Periodic Sequencing Control over Multi Communication Channels with Packet Losses

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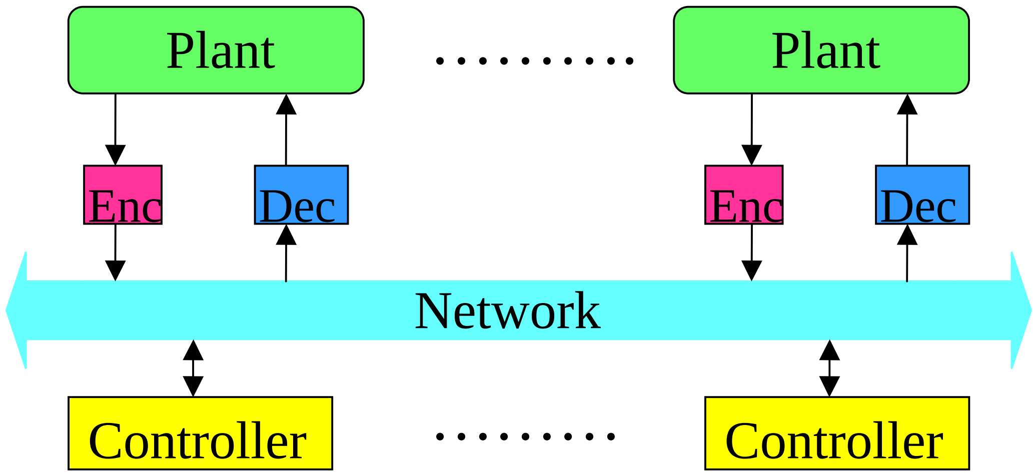
Outline

- Introduction
- Problem Formulation
- Periodic System
- Results
- Conclusion & Future Works

Introduction



Challenges :

- Band-limited channels
- Time delay
- Packet loss

QoS (Quality of Service) control:

-Wired networks ⇒use low packet loss channel is expensive in term of network costs

-Wireless networks ⇒ use high power is expensive in term of energy consumption

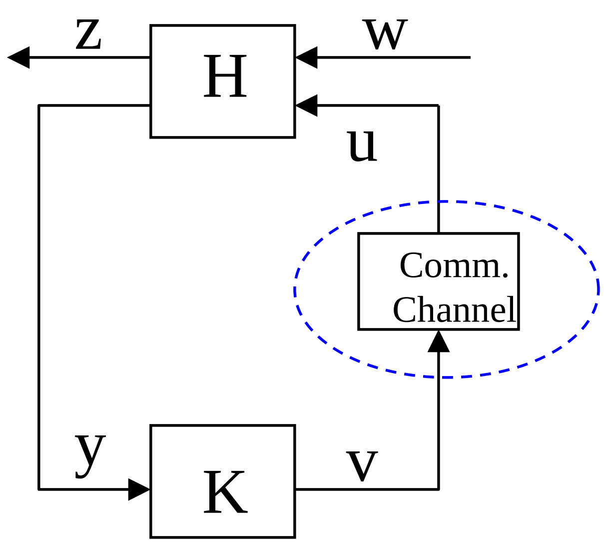
Introduction

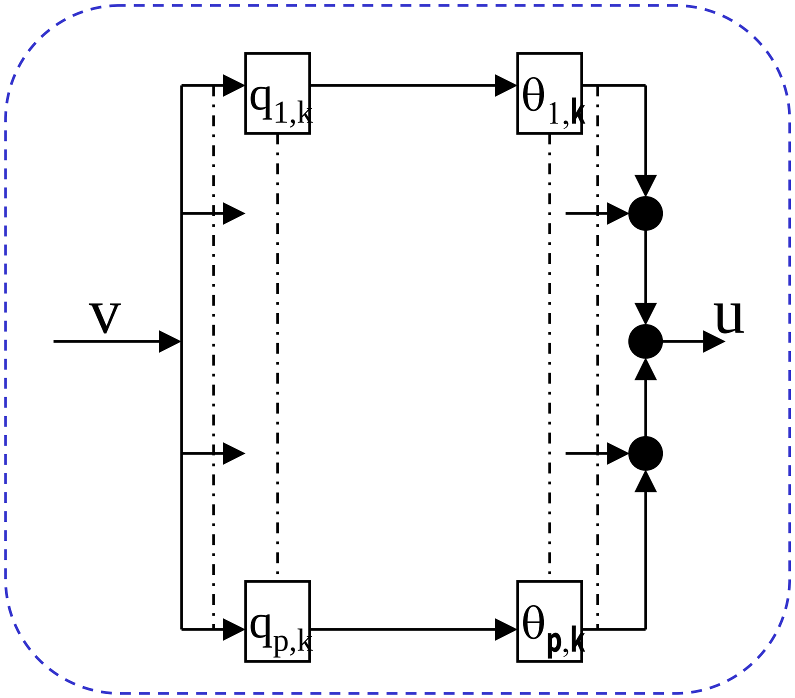
To Deal with this trade off :

