Vision-based Motion Coordination: A Passivity Approach

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Vision-based Motion Coordination of birds

Introduction

- **Motion Coordination**
  - Each agent has no global knowledge of the network state and can only plan its motion by observing its closest neighbors.
- **Technological Motivation**

  For example
  - Robotic sensor network: Monitoring and surveillance for safety
- **Previous Work**
  - A. Jadbabaie, J. Lin and A. S. Morse, IEEE TAC, 48-6, 2003
  - J. A. Fax and R. M. Murray, IEEE TAC, 49-9, 2004
  - R. O. Saber and R. M. Murray, IIEEE TAC, 49-9, 2004

  keywords: consensus, flocking, coverage, rendezvous, etc.

Introduction

- **N. Chopra and M. Spong, SICE 2005, CDC 2006**
  - Passivity-based control
  - Output synchronization for nonlinear dynamics
- **N. Moshtagh and A. Jadbabaie**
  - Coordination in three dimensional (3D) space, CDC-ECC 2005
  - Vision-based (image processing) coordination, RSS 2005
- **In this talk**
  - Passivity-based control
  - 3D configuration space
  - Vision-based control

Vision-based Motion Coordination: A Passivity Approach

Problem Statement

- **Agent Model** $(i = 1, \cdots, n)$

$$\dot{p}_i = R_i v_i \qquad (1) \qquad p_i \in \mathcal{R}^3 \quad \text{position}$$

$$\dot{R}_i = R_i \hat{\omega}_i \qquad R_i \in SO(3) \quad \text{orientation}$$

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}^{\wedge} := \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \qquad v_i \in \mathcal{R}^3 \quad \text{body velocity} \qquad \omega_i \in \mathcal{R}^3 \quad \text{angular velocity}$$

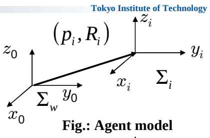


Fig.: Agent model

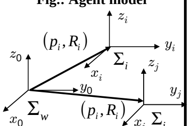


Fig.:Multi-Agent model

- **Information Network**

  Graph G : Graph consists of a pair $(V(G), E(G))$, where $V(G)$ is a finite nonempty set of nodes and $E(G) \subseteq V(G) \times V(G)$ is a set of pair of nodes, called edges.

  $G := (V, E)$: Graph

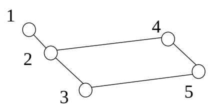
  $V := \{1, \cdots n\}$: A set of vertices indexed by a set of agents

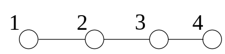
  $E \subseteq V \times V$: A set of edges to represent the neighboring relations

  Neighbors N_i : A set of agents whose information is available to the agent i

Graph Theory

- **Example**





$V(G) = \{1,2,3,4,5\}$

$E(G) = \{(1,2),(2,3),(2,4),(3,5),(4,5)\}$

$N_1 = \{2\}$

$N_2 = \{1,3,4\}$

$N_3 = \{2,5\}$

$N_4 = \{2,5\}$

$N_5 = \{3,4\}$

$V(G) = \{1,2,3,4\}$

$E(G) = \{(1,2),(2,3),(3,4),(4,5)\}$

$N_1 = \{2\}$

$N_2 = \{1,3\}$

$N_3 = \{2\}$

- **Assumptions** (A) (for simplicity)

  Graph is fixed, connective, undirected and tree.

Goal

- **Assumptions** (B) (for flocking)

  $|v_i| = 1 \quad \forall i$ each agent's speed is constant and normalized.

- **Goal** 3D Alignment

  $$R_i = R_j \quad \forall i, j \quad t \to \infty$$

  All agents attain the same orientation.

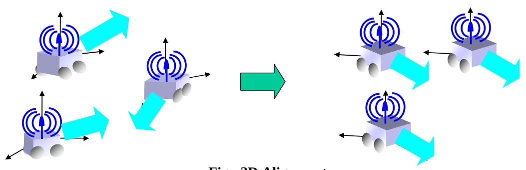


Fig.: 3D Alignment

Control Input

- **Control Input**

  $$\omega_i = \sum_{j \in N_i} e_R(R_{ij}) \qquad (2)$$

  $R_{ij} = R_i^T R_j$ Relative orientation

  $e_R(R) = \frac{1}{2}\left(R - R^T\right)^\vee$

  N_i : Agent i's neighbors

  $$\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}^\vee = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

