Quality-of-Service Control of Networked Systems with Packet Loss

パケットロスを考慮したネットワーク化システムのQoS制御

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Outline

- Introduction
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- Periodic System
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- Performances
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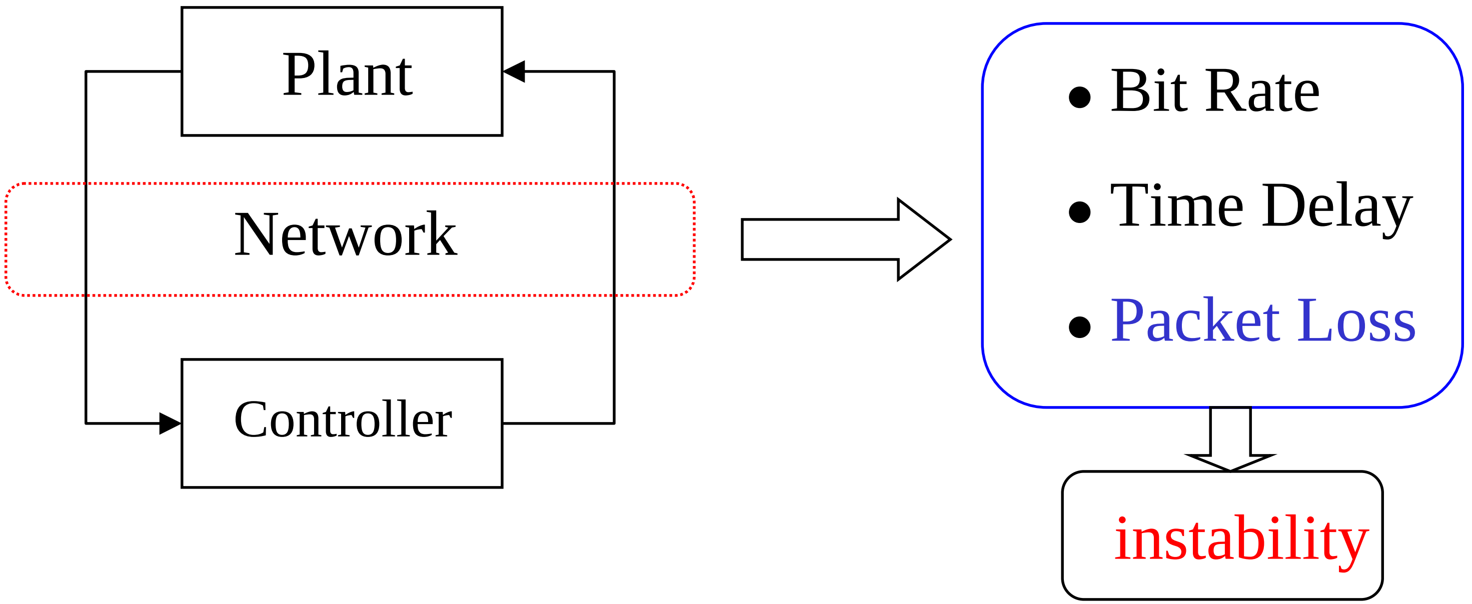
Introduction (QoS Control)



- Stability
- Performances

\+ Network Cost ⇨ Quality-of-Service

(Trade-off)

