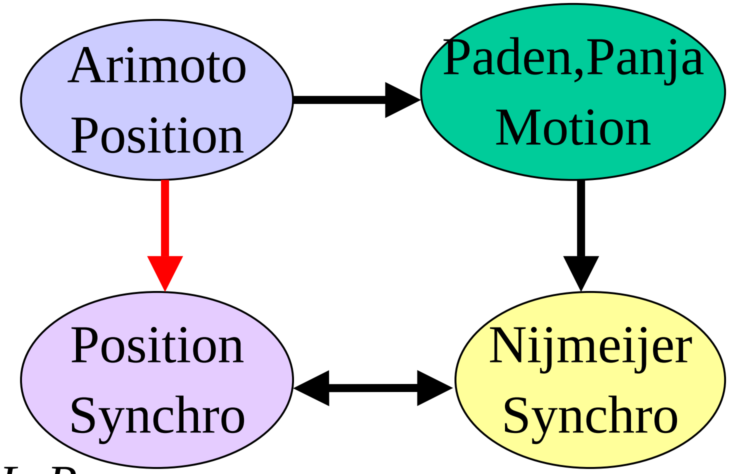
Extend synchronization control to task space

Output synchronization control by Spong

$$u_1 = -K(\dot{q}_2 - (-\dot{q}_1)) \Rightarrow \dot{q}_1 = -\dot{q}_2, q_1 = -q_2$$

Hence, If we take the input and output inverse, we may get $\langle \dot{q}_1, -\dot{q}_2 \rangle, \langle q_1, -q_2 \rangle$ synchronization

An extention of the Arimoto Position Control



Control Objects

$$\begin{cases} M_L\ddot{q}_L + C_L\dot{q}_L + g_L = \tau_L \\ M_R\ddot{q}_R + C_R\dot{q}_R + g_R = \tau_R \end{cases} \quad \cdots\cdots \quad (18)$$

Controller

$$\begin{cases} \tau_L = g_L - k_d\dot{q}_L - k_p\left(q_L - q_{rL}\right) \\ \tau_R = g_R - k_d\dot{q}_R - k_p\left(q_R - q_{rR}\right) \end{cases} \quad \cdots\cdots \quad (19)$$

There, referrence signals $q_{ri} = q_d - \left(q_i - q_j\right),\ i, j = L, R$

Then, $s_i = q_i - q_{ri} = q_i - \left(q_d - \left(q_i - q_j\right)\right) = \left(q_i - q_d\right) + \left(q_i - q_j\right) = e_i + e_{ij}$

Combine (18) and (19) ,we get the closed-loop like

$$M_i\ddot{q}_i + C_i\dot{q}_i + k_d\dot{q}_i + k_p e_i + k_p e_{ij}\ ,i, j = L, R \quad \cdots\cdots \quad (20)$$

Theorem3

Consider the closed loop system formed by the controller (19), and the robot danamics(18). Then the synchronization errors $s_i,\ i = L, R$ are asymptotically stable if the control gains k_p, k_d are positive definite

制御対象(18)、コントローラ(19)によって構成された閉ループ系(20)について考える。もしゲイン行列k_p, k_dが正定であれば、Synchronization Error $s_i,\ i = L, R$に対して、大域的に漸近安定である。

Proof of the Theorem3

Storage Function

$$V_i = \dot{q}_i^T M_i \dot{q}_i + e_i^T k_p e_i + \frac{1}{2} e_{ij}^T k_p e_{ij} \quad (e_i = q_i - q_d, e_{ij} = q_i - q_j), i, j = R, L \cdots\cdots (21)$$

Derivative of Eq(21)

$$\frac{d}{dt} V_i = \frac{d}{dt}\left(\dot{q}_i^T M_i \dot{q}_i + e_i^T k_p e_i + \frac{1}{2} e_{ij}^T k_p e_{ij} \right)$$

$$= 2\dot{q}_i^T M_i \ddot{q}_i + 2e_i^T k_p \dot{e}_i + e_{ij}^T k_p \dot{e}_{ij}$$

$$= -2\dot{q}_i^T \left(C_i \dot{q}_i + k_d \dot{q}_i + k_p e_i + k_p e_{ij} \right) + \dot{q}_i^T \dot{M}_i \dot{q}_i + 2e_i^T k_p \dot{e}_i + e_{ij}^T k_p \dot{e}_{ij}$$

$$= \underbrace{\dot{q}_i^T \left(\dot{M}_i - 2C_i \right) \dot{q}_i}_{=0 \because Skew-symmetric} - 2\dot{q}_i^T k_d \dot{q}_i - 2\dot{q}_i^T k_p e_i - 2\dot{q}_i^T k_p e_{ij} + \underbrace{2e_i^T k_p \dot{e}_i}_{=2\dot{q}_i^T k_p e_i} + \underbrace{e_{ij}^T k_p \dot{e}_{ij}}_{=\left(\dot{q}_i - \dot{q}_j\right) k_p e_{ij}}$$

$$= -2\dot{q}_i^T k_d \dot{q}_i \underline{-2\dot{q}_i^T k_p e_i} \underline{\underline{-2\dot{q}_i^T k_p e_{ij}}} \underline{+2\dot{q}_i^T k_p e_i} + \underline{\underline{\left(\dot{q}_i - \dot{q}_j \right) k_p e_{ij}}}$$

$$= -2\dot{q}_i^T k_d \dot{q}_i - \left(\dot{q}_i + \dot{q}_j \right) k_p e_{ij}$$

$$V = V_L + V_R$$

$$\frac{d}{dt} V = \frac{d}{dt} V_L + \frac{d}{dt} V_R = -2\dot{q}_L^T k_d \dot{q}_L \underline{-\left(\dot{q}_L + \dot{q}_R \right) k_p e_{LR}} - 2\dot{q}_R^T k_d \dot{q}_R \underline{-\left(\dot{q}_R + \dot{q}_R \right) k_p e_{RL}}$$

$$= -2\left(\dot{q}_L^T k_d \dot{q}_L + \dot{q}_R^T k_d \dot{q}_R \right) \le 0 \quad \because e_{LR} = -e_{RL}$$

Conclusion & Future Work

- Control of Robot Manipulators in Joint Coordinates & Task Coordinates
  - Arimoto Position Control
- Synchronization Control
  - Nijmeijer Mutual Synchronization Control
  - Extend the Arimoto Control to Multiple Manipulators
- Introduction of Coorperative Reaching control
- Future work
  - Fill up the task coordinates based control
  - Clear up the relationship of synchronization control by Nijmeijer & Spong, and extend the Arimoto controller to multiple manipulators, and then compare with it
  - Extend the synchronization control to task coordinates