- **Example** 2D

  $$R_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 \\ \sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

  $$R_{ij} = \begin{bmatrix} \cos(\theta_j - \theta_i) & -\sin(\theta_j - \theta_i) & 0 \\ \sin(\theta_j - \theta_i) & \cos(\theta_j - \theta_i) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

  $$\omega_i = e_R(R_{ij}) = \frac{1}{2}\left(R_{ij} - R_{ij}^T\right)^\vee = \begin{bmatrix} 0 \\ 0 \\ -\sin(\theta_i - \theta_j) \end{bmatrix}$$

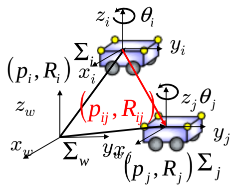


Fig.: 2D Case

Example

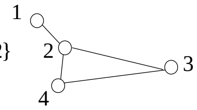
- **Example** 2D

  $$\omega_1 = \sum_{j \in N_1} e_R(R_{1j}) = e_R(R_{12}) = \begin{bmatrix} 0 \\ 0 \\ -\sin(\theta_1 - \theta_2) \end{bmatrix} \qquad N_1 = \{2\}$$

  $$\omega_2 = \sum_{j \in N_2} e_R(R_{2j}) = e_R(R_{21}) + e_R(R_{23}) + e_R(R_{24}) = \begin{bmatrix} 0 \\ 0 \\ -\sin(\theta_2 - \theta_1) - \sin(\theta_2 - \theta_3) - \sin(\theta_2 - \theta_4) \end{bmatrix} \qquad N_2 = \{1,3,4\}$$

  $$\omega_3 = \sum_{j \in N_3} e_R(R_{3j}) = e_R(R_{32}) + e_R(R_{34}) = \begin{bmatrix} 0 \\ 0 \\ -\sin(\theta_3 - \theta_2) - \sin(\theta_3 - \theta_4) \end{bmatrix} \qquad N_3 = \{2,4\}$$

  $$\omega_4 = \sum_{j \in N_4} e_R(R_{4j}) = e_R(R_{42}) + e_R(R_{43}) = \begin{bmatrix} 0 \\ 0 \\ -\sin(\theta_4 - \theta_2) - \sin(\theta_4 - \theta_3) \end{bmatrix} \qquad N_4 = \{2,3\}$$



Graph Theory

- **Graph Theory**
  - adjacency matrix
  - degree matrix
  - graph Laplacian
  - incidence matrix
  - adjacency matrix : A

    $$A := a_{ij} \begin{cases} 1: & \text{node } i \text{ and node } j \text{ are neighbors} \\ 0: & \text{otherwise} \end{cases}$$

  - degree matrix : D

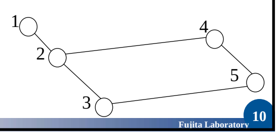
    $$D := \begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_n \end{bmatrix}$$

    d_i : degree of node i

    degree : the number of neighbors

- **Example**

  $$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$



Graph Theory

- **Graph Theory**
  - adjacency matrix
  - degree matrix
  - graph Laplacian
  - incidence matrix
  - graph Laplacian

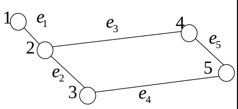
    $L := D - A$ D :degree matrix A :adjacency matrix

  - incidence matrix

    $$B := b_{ij} \begin{cases} 1: & e_j = (i,k) \\ -1: & e_j = (k,i) \\ 0: & \text{otherwise} \end{cases}$$

- **Example**

  $$L = D - A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & -1 & 0 \\ 0 & -1 & 2 & 0 & -1 \\ 0 & -1 & 0 & 2 & -1 \\ 0 & 0 & -1 & -1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$



$e_1 = (1,2)$ $e_3 = (2,4)$ $e_5 = (4,5)$

$e_2 = (2,3)$ $e_4 = (3,5)$

Theorem

Theorem 1 Consider the n agents given by (1). Under the assumptions (A) and (B), the control input (2) achieves in 3D Alignment asymptotically, namely, $R_i = R_j$, $\forall i, j$, $t \to \infty$.