Low Packet Loss ⇨ Good control performance ⇨ Network Cost Expensive

Introduction

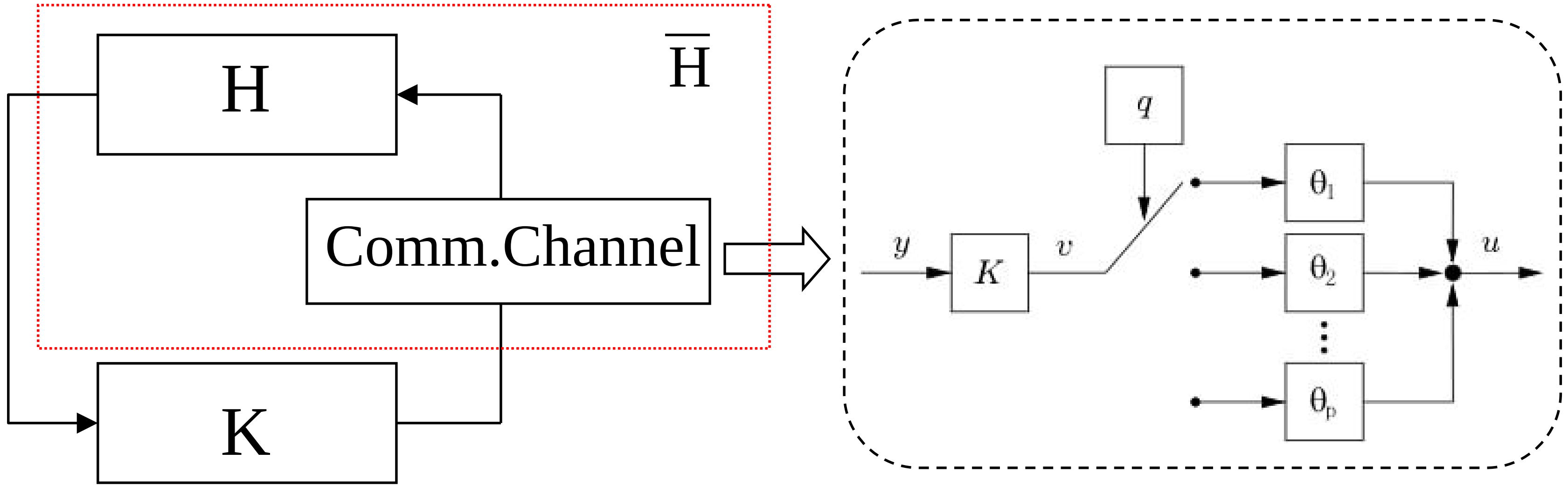
To Deal with this trade off :

- Introduce network model with many virtual channels with different packet loss probabilities
- Introduce a periodic switching control scheme.

⇓

- Designed offline ⇒ simplicity of the design
- Can be easily implemented

Problem Formulation



H

$$x_{k+1} = Ax_k + Bu_k$$

$$y_k = Cx_k$$

- $A \in \Re^{n \times n}, B \in \Re^{n \times 1}, C \in \Re^{1 \times n}$
- (A,B) controllable, (A,C) observable

Comm.ch.

$$\Theta_k = \begin{bmatrix} \theta_{1,k} \\ \vdots \\ \theta_{p,k} \end{bmatrix} \in \Re^{p \times 1}$$ ⇒ Packet loss indicator

$$\alpha_i := prob\{\theta_{i,k} = 0\}$$ ⇒ loss prob.

$$q \in I_p^N$$ ⇒ Switching pattern of period N

$$Q_k := [0 \ldots 010 \ldots 0] \in \Re^{1 \times p}$$

Only one channel can be used at one time

Problem Formulation

$\overline{\mathrm{H}}$: $$x_{k+1} = Ax_k + B\Theta_k^T Q_k^T v_k$$ $$y_k = x_k$$

Closed-loop equation:

$$x_{k+1} = (A + B\Theta_k^T Q_k^T K)x_k$$

$$x_{k+1} = A_{k,\theta(k)} x_k \begin{cases} A_{k,0} \rightarrow \text{packet is lost} \\ A_{k,1} \rightarrow \text{packet is arrived} \end{cases}$$

$$A_{k+N,\theta_k} = A_{k,\theta_k}$$ ⇒ Periodic stochastic system

For system $x_{k+1} = A_{k,\theta(k)} x_k$ which is N-periodic, the origin is said to be :

- Mean-square stable if for every initial state (x_0, θ_0),

$$\lim_{k \to \infty} E\left[\|x_k\|^2 \middle| x_0, \theta_0 \right] = 0$$

Periodic System

Periodic System : $x_{k+1} = A_k x_k$

Where $A(\cdot)$ is a periodic matrix of period N i.e $A_{k+N} = A_k$

Lemma 1 : The following statements are equivalent each other.