- Introduce network model with many channels and different packet loss probabilities
- Introduce a periodic sequencing control scheme.

- Designed offline $\Rightarrow$ simplicity of the design
- Can be easily implemented

Problem Formulation





System (H_0)

$$x_{k+1} = Ax_k + B_1 w_k + B_2 Q_k \Theta_k v_k$$

$$y_k = C_2 x_k + D_{21} w_k$$

$Q_k := q_i$

$q_i := [0 \ldots 010 \ldots 0] \in \Re^{1\times p}$ ⇨ Switching pattern

Only one channel can be used at one time

$$\Theta_k = \begin{bmatrix} \theta_{1,k} \\ \vdots \\ \theta_{p,k} \end{bmatrix}$$ ⇨ Packet loss indicator

$$\alpha_i := prob\{\theta_{i,k} = 0\}$$ ⇨ Packet loss probability

Problem Formulation

- Assume that there is *p* channels with packet loss probabilities $\alpha_i, i \in \{1,2,\ldots,p\}$
- Assume the periodic sequencing $q = \{1\ldots1\ 2\ldots.\ p\ldots p\}$, each channel is used n_i times in one period.
- Goal : Necessary condition on α_i for the stabilization of the plant. (note: apply state-feedback controller)

For system $x_{k+1} = A_{k,\theta(k)} x_k$ which is N-periodic, the origin is said to be :

- Mean-square stable if for every initial state (x_0, θ_0),

$$\lim_{k\to\infty} E\left[\|x_k\|^2 \middle| x_0, \theta_0\right] = 0$$

- Stochastically stable if for every initial state (x_0, θ_0),

$$\sum_{k=0}^{\infty} E\left[\|x_k\|^2 \middle| x_0, \theta_0\right] < \infty$$

Periodic System

Periodic System : $x_{k+1} = A_k x_k$

Where $A(\cdot)$ is a periodic matrix of period N i.e $A_{k+N} = A_k$

Lemma 1 : The following statements are equivalent each other.

- $A(\cdot)$ is stable.
- There exists a N-periodic positive definite solution of the Lyapunov inequality $P(\cdot)$

$$A_k^T P_{k+1} A_k - P_k < 0$$

S. Bittanti and P. Colaneri, IFAC Periodic control systems, 2001

Results

Proposition 1 : For the system H_0, the origin is mean square stable (stochastically stable) iff there exists an N-periodic matrix $P_l \in \Re^{n \times n}$ such that $P_l = P_l^T > 0$ and

$$\sum_{i=0}^{1} \alpha_{i,l} A_{i,l}^T P_{l+1} A_{i,l} - P_l < 0 \text{ for } l \in I_N \qquad (1)$$

(Proof)

Define the Lyapunov function as : $V_k = x_k^T P_k x_k$

Where P is time varying and $P_k^T = P_k$

The system is stochastically stable iff $E\left[\Delta V_k \middle| x_k, \theta_k\right] < 0$

If $\sum_{i=0}^{1} \alpha_{i,l} A_{i,l}^T P_{l+1} A_{i,l} - P_l < 0$ for $l \in I_N$ holds for l, then it also holds for $l+N$