(Proof)

Each agent model (1) is passive with the storage function (3).

$$E_i := \frac{1}{2}\|p_i\|^2 + \frac{1}{2}\mathrm{tr}(I_3 - R_i) \qquad (3)$$

position term   orientation term

So, relative rigid body motion between agents is passive with the storage function (4).

$$E_{ij} := \frac{1}{2}\|p_{ij}\|^2 + \frac{1}{2}\mathrm{tr}(I_3 - R_{ij}) \qquad (4) \qquad p_{ij} := R_i^T(p_i - p_j)$$

relative position term   relative orientation term

Proof

The relative orientation term of (4)

$$\phi(R_{ij}) = \frac{1}{2}\mathrm{tr}(I_3 - R_{ij}) \qquad (5)$$

The derivative of this function (5) along trajectories of the system is given as

$$\mathrm{sk}(R) := \frac{1}{2}(R - R^T)$$

$$\mathrm{sym}(R) := \frac{1}{2}(R + R^T)$$

$$e_R(R) = \mathrm{sk}(R)^{\vee}$$

$$\begin{aligned}\dot{\phi}(R_{ij}) &= -\frac{1}{2}\mathrm{tr}(\dot{R}_{ij}) \\ &= -\frac{1}{2}\mathrm{tr}(\dot{R}_{ij}R_{ij}^T R_{ij}) \\ &= -\frac{1}{2}\mathrm{tr}\left(\dot{R}_{ij}R_{ij}^T sk(R_{ij})\right) - \frac{1}{2}\mathrm{tr}\left(\dot{R}_{ij}R_{ij}^T sym(R_{ij})\right) \\ &= -\frac{1}{2}\mathrm{tr}(\dot{R}_{ij}R_{ij}^T sk(R_{ij})) \\ &= e_R^T(R_{ij})\omega_{ij} \\ &= -e_R^T(R_{ij})R_{ij}^T\omega_i - e_R^T(R_{ji})\omega_j \\ &= -e_R^T(R_{ij})\omega_i - e_R^T(R_{ji})\omega_j\end{aligned}$$

$$\left(\begin{aligned}\because\ &\mathrm{tr}\left(\dot{R}_{ij}R_{ij}^T sym(R_{ij})\right) \\ &= \mathrm{tr}\left(\dot{R}_{ij}R_{ij}^T sym(R_{ij})\right)^T \\ &= \mathrm{tr}\left(sym(R_{ij})^T(\dot{R}_{ij}R_{ij}^T)^T\right) \\ &= \mathrm{tr}\left(sym(R_{ij})(-\dot{R}_{ij}R_{ij}^T)\right) \\ &= -\mathrm{tr}\left(\dot{R}_{ij}R_{ij}sym(R_{ij}^T)\right) \\ &= 0\end{aligned}\right)$$

$$\left(\because\ \frac{1}{2}tr(\hat{a}\hat{b}) = -a^T b\right)$$

$$\left(\because\ \omega_{ij} = -R_{ij}^T\omega_i + \omega_j\right)$$

$$\left(\because\ R_{ij}e_R(R_{ij}) = e_R(R_{ij})\right)$$

Proof

Define the potential function as the sum of the relative orientation term of (4).

$$V = \frac{1}{2}\sum_{i=1}^{n}\sum_{j\in N_i}\mathrm{tr}(I_3 - R_{ij}) = \frac{1}{2}\mathrm{tr}\left(R^T(L\otimes I_3)R\right) \quad (6) \qquad R := \begin{bmatrix}R_1^T & \cdots & R_n^T\end{bmatrix}^T$$

I_3 : Identity Matrix

So, the derivative of the potential function (6) is given as

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{n}\sum_{j\in N_i}\left(-e_R^T(R_{ij})\omega_i - e_R^T(R_{ji})\omega_j\right) \\ &= \sum_{i=1}^{n}\sum_{j\in N_i}\left(-e_R^T(R_{ij})\omega_i + e_R^T(R_{ij})\omega_j\right) \quad (7) \qquad \left(\because -e_R^T(R_{ji}) = e_R^T(R_{ij})\right)\end{aligned}$$

Since the graph is undirected, the presence of the term $-e_R^T(R_{ij})\omega_i + e_R^T(R_{ij})\omega_j$ in (7) implies that the term $-e_R^T(R_{ji})\omega_j + e_R^T(R_{ji})\omega_i$ also exists in (7).