- $A(\cdot)$ is stable.
- There exists a N-periodic positive definite solution of the Lyapunov inequality $P(\cdot)$

$$A_k^T P_{k+1} A_k - P_k < 0$$

S. Bittanti and P. Colaneri, IFAC Periodic control systems, 2001

$$A(\cdot) = A(0)A(1)\cdots A(N-1)$$

$$A_k \rightarrow A_{k,\theta(k)}$$

Stability Condition

Each channel is used n_i times $\left(\sum_{i=1}^{p} n_i = N\right)$

Proposition 1 : For the system $\overline{H}$, the origin is mean square stable iff there exists an N-periodic matrix $P_l \in \Re^{n\times n}$ such that $P_l = P_l^T > 0$ and

$$\sum_{i=0} \alpha_{i,l} A_{i,l}^T P_{l+1} A_{i,l} - P_l < 0 \text{ for } l \in I_N \qquad (1)$$

(Proof)

Define the Lyapunov function as : $V_k = x_k^T P_k x_k$

Where P is time varying and $P_k^T = P_k$

The system is mean square stable iff $E\left[\Delta V_k \middle| x_k, \theta_k\right] < 0$

If $\sum_{i=0}^{1} \alpha_{i,l} A_{i,l}^T P_{l+1} A_{i,l} - P_l < 0$ for $l \in I_N$ holds for l, then it also holds for $l+N$

Proof

Closed-loop equation :

$$x_{k+1} = A_{\theta(k),k}$$

Applying lemma 1 and compute the expected value of the difference :

$$\begin{aligned}
E[\Delta V_k] &= E\left[V_{k+1} \middle| x_k, \theta_{k-1}\right] - V_k \\
&= E\left[x_{k+1}^T P_{k+1} x_{k+1} \middle| x_k, \theta_{k-1}\right] - x_k^T P_k x_k \\
&= E\left[x_k^T A_{i,k}^T P_{k+1} A_{i,k} x_k \middle| x_k, \theta_{k-1}\right] - x_k^T P_k x_k \\
&= x_k^T E\left[A_{i,k}^T P_{k+1} A_{i,k} \middle| x_k, \theta_{k-1}\right] x_k - x_k^T P_k x_k \\
&= x_k^T \left(\alpha_k A_{0,k}^T P_{k+1} A_{0,k} + (1-\alpha_k) A_{1,k}^T P_{k+1} A_{1,k}\right) x_k - x_k^T P_k x_k \\
&= x_k^T \left(\sum_{i=0}^{1} \alpha_{i,k} A_{i,k}^T P_{k+1} A_{i,k} - P_k\right) x_k < 0
\end{aligned}$$

Necessary condition on loss prob.

Proposition 2 : The necessary condition on the packet loss probabilities α_i so that there exists a controller that stabilizes the plant is given by

$$\left(\prod_{i=1}^{p} \alpha_i^{\frac{n_i}{2}}\right) \max\left|\lambda(A)\right|^N < 1$$

Where $\lambda(\cdot)$ denotes an eigenvalue

(Proof)

By proposition 1, The system is stable iff there exists an N-periodic $P_k >$ 0 such that N inequalities in (1) hold.

$$\alpha_{0,l} A_{l,0}^T P_{l+1} A_{l,0} - P_l < \sum_{i=0}^{1} \alpha_{i,l} A_{l,i}^T P_{l+1} A_{l,i} - P_l < 0$$

Proof

The Necessary cond. for the system to be stable is that $\exists P_k > 0$ such that:

$$\alpha_{0,l} A_{l,0}^T P_{l+1} A_{l,0} - P_l < 0 \text{ for } l \in I_N$$

Straightforward calculation gives us :

$$\alpha_1^{n_1} \alpha_2^{n_2} \ldots \alpha_p^{n_p} \left(A^T\right)^N P_0 A^N - P_0 < 0, P_0 > 0$$

The solution of P_0 exists iff $\alpha_1^{n_1} \alpha_2^{n_2} \ldots \alpha_p^{n_p} A^N$ is a stable matrix

i.e. $\left(\prod_{i=1}^{p} \alpha_i^{\frac{n_i}{2}}\right) \max\left|\lambda(A)\right|^N < 1$

Performance –1 (Guaranteed Decay Rate)