Proof

System H_0 can be written as :

$$x_{k+1} = Ax_k + B_1 w_k + B_2 Q_k \Theta_k K x_k \quad (C_2 = I, D_{21} = 0)$$
$$= \left(A + B_2 Q_k \Theta_k K\right) x_k + B_1 w_k$$

Applying lemma 1 and compute the expected value of the difference :

$$E\left[\Delta V_k\right] = E\left[V_{k+1} \middle| x_k, \theta_{k-1}\right] - V_k$$
$$= E\left[x_{k+1}^T P_{k+1} x_{k+1} \middle| x_k, \theta_{k-1}\right] - x_k^T P_k x_k$$
$$= E\left[x_k^T A_{i,k}^T P_{k+1} A_{i,k} x_k \middle| x_k, \theta_{k-1}\right] - x_k^T P_k x_k$$
$$= x_k^T E\left[A_{i,k}^T P_{k+1} A_{i,k} \middle| x_k, \theta_{k-1}\right] x_k - x_k^T P_k x_k$$
$$= x_k^T \left(\alpha_k A_{0,k}^T P_{k+1} A_{0,k} + (1 - \alpha_k) A_{1,k}^T P_{k+1} A_{1,k}\right) x_k - x_k^T P_k x_k$$
$$= x_k^T \left(\sum_{i=0}^{1} \alpha_{i,k} A_{i,k}^T P_{k+1} A_{i,k} - P_k\right) x_k < 0$$

Results

Proposition 2 : The necessary condition on the packet loss probabilities α_i so that there exists a controller that stabilizes the plant is given by

$$\left(\prod_{i=1}^{p} \alpha_i^{\frac{n_i}{2}} \right) \max |\lambda(A)|^N < 1$$

Where $\lambda(\cdot)$ denotes an eigenvalue

(Proof)

By proposition 1, The system is stable iff there exists an N-periodic $P_k > 0$ such that N inequalities in (1) hold. If the uncontrolled system is stable, we also can guarantee that the controlled system is also stable.

Straightforward calculation leads to the inequality :

Proof

$$\alpha_1^{n_1}\alpha_2^{n_2}\ldots\alpha_p^{n_p}\left(A^T\right)^N P_0 A^N - P_0 < 0, P_0 > 0$$

The solution of P_0 exists iff $\alpha_1^{n_1}\alpha_2^{n_2}\ldots\alpha_p^{n_p} A^N$ is a stable matrix

i.e. $\left(\prod_{i=1}^{p}\alpha_i^{\frac{n_i}{2}}\right)\max\left|\lambda(A)\right|^N < 1$

Performance-Guaranteed Decay Rate

- Assume there are two channels : $\alpha_1 = 0$ and α_2
- Find the largest periodic N such that :

$$\sup_{\text{(for any packet loss realization)}} \left(\|x_k\|^2\right) \le c^{-k}\|x_0\|^2, c \ge 1$$

- The maximum periodic N is given by :

$$N \le \frac{\log\left(\dfrac{\|A\|}{\|A_c\|}\right)}{\log\left(\sqrt{c}\|A\|\right)}$$

Results for Scalar Case

- By Applying the same computation as in proposition 2, the necessary and sufficient condition for the system to be stable is given by

$$\prod_{i=1}^{p}\left(\alpha_i a^2+(1-\alpha_i)a_c^2\right)^{\frac{n_i}{2}}<1$$

- Assume that we have two channels $(\alpha_1=0 \text{ and } \alpha_2)$ which are used n_1 and n_2 times respectively. The relation between n_1 and n_2 is given by

$$n_1<-\frac{n_2}{2}\frac{\log\left(\alpha_2 a^2+(1-\alpha_2)a_c^2\right)}{\log(a_c)}$$

- Assume that we have two channels $(\alpha_1=0 \text{ and } \alpha_2)$ which are switched every time step. The necessary & sufficient condition for the stability is given by

$$\alpha_2<\frac{1}{(a^2-a_c^2)}\left(\frac{1}{a_c^2}-a_c^2\right)$$

Conclusions & Future Work

- The necessary condition for the stability of periodic sequencing system and also the performance measure are introduced.
- Next step, consider scheduling scheme and apply the model to multi-agent or sensor networks problems