Further $-e_R^T(R_{ji})\omega_j + e_R^T(R_{ji})\omega_i = e_R^T(R_{ij})\omega_j - e_R^T(R_{ij})\omega_i$ $\left(\because -e_R^T(R_{ji}) = e_R^T(R_{ij})\right)$

Consequently, the equation (7) can be written

$$\begin{aligned}&= 2\sum_{(i,j)\in E}\left(-e_R^T(R_{ij})\omega_i + e_R^T(R_{ij})\omega_j\right) \\ &= -2e_R^T(B\otimes I_3)^T\omega \qquad (8)\end{aligned}$$

$$\omega := \begin{bmatrix}\omega_1 \\ \vdots \\ \omega_n\end{bmatrix} \quad e_R := \begin{bmatrix}e_R(R_{ij}) \\ \vdots\end{bmatrix} \ \forall(i,j)\in E$$

B : incidence matrix

Proof

The control input (2) can be written as

$$\omega_i = \sum_{j\in N_i} e_R(R_{ij}) \ \Rightarrow\ \omega = (B\otimes I_3)e_R \qquad (9)$$

$$\omega := \begin{bmatrix}\omega_1 \\ \vdots \\ \omega_n\end{bmatrix} \qquad e_R := \begin{bmatrix}e_R(R_{ij}) \\ \vdots\end{bmatrix} \ \forall(i,j)\in E$$

B : incidence matrix

Substitute (9) to (8).

$$\dot{V} = -2e_R^T(B\otimes I_3)^T(B\otimes I_3)e_R \le 0$$

Using LaSalle's Invariance Principle

$$\dot{V} = 2e_R^T(B\otimes I_3)^T(B\otimes I_3)e_R = 0$$

$$(B\otimes I_3)e_R = 0 \qquad (10)$$

Since the graph is tree, rank $B = n-1$ and the number of the edges is $n-1$. These imply rank$(B\otimes I_3) = 3(n-1)$ and $e_R \in R^{3(n-1)\times 1}$.

Hence $\quad (B\otimes I_3)e_R = 0 \ \Rightarrow\ e_R = 0 \qquad (11)$

$e_R = 0$ means the orientation of every agent converge to that of it's neighbor. The connectivity then implies 3D Alignment.

Each agent converges to the same orientation

Q.E.D.

Vision-based Motion Coordination

- **Agent Model**

$$\dot{p}_i = R_i v_i$$

$$\dot{R}_i = R_i\hat{\omega}_i \qquad (i = 1,\cdots,n) \qquad (1)$$

- **Control Input**

$$\omega_i = \sum_{j\in N_i} e_R(R_{ij}) \qquad (2)$$

$R_{ij} := R_i^T R_j$ relative orientation

$$e_R(R) := \frac{1}{2}\left(R - R^T\right)^{\vee}$$

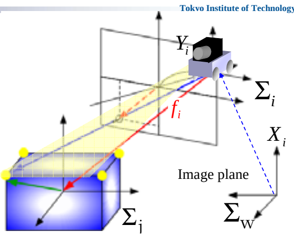


Fig : Pinhole Camera

We can't directly measure relative orientation because the only 2D information can directly be measured using a vision sensor. We consider a nonlinear observer to estimate the position and orientation of the other agents in 3D configuration space.

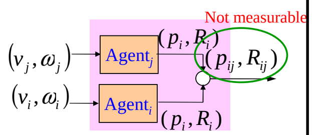


Fig : Block Diagram of relative rigid body motion

Camera Model and Image Information

- **Relative Feature Points**

$$p_{ci} := \begin{bmatrix}x_{ci} & y_{ci} & z_{ci}\end{bmatrix}^T$$

$$= p_{ij} + R_{ij}\,p_{oi} \qquad (12)$$

- **Perspective Projection**

$$f_i = \frac{\lambda}{z_{ci}}\begin{bmatrix}x_{ci} \\ y_{ci}\end{bmatrix} \qquad (13)$$

(f_i depends on p_{ci}.)

- **Image Information** (m points)

$$f = \begin{bmatrix}f_1 \\ \vdots \\ f_m\end{bmatrix} \qquad (14)$$

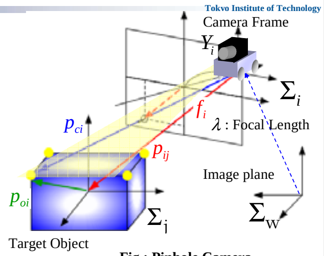


Fig : Pinhole Camera

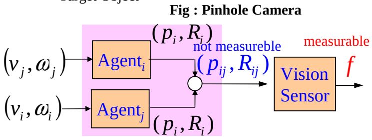


Fig : Block Diagram of RRBM with Camera

Image information f includes the relative rigid body motion (p_{ij}, R_{ij}).