- Assume there are two channels : $\alpha_1 = 0$ and α_2
- Find the minimum data rate over ch.1 or largest periodic N such that :

$$\sup_{\substack{\text{(for any packet}\\ \text{loss realization)}}} \left(\|x_k\|^2\right) \leq c^{-k}\|x_0\|^2, c \geq 1$$

- Consider q = [1,2,…,2]. By Assuming worst case i.e $\alpha_2 = 0$ in one period, the maximum periodic N is given by :

$$N \leq \frac{\log\left(\frac{\|A\|}{\|A_c\|}\right)}{\log\left(\sqrt{c}\|A\|\right)}$$

Performance - 2

Assume p=2 with loss prob. α_1, α_2

For scalar system, State convergence rate for periodic switching is better than for mean loss prob.

Proof:

For periodic switching, q = [1,…,2,…] , $n_1 = n_2 = n$,
Expectation of the norm of the state in one period:

$$\overline{E} = E\left[x_{2n}^T x_{2n} \middle| x_0\right] = x_0\left[(\alpha_1 a^2 + (1-\alpha_1)a_c^2)(\alpha_2 a^2 + (1-\alpha_2)a_c^2)\right]^n x_0$$

Next, for mean loss prob i.e $\alpha_1 = \alpha_2 = \frac{\alpha_1 + \alpha_2}{2}$, expectation of the norm in one period :

Performance - 2

$$\underline{E} = E\left[x_{2n}^T x_{2n} \middle| x_0\right] = x_0\left[\left(\frac{\alpha_1+\alpha_2}{2}\right)a^2 + (1-\left(\frac{\alpha_1+\alpha_2}{2}\right))a_c^2\right]^{2n} x_0$$

Assume n = 1,

$$\underline{E} - \overline{E} = x_0(\alpha_1-\alpha_2)^2\left[a^2-a_c^2\right]^2 x_0 > 0$$

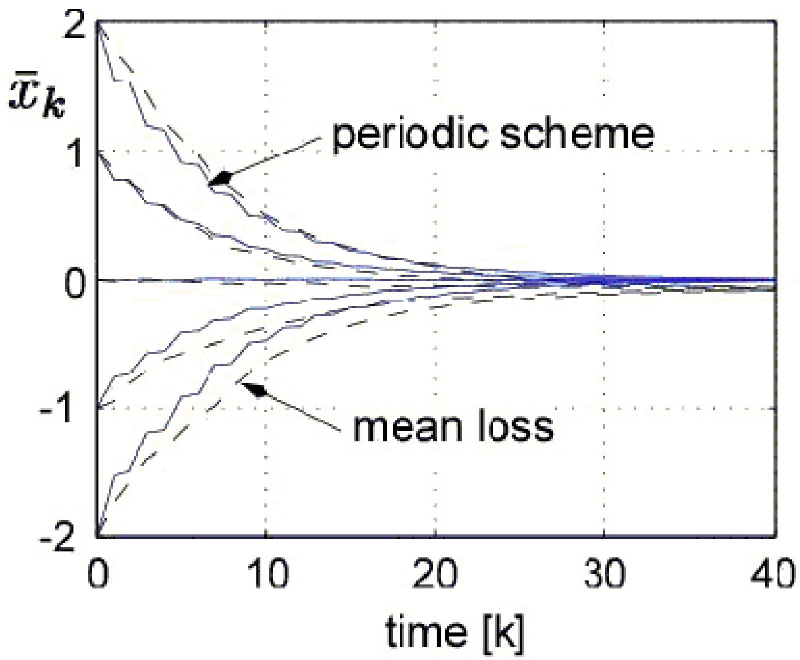
The above inequality is also satisfied for n > 1

Simulation

Multi-Agent System :

$$x_{k+1} = (I - \varepsilon L)x_k$$

- 5 Agents, fully connected undirected graph
- $\alpha_1 = 0.1, \alpha_2 = 0.9$, q = [1,2], simulation 1000 times



Conclusion & Future work

- Periodic switching scheme for QoS control is proposed. Stability condition is derived and performance of periodic switching is studied.
- Optimal switching scheme and proof for MAS are left as future work