Vision-based Motion Coordination

- **Visual Observer** (M. Fujita, H. Kawai, M. Spong, IEEE TCST, 2007)

$$\begin{bmatrix}\overline{R_{ij}^T\dot{p}_{ij}} \\ \overline{(R_{ij}^T\dot{R}_{ij})}^{\vee}\end{bmatrix} = -\begin{bmatrix}\bar{R}_{ij}^T & -\overline{R_{ij}^T\hat{p}_{ij}} \\ 0 & \bar{R}_{ij}^T\end{bmatrix}\begin{bmatrix}v_i \\ \omega_i\end{bmatrix} + KJ^{\dagger}(f - \bar{f}) \qquad (15)$$

$K > 0$: Observer Gain

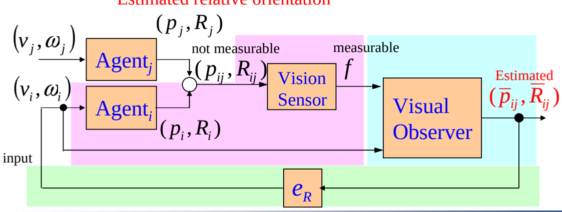
f : Image Information

J : Image Jacobian

- **Control Input**

$$\omega_i = \sum_{j\in N_i} e_R(\bar{R}_{ij}) \qquad (16)$$

Estimated relative orientation



Leader Less and Leader Following

- **Leader Less and Leader Following**

We consider the case that on additional agent, labeled 0, acts as leader. Agent 0 moves with the constant velocity $v = 1$ (same as others) and a fixed orientation R_0 .

Here we find a control law that results in a stable formation of the group while following the leader.

We consider the input of each agent in the leaderless case.

$$\omega_i = \sum_{j \in N_i} e_R(R_{ij}) \qquad (2)$$

We can separate the leader from other agents and write

$$\omega_i = \sum_{j \in N_i} e_R(R_{ij}) + c_i e_R(R_{i0}) \qquad (17)$$

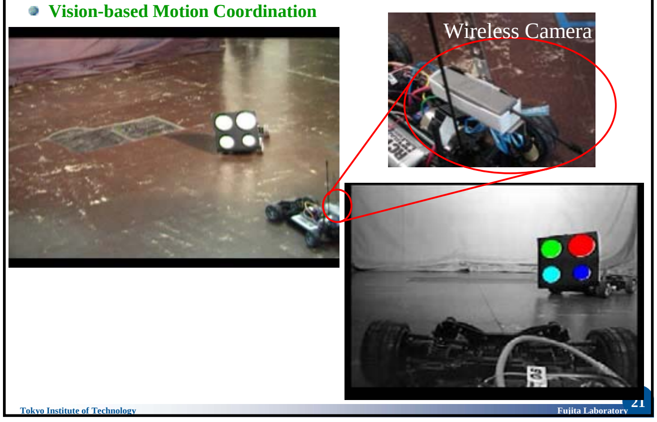
where $c_i = 1$ if agent i and the leader are neighbors and $c_i = 0$ otherwise.

In order to show that the error is asymptotically stable, diagonally dominant matrix is necessary.

Vision-based Motion Coordination of birds

Experiment

- **Vision-based Motion Coordination**



Thank you

Thank you

Appendix

Appendix

Nonlinear Observer

Estimated RRBM $(\bar{p}_{ij}, \bar{R}_{ij})$ **Estimated Body Velocity** $(\bar{v}_i, \bar{\omega}_i)$

- **Model of Estimated Relative Rigid Body Motion**

$$\begin{bmatrix} \bar{v}_i \\ \bar{\omega}_i \end{bmatrix} = \begin{bmatrix} \overline{R_{ij}^T \dot{p}_{ij}} \\ \overline{(R_{ij}^T \dot{R}_{ij})^\vee} \end{bmatrix} = -\begin{bmatrix} \bar{R}_{ij}^T & -\overline{R_{ij}^T \hat{p}_{ij}} \\ 0 & \bar{R}_{ij}^T \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} + u_e \quad (18)$$

(This model is reproduced just as Luenberger observer for linear systems.)

u_e : Input for Estimation Error (It will be proposed afterwards.)

- **Estimated Image Information**

$$\bar{p}_{ci} = \bar{p}_{ij} + \bar{R}_{ij} p_{oi}, \quad \bar{f}_i := \frac{\lambda}{\bar{z}_{ci}} \begin{bmatrix} \bar{x}_{ci} \\ \bar{y}_{ci} \end{bmatrix} \quad (19)$$

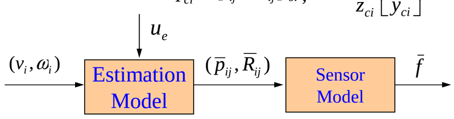


Fig. 8: Block Diagram of Estimated Relative Rigid Body Motion

Nonlinear Observer

- **Estimation Error (Error between Estimated State and Actual One)**

$$\begin{bmatrix} p_{ijee} \\ R_{ijee} \end{bmatrix} = \begin{bmatrix} \bar{R}_{ij}^T (p_{ij} - \bar{p}_{ij}) \\ \bar{R}_{ij}^T R_{ij} \end{bmatrix} \quad (20) \qquad e_e := \begin{bmatrix} p_{ijee} \\ e_R(R_{ijee}) \end{bmatrix} \quad (21)$$

(Vector Form)

Relation between Estimation Error and Image Information

$$f - \bar{f} = Je_e \quad (22)$$

J : Image Jacobian

Agent Model

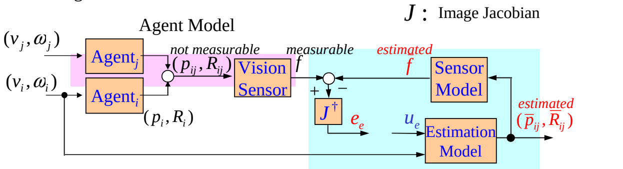


Fig.: Block Diagram of Agent Model and Visual Observer Visual Observer

- **Estimation Error System**

$$\begin{bmatrix} \overline{R_{ij}^T \dot{p}_{ij}} \\ (R_{ij}^T \dot{R}_{ij})^\vee \end{bmatrix} = -\begin{bmatrix} \bar{R}_{ij}^T & -\overline{R_{ij}^T \hat{p}_{ij}} \\ 0 & \bar{R}_{ij}^T \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} + u_e \;(18) \Leftrightarrow \begin{bmatrix} R_{ee}^T \dot{p}_{ee} \\ (R_{ee}^T \dot{R}_{ee})^\vee \end{bmatrix} = -\begin{bmatrix} R_{ee}^T & -R_{ee}^T \hat{p}_{ee} \\ 0 & R_{ee}^T \end{bmatrix} u_e + \begin{bmatrix} v_j \\ \omega_j \end{bmatrix} \;(23)$$

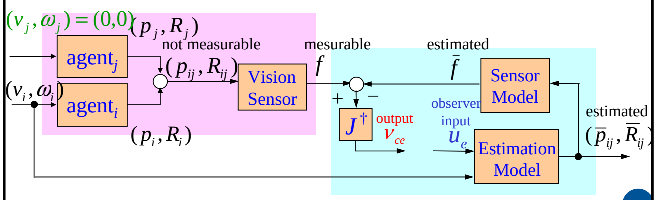
Property of Visual Observer

Lemma 1 **If the target is static $(v_j, \omega_j) = (0,0)$, then the visual observer (23) satisfies**

$$\int_0^T u_e^T \nu_{ce} \, d\tau \geq -\beta_e \quad (24)$$

where $\nu_{ce} := \begin{bmatrix} -p_{ee} \\ -e_R(R_{ee}) \end{bmatrix}$, β_{ce} **is a positive scalar.**

passivity



Property of Visual Observer

- **Storage Function** $V = \frac{1}{2} \| p_{ijee} \|^2 + \phi(R_{ijee}) \quad (25)$

$\left(\phi(R_{ijee}) := \frac{1}{2} \mathrm{tr}(I - R_{ijee}) \right.$ Error Function of Rotation Matrix $\left. \right)$

(Proof) Differentiating the storage function (25) with respect to time along the trajectories of the visual observer

$$\dot{V} = -\begin{bmatrix} p_{ijee} & e_R^T(R_{ijee}) \end{bmatrix}^T \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} u_e - p_{ijee}^T \hat{\omega}_e p_{ijee} \qquad u_e = \begin{bmatrix} v_e \\ \omega_e \end{bmatrix}$$

$= \nu_e^T$ skew-symmetric matrices

$$= u_e^T \nu_e$$

Integrating both sides from 0 to T, we can obtain

$$\int_0^T u_e^T \nu_e \, d\tau \geq V(T) - V(0) \geq -V(0) := -\beta_e \qquad \text{(Q.E.D.)}$$